Selected Chapters of Biomedical Instrumentation

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INTRODUCTION

Below are examples of electronic instrumentation. Recognizing technical issues related with each example, you will be able to find solutions while you study the book.

Example 0.0.1 *You measure a temperature of an object at 37 °C using a thermistor. The room temperature is 20 °C. The thermistor has a resistance* $R(T)$ *that changes with temperature as* $R(T) = 100e^{-\alpha(T-20)}\ \Omega$. *You attach the thermistor to the object and wait for 5 min so that the thermistor reaches the same temperature as the object. What temperature do you expect to get? Discuss the result.*

Example 0.0.2 *You are provided with a cylindrical container filled with 1% saline. You place a 1.5 V dc battery at its center. Current spreads throughout the container from the positive to negative terminals of the battery. You want to measure a voltage difference between two points inside the container. How can you sense the voltage difference? Discuss the result.*

Example 0.0.3 *You design a biopotential amplifier for ECG measurement. ECG signal has a dynamic range of* $\pm$*5 mV and bandwidth of 0.01 to 100 Hz when it appears between a pair of electrodes on the skin. The amplifier is placed between the electrode pairs and an ADC. The ADC has a dynamic range of* $\pm$*5 V. The electrode-skin interface is modeled as a dc contact potential of* $\pm$*300 mV and contact impedance of 50 k*Ω*. The third electrode on the right leg is connected to the isolated circuit ground. Design the amplifier. The noise level is 0.01 mV*$_{RTI}$*, discuss specifications of the ADC and the resolution of the measurement.*

Example 0.0.4 *You design a pressure measuring device. The pressure ranges from 0 to 500 mmHg. The sensor has a sensitivity of 0.01 mV/mmHg/V. You excite the sensor by dc 2 V for the sensitivity of 0.02 mV/mmHg. Using a differential voltage amplifier with a gain of 100, you get the output voltage in the range of 0 to 1 V. The required resolution is 1 mmHg. Discuss the minimum requirement on the analog-to-digital converter and noise level at the output of the voltage amplifier.*

Example 0.0.5 *You design a patient monitor for real-time monitoring of vital signs including ECG, blood pressure, temperature, respiration and pulse oximetry. Draw a block diagram and generate specifications.*

Example 0.0.6 *You design a wireless portable device that detects the heart rate. It acquires a single channel ECG signal*

1

Electromagnetism

1.1 Charged Particle and Charge Density

Charged particles may include electrons, holes, unbounded ions and molecules. We characterize a charged particle by its mass, charge, size and position. They are mobile and their movements at temperature greater than 0 K are random. Bounded ions or atoms and molecules may may have net charges. They are immobile but some of them can be rotated. Polar molecules do not have a net charge but form dipoles separating positive and negative charges in the space.

Inside a material, there could be N isolated charges Q_i for $i = 1, 2, \cdots, N$. When they are closely packed to form a cluster of charges, we may represent it as a volume charge density ρ:

$$\rho(\mathbf{r}) = \lim_{\Delta v \to 0} \frac{\sum_{i \in \Delta v(\mathbf{r})} Q_i}{\Delta v(\mathbf{r})} \tag{1.1}$$

where Q_i is the charge in C of a charged particle within the volume $\Delta v(\mathbf{r})$ centered at $\mathbf{r}$. We can find the total charge Q in a volume V as

$$Q = \int_V \rho(\mathbf{r}) d\mathbf{r}. \tag{1.2}$$

1.2 Electric Field and Flux

We consider an empty space $\mathcal{S}_0$. There is absolutely nothing in $\mathcal{S}_0$. We set the Cartesian coordinate and set a point in the space as the origin $\mathbf{O} = (0, 0, 0)$. We place a particle with an infinitesimally small mass but a finite electric charge Q at the origin. We denote the space with the point charge as $\mathcal{S}_Q$. Is there any difference between $\mathcal{S}_0$ and $\mathcal{S}_Q$ other than the existence of the point charge? What is the consequence of the existence of the point charge?

In the empty space $\mathcal{S}_0$, there is nothing influencing anything. In $\mathcal{S}_Q$, the point charge has its influence everywhere since another point charge Q_r at $\mathbf{r} = (x, y, z)$ feels the force

$$\mathbf{F} = \frac{1}{4\pi\epsilon} \frac{QQ_r}{|\mathbf{r}|^2} \mathbf{a}_r \tag{1.3}$$

where ϵ is a constant and $\mathbf{a}_r$ is the unit vector in the direction of $\mathbf{r}$. The force in (1.3) is the Coulomb force and it is an observed law of nature. The space $\mathcal{S}_Q$ is different from the empty

Selected Chapters of Biomedical Instrumentation Eung Je Woo

space $\mathcal{S}_0$ in the sense that there exists an electric field in $\mathcal{S}_Q$ as

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\frac{Q}{|\mathbf{r}|^2}\mathbf{a}_r \tag{1.4}$$

where $\mathbf{E}$ is the electric field intensity. The influence of the point charge at the origin gets smaller as we move away from it since the electric field intensity is inversely proportional to the square of the distance from the point charge.

We call the point charge the source charge and the position of the source charge is the source point. The point $\mathbf{r}$ where we evaluate the field strength is called the field point. Inside $\mathcal{S}_Q$ with the electric field $\mathbf{E}$, we feel a force $\mathbf{F}$ at the field point $\mathbf{r}$ as

$$\mathbf{F}(\mathbf{r}) = Q_r\mathbf{E}(\mathbf{r}) \tag{1.5}$$

for any second point charge Q_r at $\mathbf{r}$.

We consider a sphere with its center at the origin and radius of r. We evaluate the following integral:

$$\oint_S \mathbf{E}(\mathbf{r})\cdot\mathbf{ds_r} = \frac{1}{4\pi\epsilon}\frac{Q}{r^2}\times 4\pi r^2 = \frac{Q}{\epsilon} \tag{1.6}$$

where S is the closed surface of the sphere and $\mathbf{ds_r}$ is the outward normal surface element vector on S at $\mathbf{r}$. It means that the total sum all electric field strength $\mathbf{E}(\mathbf{r})$ acting on the surface of the sphere is Q/ϵ. This suggest the concept of the flux. The point charge at the origin radiates the electric flux omnidirectionally. The total sum of the electric flux must equal the source charge Q. We define the electric flux density $\mathbf{D}$ as

$$\mathbf{D}(\mathbf{r}) = \epsilon\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi}\frac{Q}{|\mathbf{r}|^2}\mathbf{a}_r. \tag{1.7}$$

This definition suits the concept of the flux and flux density since

$$\oint_S \mathbf{D}(\mathbf{r})\cdot\mathbf{ds} = Q. \tag{1.8}$$

We consider a space $\mathcal{S}$ with N point charges Q_i at $\mathbf{r}_i = (x_i, y_i, z_i)$. From these N source charges at N source points, we may compute N electric field intensities $\mathbf{E}_i$ at a field point $\mathbf{r}$. The total electric field intensity $\mathbf{E}$ at $\mathbf{r}$ is the sum of them as

$$\mathbf{E}(\mathbf{r}) = \sum_{i=1}^{N}\mathbf{E}_i(\mathbf{r}) = \frac{1}{4\pi\epsilon}\sum_{i=1}^{N}\frac{Q_i}{|\mathbf{r}-\mathbf{r}_i|^2}\mathbf{a}_{r_i,r} = \frac{1}{\epsilon}\mathbf{D}(\mathbf{r}) \tag{1.9}$$

where $\mathbf{a}_{r_i,r}$ is a unit vector in the direction from $\mathbf{r}_i$ to $\mathbf{r}$. For a volume charge distribution $\rho(\mathbf{r})$, we have the following expression:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon}\int_V \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}d\mathbf{r}' = \frac{1}{\epsilon}\mathbf{D}(\mathbf{r}) \tag{1.10}$$

where the total source charge is $Q = \int_V \rho(\mathbf{r})d\mathbf{r}$.

1.3 Potential and Voltage

We consider a domain $\mathcal{D}$ with the electric field intensity $\mathbf{E}(\mathbf{r})$. We take a point charge Q_r and put it at $\mathbf{r}_1$. We move it from $\mathbf{r}_1$ to $\mathbf{r}_2$ following a path $C_{\mathbf{r}_1 \to \mathbf{r}_2}$. Since there is a force acting on the charge Q_r, we do work W as

$$W = -\int_{C_{\infty \to \mathbf{r}_1}} Q_r \mathbf{E}(\mathbf{r}) \cdot \mathbf{dl_r} \tag{1.11}$$

where $\mathbf{dl_r}$ is a line element tangential to the curve $C_{\mathbf{r}_1 \to \mathbf{r}_2}$ at $\mathbf{r}$. We need the minus sign since we move the charge against the force. We define the potential difference V between $\mathbf{r}_2$ and $\mathbf{r}_1$ as the work per unit charge:

$$V_{\mathbf{r}_2,\mathbf{r}_1} = \frac{W}{Q_r} = -\int_{C_{\mathbf{r}_1 \to \mathbf{r}_2}} \mathbf{E}(\mathbf{r}) \cdot \mathbf{dl_r}. \tag{1.12}$$

We let $\mathbf{r}_1 \to \infty$ and set $\mathbf{r} = \mathbf{r}_2$ as an arbitrary point in the domain. The potential V at $\mathbf{r}$ with respect to the reference point at ∞ is

$$V(\mathbf{r}_1) = \frac{W}{Q_r} = -\int_{C_{\infty \to \mathbf{r}_1}} \mathbf{E}(\mathbf{r}) \cdot \mathbf{dl_r}. \tag{1.13}$$

We may draw a line or surface where the potential $V(\mathbf{r})$ is the same. They are an equipotential line or equipotential surface.

In the domain $\mathcal{D}$, we consider many equipotential lines. At a point $\mathbf{r}$ on an equipotential line, we evaluate the potential $V(\mathbf{r})$. We move from $\mathbf{r}$ to $\mathbf{r} + \mathbf{dr}$ and evaluate the potential $V(\mathbf{r} + \mathbf{dr})$ at $\mathbf{r} + \mathbf{dr}$. There are infinite many directions we can take to move to $\mathbf{r} + \mathbf{dr}$. We define the gradient of ∇V as

$$\nabla V = \frac{dV}{\mathbf{dr}}|_{max} = \frac{dV}{dx}\mathbf{a}_x + \frac{dV}{dy}\mathbf{a}_y + \frac{dV}{dz}\mathbf{a}_z. \tag{1.14}$$

From the vector calculus, we have

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r}). \tag{1.15}$$

1.4 Current and Its Continuity

1.5 Magnetic Field and Flux

1.6 Static Electromagnetic Field

1.7 Time-varying Electromagnetic Field

1.8 Power and Energy

1.9 Electromagnetic Spectrum

We summarize the electromagnetic spectrum from 20 Hz to 10^{19} Hz or higher (γ-ray). For the definition of the sinusoidal frequency, see chapter 4. When we consider an electromagnetic wave which propagates in the space, we have the following relation:

$$v = \lambda f \tag{1.16}$$

Table 1.1 Electromagnetic spectrum

Wave	Band	Frequency or Energy	Wavelength
Audio frequency		20–20,000 Hz	10^7 m at 30 Hz
Radio frequency	Low frequency	30–300 kHz	1–10 km
	Medium frequency	0.3–3 MHz	0.1–1 km
	High frequency	3–30 MHz	10–100 m
	Very high frequency (VHF)	30–300 MHz	1–10 m
	Ultra high frequency (UHF)	0.3–3 GHz	0.1–1 m
Microwave	Centimeter wave	3–30 GHz	1–10 cm
	Millimeter wave	30–300 GHz	1–10 mm
	Submillimeter wave	0.3–3 THz	0.1–1 mm
Infrared	Far infrared	3–30 THz	10–100 μm
	Intermediate infrared	30–300 THz	1–10 μm
Visible light	Red light		
	Orange light		
	Yellow light		
	Green light		
	Blue light		
	Cyan light		
	Violet light		
Ultra violet			$10^{-8} - -10^{-7}$ m
X-ray			$10^{-11} - -10^{-9}$ m
γ-ray			$10^{-16} - -10^{-11}$ m

where v is the wave speed in m/s, λ is the wavelength in m and f is the frequency in Hz. For electromagnetic waves, the velocity is almost 3^8 m/s, which is the speed of light in the vacuum.

2

Discrete Passive Device

2.1 Conductor and Conductivity

We assume an electrically conducting material occupying a domain $\mathcal{D}$. Inside the material, there exist mobile charges with a volume charge distribution $\rho(\mathbf{r})$ C/m^3. Mobile charges could be electrons, holes, positive ions (cations) and negative ions (anions). We consider a mobile particle with its charge q and mass m at $\mathbf{r}$. We assume that there is a constant external electric field $\mathbf{E}(\mathbf{r}) = E\mathbf{a}_x$ applied to the domain. The particle feels the force

$$\mathbf{F}(\mathbf{r}) = qE\mathbf{a}_x = m\mathbf{a} \tag{2.1}$$

where $\mathbf{a}$ is the acceleration. This means that the particle begins to move toward the x-direction with an acceleration of $\mathbf{a} = (qE/m)\mathbf{a}_x$. The particle is accelerated until it hits an immobile element inside the material. Loosing its kinetic energy during the collision, it restart the same motion until the next collision. The particle reaches an average velocity $\mathbf{v}_d$ called the drift velocity as

$$\mathbf{v}_d = \mu\mathbf{E} = \mu E\mathbf{a}_x \tag{2.2}$$

where μ is the mobility. The mobility is a material property determined by its composition and structure and changes with temperature.

We assume a across-sectional area perpendicular to the x-axis with its area A. The total amount of charge passing through the unit are between time t and $t + dt$ is

$$dQ(\mathbf{r}) = \rho(\mathbf{r})A\mathbf{a}_x\mathbf{v}_d dt = \rho(\mathbf{r})A\mu E dt. \tag{2.3}$$

The current flowing through the area is

$$I(\mathbf{r}) = \frac{dQ(\mathbf{r})}{dt} = \rho(\mathbf{r})\mu EA. \tag{2.4}$$

The current density is

$$\mathbf{J}(\mathbf{r}) = \frac{I(\mathbf{r})}{A}\mathbf{a}_x = \rho(\mathbf{r})\mu E\mathbf{a}_x = \rho(\mathbf{r})\mu\mathbf{E}. \tag{2.5}$$

In general,

$$\mathbf{J}(\mathbf{r}) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r}) \tag{2.6}$$

where σ is the conductivity. Note that the conductivity is determined by the charge density and the mobility.

Selected Chapters of Biomedical Instrumentation Eung Je Woo

2.2 Resistor

We consider a homogeneous cylindrical object with its conductivity σ, length L and across-sectional area A. With a constant external electric field $\mathbf{E} = E\mathbf{a}_x$. The current density on a cross-section perpendicular to the x-axis is

$$\mathbf{J} = J\mathbf{a}_x = \sigma E \mathbf{a}_x. \tag{2.7}$$

The total current is

$$I = JA = \sigma EA. \tag{2.8}$$

From $\mathbf{E} = -\nabla V$, the voltage difference between the left and right surfaces is $V = EL$ and $E = V/L$. We obtain the relation between the voltage and current as

$$V = \frac{1}{\sigma}\frac{L}{A}I = RI \tag{2.9}$$

where the resistance R in ohm (Ω) is

$$R = \frac{1}{\sigma}\frac{L}{A} = \frac{V}{I}. \tag{2.10}$$

A resistor is a linear passive device satisfying the Ohm's law in (2.10). Figure **??** shows its circuit symbol.

Figure **??** shows resistors used in electrical circuits. A resistor is specified by its resistance in Ω, the tolerance in %, power in W and type. For example, a 1% metal-film resistor has its resistance of 1 kΩ and power of 0.25 W. The actual resistance is between 990 Ω and 1010 Ω. Within $P = VI \leq 0.25$ W, this resistor follows Ohm's law in (2.10).

Injecting a known dc current I and measuring the induced dc voltage, we may find the resistance R as is done in an electrical multimeter. If we have geometrical information of L and A, we can find the conductivity σ. For some material like biological tissues, we denote the conductivity as σ_ω to emphasize its frequency dependence. We may measure σ_ω by injecting a sinusoidal current $i(t) = I\cos\omega t$ to measure the induced ac voltage $v(t) = V\cos\omega\, t$ where t is the time in second (s). Assuming a linear component, the resistance R at ω also follows Ohm's law as

$$v(t) = Ri(t) = RI\cos\omega t = \frac{1}{\sigma_\omega}\frac{L}{A}I\cos\omega t. \tag{2.11}$$

Note that the current and voltage are in phase. Repeating this measurement for multiple frequencies, we may get a conductivity spectrum which plots conductivity σ_ω as a function of frequency ω.

2.3 Dielectric and Permittivity

2.4 Capacitor

We consider a dielectric sandwiched by two parallel metal plates. When we apply a dc voltage V between the plates, it induces an electric field inside the dielectric. The dielectric contains immobile charges and their polarization or rotations in the electric field produces surface charges Q and $-Q$ in coulomb (C). The induced charge is proportional to the applied voltage as

$$Q = CV \tag{2.12}$$

where the proportionality constant C is called the capacitance in C/V or farad (F). The capacitance C between two plates is given by

$$C = \epsilon \frac{A}{d} \tag{2.13}$$

where ϵ is the permittivity in F/m, A the surface area and d the gap between the plates. The permittivity is a material property determined by the polarization of the dielectric under an external electric field. For most dielectrics including biological tissues, the permittivity changes with frequency and we denote it as ϵ_ω.

If we assume a perfect dielectric, there is no mobile charge and its conductivity σ is 0 S/m. Injecting dc current I through the dielectric, we get 0 dc voltage across it. If we apply a sinusoidal voltage $v(t) = V \sin \omega t$, there occurs an ac displacement current through the dielectric due to time-varying polarizations with the frequency ω:

$$i(t) = C\frac{dv(t)}{dt} = \omega C V \cos \omega t = I \cos \omega t. \tag{2.14}$$

Note that the current and voltage are out of phase by 90° or the voltage is in the quadrature of the current. Assuming that there is no polarization initially, we can express the induced voltage $v(t)$ subject to an injection current $i(t)$ as

$$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau = \frac{I}{\omega C}\sin \omega t = \frac{Id}{\omega \epsilon_\omega A}\cos\left(\omega t - \frac{\pi}{2}\right). \tag{2.15}$$

With known ω and I, we may find the capacitance C in F which equals to A·s/V or s/Ω. If we have geometrical information of A and d, we can find the permittivity ϵ_ω in F/m. Repeating this measurement for multiple frequencies, we may get a permittivity spectrum which plots permittivity ϵ_ω as a function of frequency ω.

3

Signal Source and Power Supply

3.1 Circuit Ground and Earth Ground

Since voltage is a potential difference between two points, we must specify a voltage at a point with respect to a reference point. The choice is arbitrary and depends on a given situation. For example, we measure voltages inside one battery-powered mobile device with respect to a chosen reference point in it. Inside another mobile device, we measure voltages with respect to a reference point in this second device. It is meaningless to mention a voltage difference between a point in one device and a point in another in figure 3.1(a). If we connect the two separate reference points in two devices together, then the voltage difference across the devices makes sense as in figure 3.1(b). With the reference points tied together, all voltages in any phone with respect to its reference point do not change.

Considering its existence everywhere on the earth, the earth ground is the ideal choice as a common reference to denote a voltage difference everywhere. The earth ground is the universal voltage reference point while a circuit ground is a local voltage reference point within a circuit or device. When a local circuit ground is not connected to the earth ground, we say that the circuit or device is floating or isolated from the earth as in figure 3.1(a) and (b). When the local reference point is connected to the earth ground, it is earth grounded and may share the earth ground with other devices in figure 3.1(c). For multiple earth grounded devices, we can compare voltages in different devices.

Selected Chapters of Biomedical Instrumentation Eung Je Woo

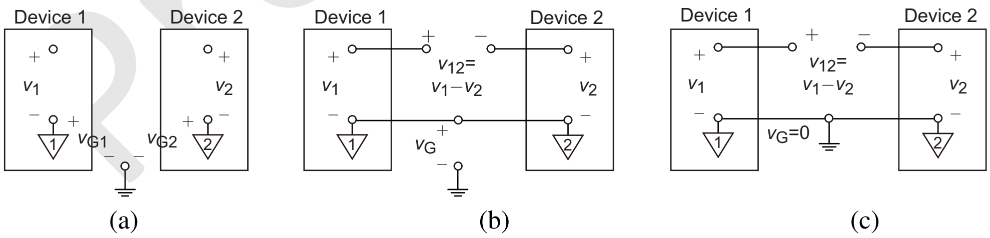

Figure 3.1 (a) Two floating mobile devices with separate circuit grounds. (b) Two floating mobile devices with one common circuit ground. (c) Two mobile devices with the common earth ground.

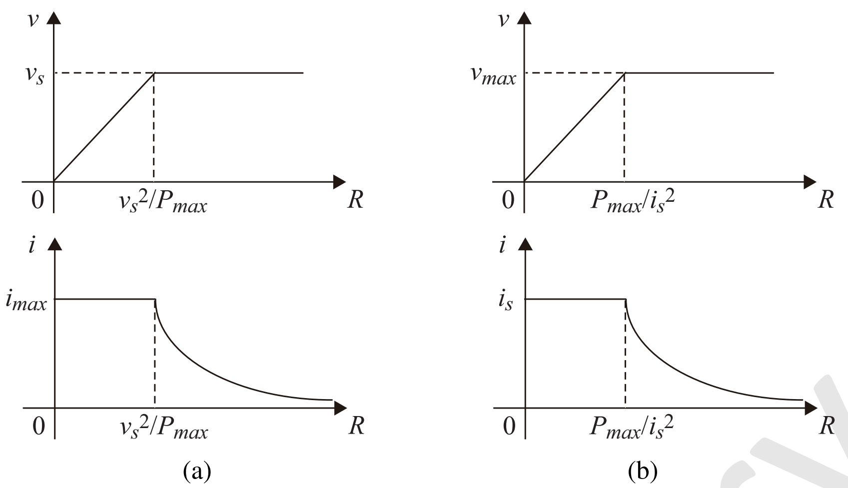

Figure 3.2 Voltage and current as functions of a resistance R: (a) voltage source and (b) current source.

3.2 Constant Voltage Source

A constant voltage source v_s has two terminals. The voltage v between the two terminals equals the source voltage v_s when nothing is connected to them. This is called the open-circuit voltage v_{open}. For an ideal voltage source v_{ideal}, the voltage across the terminals is always $v = v_{ideal} = v_{open}$. The exception is the case when we short circuit the terminals where $v = 0$.

We consider a practical voltage source v_s with a maximum power of P_{max}. When we connect a resistor R, there flows a current $i = v_s/R$ through R as long as $R \geq v_s^2/P_{max}$. When $R < v_s^2/P_{max}$, the current is fixed as $i_{max} = P_{max}/v_s$ and the voltage across the terminal is $v = Ri_{max} = RP_{max}/v_s$, which decreases linearly as R decreases.

For $R \geq v_s^2/P_{max}$, the voltage source supplies current $i \leq i_{max}$ so that $Ri = v_s$. The load is the amount of current $i = v_s/R$ so that $v = v_s$. As R is reduced, the load gets bigger. When $R = v_s^2/P_{max}$, the voltage source sees the biggest load of $i = i_{max} = P_{max}/v_s$. When $R < v_s^2/P_{max}$, the voltage source is seeing a load that is bigger than the largest load it can support, that is, it is overloaded. When overloaded, the voltage source outputs the maximum current it can provide regardless of the resistance R. For the overloaded case, the terminal voltage decreases linearly with the decrease in R since $v = Ri_{max}$ for a fixed i_{max}. The voltage source is not anymore a voltage source. Instead, it behaves like a current source.

A more practical constant voltage source has a source impedance or resistance. We will study loading effects related with the source resistance in chapter 7, 9 and 20.

Exercise 3.2.1 *Explain plots of v and i versus R in figure 3.2(a), which illustrates behavior of a constant voltage source.*

3.3 Constant Current Source

A constant current source i_s has two terminals and a load between them. The current i through the load equals the source current i_s when the terminals are shorted. This is called the short-circuit current i_{short}. For an ideal current source i_{ideal}, the current through the terminals is always $i = i_{ideal} = i_{short}$ regardless of the inserted component. The exception is the case when we open circuit the terminals where $i = 0$.

We consider a practical current source i_s with a maximum power of P_{max}. When we connect a resistor R, there occurs a voltage $v = Ri_s$ across R as long as $R \leq P_{max}/i_s^2$. When $R > P_{max}/i_s^2$, the voltage is fixed as $v_{max} = P_{max}/i_s$ and the current through the resistor is $i = v_{max}/R = P_{max}/(Ri_s)$, which decreases as R increases.

For $R \leq P_{max}/i_s^2$, the current source induces voltage $v \leq v_{max}$ so that $R/v = i_s$. The load is the amount of voltage $v = Ri_s$ so that $i = i_s$. As R is increased, the load gets bigger. When $R = P_{max}/i_s^2$, the current source sees the biggest load of $v = v_{max} = P_{max}/i_s$. When $R > P_{max}/i_s^2$, the current source is seeing a load that is bigger than the largest load it can support, that is, it is overloaded. When overloaded, the current source provides the maximum voltage it can induce regardless of the resistance R. For the overloaded case, the terminal current decreases with the increase in R since $i = v_{max}/R$ for a fixed v_{max}. The current source is not anymore a current source. Instead, it behaves like a voltage source.

A more practical constant current source has a source impedance or resistance. We will study loading effects related with the source resistance in chapter 7, 9 and 20.

Exercise 3.3.1 *Explain plots of v and i versus R in figure 3.2(b), which illustrates behavior of a constant current source.*

Exercise 3.3.2 *Discuss the duality between voltage and current sources.*

3.4 Ac Line Power

Electricity is delivered from a power plant to a user in ac 50 or 60 Hz to reduce loss during power transmission. The voltage ranges from 100 to 220 V. The voltage is referenced to the earth ground. Electrical power distribution involves construction of complicated network, which is one of the most important infrastructures of our society. Ac electricity is available from receptacles almost everywhere.

The ac voltage is denoted as

$$v(t) = V \cos \omega t \ \ \text{V} \tag{3.1}$$

where the amplitude A and frequency $f = \omega/2\pi$ may change within a given tolerance. There must be current

$$i(t) = I \cos(\omega t + \theta) \ \ \text{A} \tag{3.2}$$

with its amplitude I and phase angle θ flowing through a load such as a lamp. The delivered power in a form of light or heat, for example, is

$$p(t) = v(t)i(t) = VI \cos \omega t \cos(\omega t + \theta) = \frac{VI}{2} \{\cos \theta + \cos(2\omega t + \theta)\} \ \ \text{W}. \tag{3.3}$$

Exercise 3.4.1 *The current in (3.2) has a phase shift of θ with respect to the voltage in (3.1). Explain the reason.*

Exercise 3.4.2 *Compute the mean power as*

$$P_{mean} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt$$

where $T = 2\pi/\omega$ and t_0 is an arbitrary positive real number. Discuss changes in the delivered power as θ changes.

Exercise 3.4.3 *Define the real, reactive and apparent powers and power factor.*

3.5 Dc Power Supply

Most electrical devices use dc voltages and currents for their normal operations. A dc power supply convert ac line voltage into dc voltages. With respect to a circuit ground, positive and negative dc voltages in the range of 0 to 30 V are commonly used. We will study electric circuits for a dc power supply in chapter 10.

4

Analog Signal

4.1 Single-ended and Differential Signal

A voltage signal is single-ended when it is expressed with respect to a circuit ground. The reference point is often called the circuit ground or simply ground. We consider two single-ended signals with respect to the same circuit ground. The differential signal is the voltage difference between these two single-ended signals. In figure 3.1(b) and (c), v_1 and v_2 are single-ended signals whereas v_{12} is a differential signal.

We denote two single-ended signals by v_1 and v_2. The differential or differential-mode signal v_{dm} is

$$v_{dm} = v_2 - v_1 \tag{4.1}$$

and the common-mode signal v_{cm} is

$$v_{cm} = \frac{1}{2}(v_1 + v_2). \tag{4.2}$$

It is easy to show that

$$v_1 = v_{cm} - \frac{1}{2}v_{dm}, \tag{4.3}$$

$$v_2 = v_{cm} + \frac{1}{2}v_{dm}. \tag{4.4}$$

In most cases, it is the differential-mode signal v_{dm} that conveys signal information. The common-mode signal v_{cm} is often considered as a bias, baseline or background that should be rejected. When v_1 and v_2 share common noise or artifact, dealing with the differential-mode signal is advantageous.

4.2 Continuous-time Continuous-amplitude Signal

4.3 Discrete-time Continuous-amplitude Signal

4.4 Power and Energy

We consider an analog signal $s(t)$. Its instantaneous power $p(t)$ is

$$p(t) = s^2(t). \tag{4.5}$$

Selected Chapters of Biomedical Instrumentation Eung Je Woo

The mean power P_s is either

$$P_s = \frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt \tag{4.6}$$

or

$$P_s^{\infty} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s^2(t)dt. \tag{4.7}$$

The energy of the signal $s(t)$ is either

$$E_s = \int_{t_0}^{t_0+T} s^2(t)dt \tag{4.8}$$

or

$$E_s^{\infty} = \lim_{T\to\infty}\int_{-T/2}^{T/2} s^2(t)dt. \tag{4.9}$$

Exercise 4.4.1 *Suggest examples of a signal with a finite energy, that is,* $E_s^{\infty} < \infty$. *In this case,* $P_s^{\infty} = 0$.

Exercise 4.4.2 *Suggest examples of a signal with a finite power, that is,* $P_s^{\infty} < \infty$. *In this case,* $E_s^{\infty} = 0$.

4.5 Constant Signal

A constant or dc signal is

$$s(t) = c \tag{4.10}$$

where c is a constant.

4.6 Unit Step Signal

The unit step signal is denoted as $u(t)$:

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0. \end{cases} \tag{4.11}$$

Exercise 4.6.1 *Check if* $u(t)$ *is continuous and differentiable at* $t = 0$.

4.7 Unit Impulse Signal

The unit impulse is denoted as $\delta(t)$:

$$\delta(t) = \frac{du(t)}{dt} \quad \text{and} \quad \int_{-\infty}^{\infty}\delta(t)dt = 1. \tag{4.12}$$

Note that

$$u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau = \int_{0}^{\infty}\delta(t-\tau)d\tau.$$

Exercise 4.7.1 *Derive $\delta(t)$ and $u(t)$ using*

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 < t < \Delta \\ 0, & \textit{otherwise}. \end{cases} \tag{4.13}$$

Exercise 4.7.2 *Discuss the property of $s(t)\delta(t-t_0) = s(t_0)\delta(t-t_0)$.*

4.8 Exponential Signal

A real exponential signal is

$$s(t) = Ae^{\tau t} \tag{4.14}$$

where both A and τ are real. If they are complex,

$$s(t) = (A + \mathrm{i}B)e^{(\sigma+\mathrm{i}\omega)t} = Me^{\mathrm{i}\theta}e^{\sigma t}e^{\mathrm{i}\omega t} = Me^{\sigma t}\left\{\cos(\omega t+\theta) + \mathrm{i}\sin(\omega t+\theta)\right\} \tag{4.15}$$

where $M = \sqrt{A^2+B^2}$ and $\theta = \tan^{-1}\frac{B}{A}$. The complex exponential signal is

$$s(t) = e^{\mathrm{i}\omega t} = \cos\omega t + \mathrm{i}\sin\omega t. \tag{4.16}$$

Note that

$$e^{\mathrm{i}\omega(t+T)} = e^{\mathrm{i}\omega T}e^{\mathrm{i}\omega t} = e^{\mathrm{i}\omega t}$$

when we choose the fundamental period T as

$$T = \frac{2\pi}{|\omega|}.$$

The fundamental frequency f is

$$f = \frac{1}{T}.$$

4.9 Sinusoidal Signal

A sinusoid is expressed as

$$s(t) = A\cos(\omega t+\theta) \quad \text{or} \quad s(t) = A\sin(\omega t+\theta) \tag{4.17}$$

where A is the amplitude, ω is the angular frequency in radian per second (rad/s) with $\omega = 2\pi f$ where f is the frequency in Hertz (Hz or 1/s), and θ is the phase angle in radian (rad). Figure **??** plots two sinusoidal signals. The sinusoid is periodic with a period

$$T = \frac{1}{f} \quad \left(f = \frac{1}{T} \quad \text{or} \quad Tf = 1\right) \tag{4.18}$$

since for any integer n,

$$s(t+nT) = A\cos(\omega t + 2\pi n + \theta) = A\cos(\omega t+\theta) = s(t).$$

Note that the frequency denotes the number of sinusoidal cycles within 1 s and the period is the duration of one sinusoidal cycle.

Exercise 4.9.1 *Consider a circle with its center at the origin of the xy-plane. The radius is A. You start from the point $(A, 0)$ and run along the circumference at a constant angular velocity of ω rad/s. At time t, your position is denoted as $(x(t), y(t))$. Find expressions for $x(t)$ and $y(t)$ and plot them. Find an equation of the trajectory of motion and plot it.*

Exercise 4.9.2 *Repeat the exercise 4.9.1 with the following starting positions:*

1. $(0, A)$.
2. $(-A, 0)$.
3. $(0, -A)$
4. $(A\cos\theta, A\sin\theta)$.

We consider the xy-plane as the complex plane, which is often called the Argand plane or z-plane. As shown in figure **??**, we denote a point on the circle as a complex number $\mathbf{C} = A\cos\theta + \mathrm{i}A\sin\theta = A\angle\theta$. Denoting the starting point in the exercise 4.9.1 as $\mathbf{C}$, we can express its position $x(t)$ as

$$x(t) = \Re\{\mathbf{C}e^{\mathrm{i}\omega t}\} = A\cos(\omega t + \theta). \tag{4.19}$$

Exercise 4.9.3 *Prove (4.19) using Euler's formula $e^{\mathrm{i}\omega t} = \cos\omega t + \mathrm{i}\sin\omega t$.*

The peak value of the sinusoid in (4.17) is $S_p = A$ and its peak-to-peak value is

$$S_{pp} = 2A. \tag{4.20}$$

We evaluate the mean value as

$$S_{mean} = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} A\cos(\omega t + \theta)dt = 0. \tag{4.21}$$

Its root-mean-square (rms) value is

$$S_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} A^2\cos^2(\omega t + \theta)dt} = \frac{A}{\sqrt{2}}. \tag{4.22}$$

The power is

$$p(t) = s^2(t) = A^2\cos^2(\omega t + \theta) \tag{4.23}$$

and we may evaluate the peak power as $P_p = A^2$. Its mean power is

$$P_{mean} = \frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} A^2\cos^2(\omega t + \theta)dt = \frac{A^2}{2}. \tag{4.24}$$

The energy is computed as the product of the mean power by the time duration.

Exercise 4.9.4 *Prove (4.21) and (4.22).*

Exercise 4.9.5 *For a sinusoid $s(t) = A\cos(\omega t + \theta) + B$, find its peak, peak-to-peak, mean and rms values. Find its peak and mean power.*

4.10 Rectangular Signal

Many sinusoids are added to be a waveform similar to a rectangular wave

Fundamental frequency

Harmonics

Duty cycle

Other issues are same as in the previous section on sinusoid

4.11 Ramp Signal

Many sinusoids are added to be a waveform similar to a ramp wave

Fundamental frequency

Harmonics

Other issues are same as in the previous section on sinusoid

4.12 Triangular Signal

Many sinusoids are added to be a waveform similar to a triangular wave

Fundamental frequency

Harmonics

Other issues are same as in the previous section on sinusoid

4.13 Hybrid Signal

5

System and Transfer Function

5.1 System in General

System transforms input signals into output signals System is a function mapping input signals into output signals

System interconnections (1) Series or cascade interconnection (2) Parallel interconnection (3) Feedback interconnection (4) Combination

Basic system properties (1) Memoryless vs. with memory (2) Invertibility and inverse system (3) Causality (4) Stability (boundedness) (5) Time invariance (6) Linearity

5.2 Linear System

For a single input and single output system, its input-output relation is expressed as a nth order ordinary differential equation:

$$a_n \frac{d^n y(t)}{dt^n} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + \cdots + b_1 \frac{dx(t)}{dt} + b_0 x(t). \tag{5.1}$$

5.3 Operational Transfer Function

Using the Laplace operator s as

$$\mathcal{L}\left\{\frac{d^k(\cdot)}{dt^k}\right\} = s^k,$$

we may express (5.1) as

$$\left(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0\right) y(t) = \left(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0\right) x(t). \tag{5.2}$$

The operational transfer function $H(s)$ is

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{5.3}$$

where $Y(s) = \mathcal{L}\{y(t)\}$ and $X(s) = \mathcal{L}\{x(t)\}$.

5.4 Frequency Transfer Function

From the theory of Fourier transform, we have

$$\mathcal{F}\left\{\frac{d^k(\cdot)}{dt^k}\right\} = (\mathrm{i}\omega)^k.$$

The frequency transfer function $H(\mathrm{i}\omega)$ is

$$H(\mathrm{i}\omega) = \frac{Y(\mathrm{i}\omega)}{X(\mathrm{i}\omega)} = \frac{b_m(\mathrm{i}\omega)^m + b_{m-1}(\mathrm{i}\omega)^{m-1} + \cdots + b_1(\mathrm{i}\omega) + b_0}{a_n(\mathrm{i}\omega)^n + a_{n-1}(\mathrm{i}\omega)^{n-1} + \cdots + a_1(\mathrm{i}\omega) + a_0} \tag{5.4}$$

where $Y(\mathrm{i}\omega) = \mathcal{F}\{y(t)\}$ and $X(\mathrm{i}\omega) = \mathcal{F}\{x(t)\}$.

5.5 Zeroth Order System

A zeroth order system with $n = 0, m = 0$ has the following transfer functions:

$$H(s) = H(\mathrm{i}\omega) = \frac{b_0}{a_0} = K$$

where K is the gain of the system.

5.6 First Order System

For a first order system with $n = 1, m = 0$,

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t), \tag{5.5}$$

$$H(s) = \frac{1}{a_1 s + a_0} = \frac{K}{\tau s + 1}, \quad \text{and} \tag{5.6}$$

$$H(\mathrm{i}\omega) = \frac{1}{a_1(\mathrm{i}\omega) + a_0} = \frac{K}{\tau(\mathrm{i}\omega) + 1} \tag{5.7}$$

where we assume $b_0 = 1$ without loss of generality.

5.7 Second Order System

For a second order system with $n = 2, m = 0$,

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t), \tag{5.8}$$

$$H(s) = \frac{1}{a_2 s^2 + a_1 s + a_0} = \frac{K}{\frac{s^2}{s_0^2} + 2\zeta s_0 s + 1}, \quad \text{and} \tag{5.9}$$

$$H(\mathrm{i}\omega) = \frac{1}{a_1(\mathrm{i}\omega) + a_0} = \frac{K}{\tau(\mathrm{i}\omega) + 1} \tag{5.10}$$

where we assume $b_0 = 1$ without loss of generality.

5.8 Higher Order System

7

Basic Circuit Theory

7.1 Series and Parallel Connection

Multiple components are connected in series if current passing through them is same. The voltage across all of them is the sum of voltages across each one of them. Multiple components are connected in parallel when voltage across each one of them is same for all. The current passing through each one of them is added to a total current.

For N components in series, the total impedance $\mathbf{Z}_t$ is

$$\mathbf{Z}_t = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N \tag{7.1}$$

where $\mathbf{Z}_i$ is the impedance of the ith component. For N components in parallel,

$$\frac{1}{\mathbf{Z}_t} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}. \tag{7.2}$$

Exercise 7.1.1 *Prove (7.1) and (7.2).*

7.2 Kirchhoff's Laws

7.2.1 Kirchhoff's Current Law

At a node of a circuit, the sum of all currents flowing into or out of the node is zero, that is,

$$\sum_{j=1}^{N} i_j = 0. \tag{7.3}$$

This means the continuity of current.

7.2.2 Kirchhoff's Voltage Law

Around a loop of a circuit, we evaluate voltages across each component. The sum of all voltages around a loop is zero, that is,

$$\sum_{j=1}^{N} v_j = 1 \tag{7.4}$$

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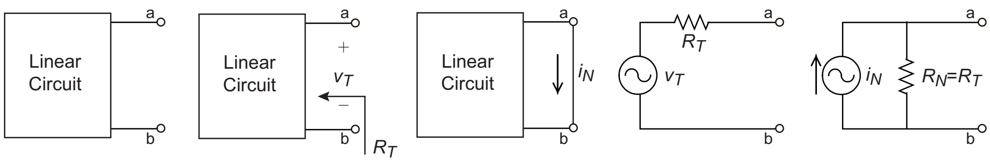

(a) (b) (c) (d) (e)

Figure 7.1 (a) Linear circuit with a port of two terminals a and b. (b) Open-circuit voltage v_T and equivalent resistance R_T. (c) Short-circuit current i_N. (d) Thevenin equivalent circuit. (e) Norton equivalent circuit.

7.3 Superposition

For a linear circuit, the principle of superposition holds. Consider a linear circuit with multiple sources. A voltage (or current) in the circuit is the sum of voltages (or currents) subject to each one of the sources with all other sources turned off. Turning a voltage source v_s off means setting $v_s = 0$, that is, short circuit the voltage source. To turn a current source i_s off, open circuit the current source since $i_s = 0$.

7.4 Thevenin and Norton Equivalent Circuit

For any linear circuit, we may define a port with two terminals as in figure 7.1. When we see the circuit from the port, it appears as a voltage (or current) source with a source impedance (or resistance). The equivalent voltage and current source circuits are called the Thevenin and Norton equivalent circuits, respectively. The Thevenin voltage source equals the open-circuit voltage at the port and the Norton current source equals the short-circuit current through the terminal. The source impedance (or resistance) is the equivalent impedance (or resistance) seen at the port with all sources in the linear circuit turned off. Note that in figure 7.1,

$$i_N = \frac{v_T}{R_T} \quad \text{and} \quad R_N = R_T.$$

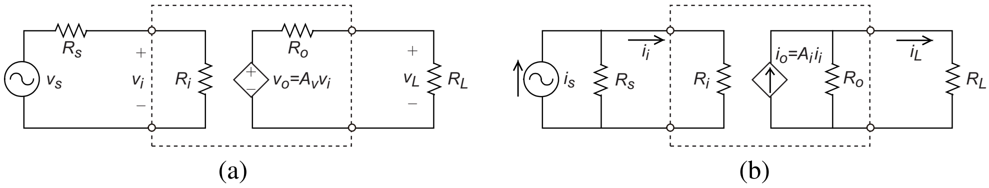

Figure 7.2 (a) Voltage and (b) current amplifiers. Each amplifier is included in the dotted box.

7.5 RC Circuit

7.5.1 Transient Analysis

7.5.2 Steady-state Sinusoidal Analysis

7.6 RLC Circuit

7.6.1 Transient Analysis

7.6.2 Steady-state Sinusoidal Analysis

7.7 Circuit for Differential Signal

7.8 Amplifier

Figure 7.2(a) and (b) show structures of the voltage and current amplifiers, respectively. We represent a source signal as its Thevenin or Norton equivalent circuit with a source resistance R_s. We can view the input resistance R_i of the amplifier as a sensor to sense the strength of the source signal. The sensed small signal at the input is magnified by the gain and the amplified signal appears at the output of the amplifier. The output signal is again represented by its Thevenin or Norton equivalent circuit with a output resistance R_o. At the output, we may consider a load R_L to which the amplifier sends out its signal. The required power for the amplification by open-loop gains A_v and A_i is supplied by the dc power supply. We may view the amplifier as a power-transfer device that moves dc power from the dc power supply to ac signal power at the output.

For the voltage amplifier in figure 7.2(a), we may derive the total voltage gain H_v as

$$H_v = \frac{R_i}{R_s + R_i} \frac{R_L}{R_o + R_L} A_v v_s. \tag{7.5}$$

The first fraction quantifies the loading effect at the input and the second at the output. To minimize these loading effects, it is desirable to have high R_i and low R_o. For the current amplifier in figure 7.2(b), the total current gain H_i is

$$H_i = \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L} A_i i_i. \tag{7.6}$$

The first and second fractions quantify the loading effects at the input and output, respectively. To minimize these loading effects, it is desirable to have low R_i and high R_o. Open-loop gains A_v and A_i are greater than one and this is accomplished by using an

Table 7.1 Classification of amplifiers. Z_i and Z_o are input and output impedances of the amplifier.

Type	Input	Output	Gain	Z_i	Z_o
Voltage amplifier	Voltage	Voltage	V/V	High	Low
Current amplifier	Current	Current	A/A	Low	High
Trans-admittance amplifier	Voltage	Current	A/V or S	High	High
Trans-impedance amplifier	Current	Voltage	V/A or Ω	Low	Low

active component. Active components such as transistors and operational amplifiers transfer dc power to ac signal power for signal amplifications.

Example 7.8.1 *Draw a diagram of a trans-conductance amplifier. Derive the total gain and conditions to minimize loading effects.*

Example 7.8.2 *Draw a diagram of a trans-resistance amplifier. Derive the total gain and conditions to minimize loading effects.*

When we include reactance terms, the total gain should be expressed as a complex number whose value changes with frequency. In such a case, the trans-conductance and trans-resistance amplifiers should be called as the trans-admittance and trans-impedance amplifiers, respectively. Table 7.1 summarizes four amplifier types. Operational amplifiers are basic building blocks for amplifier designs in instrumentation.

9

Linear Analog Circuit

9.1 Inverting Amplifier

Figure 9.1(a) and (b) show inverting amplifiers. It is a voltage amplifier for a single-ended signal with gains of

$$\mathbf{H}_v(f) = -\frac{\mathbf{Z}_2(f)}{\mathbf{Z}_1(f)} \quad \text{or} \quad H_v = -\frac{R_2}{R_1} \tag{9.1}$$

for (a) and (b), respectively. Its input impedance is $\mathbf{Z}_1(f)$ or R_1 and output impedance is very small.

When the inverting amplifier is connected to a signal source as in figure 9.1(c), there occurs a loading effect at the input since gains are altered to

$$H_v = -\frac{R_2}{R_s + R_1} \quad \text{or} \quad \mathbf{H}_v(f) = -\frac{\mathbf{Z}_2(f)}{\mathbf{Z}_s(f) + \mathbf{Z}_1(f)} \tag{9.2}$$

in general. The loading effect at the output is negligible.

Example 9.1.1 *We designed an inverting amplifier in figure 9.1(b) with R_1 =1 kΩ and R_2 =100 kΩ for the gain of -100. When we amplify a source signal $v_s(t) = 10\cos(2\pi \times 1000t)$ mV with R_s =100 Ω, find $v_i(t)$ and $v_o(t)$. Plot $v_s(t), v_i(t)$ and $v_o(t)$ for $0 \leq t \leq$ 5 ms.*

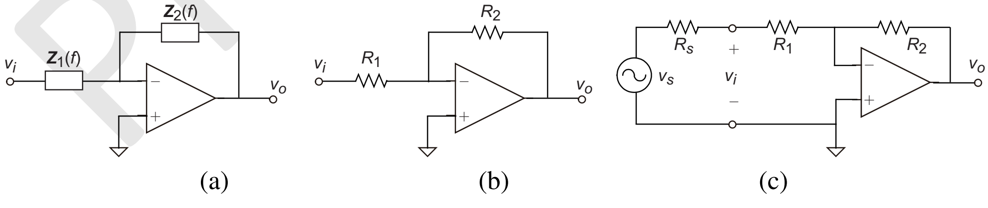

Figure 9.1 Inverting amplifiers with gains of (a) $-\frac{\mathbf{Z}_2(f)}{\mathbf{Z}_1(f)}$ and (b) $-\frac{R_2}{R_1}$, respectively. (c) shows a loading effect at the input.

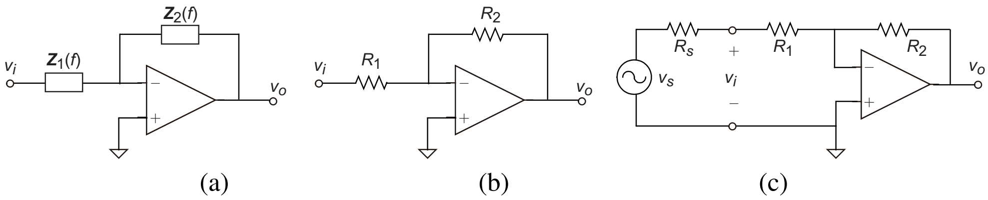

Figure 9.2 Noninverting amplifiers with gains of (a) $1 + \frac{\mathbf{Z}_2(f)}{\mathbf{Z}_1(f)}$ and (b) $1 + \frac{R_2}{R_1}$, respectively. (c) shows that there is no loading effect at the input.

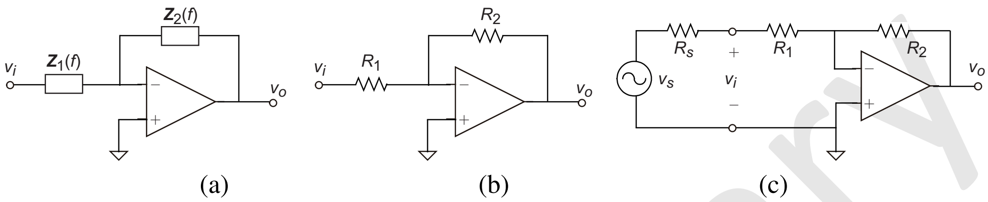

Figure 9.3 Voltage follower or buffer with the gain of 1.

9.2 Noninverting Amplifier

Figure 9.2(a) and (b) show noninverting amplifiers. It is a voltage amplifier for a single-ended signal with gains of

$$\mathbf{H}_v(f) = 1 + \frac{\mathbf{Z}_2(f)}{\mathbf{Z}_1(f)} \quad \text{or} \quad H_v = 1 + \frac{R_2}{R_1} \tag{9.3}$$

for (a) and (b), respectively. Its input impedance is very large and output impedance is very small. When the noninverting amplifier is connected to a signal source as in figure 9.2(c), there is no loading effect at the input and also output.

Example 9.2.1 *We designed a noninverting amplifier in figure 9.2(b) with R_1 =1 kΩ and R_2 =99 kΩ for the gain of 100. When we amplify a source signal $v_s(t) = 10\cos(2\pi \times 1000t)$ mV with R_s =100 Ω, find $v_i(t)$ and $v_o(t)$. Plot $v_s(t), v_i(t)$ and $v_o(t)$ for $0 \leq t \leq 5$ ms.*

9.3 Voltage Follower or Buffer

Figure 9.3(a) shows a voltage follower or buffer. It is a voltage amplifier for a single-ended signal with the gain of 1. It is equivalent to the noninverting amplifier in figure 9.2(b) with $R_1 = \infty$. Its input impedance is very large and output impedance is very small.

Example 9.3.1 *Prevent the loading effect in figure 9.2(c) by using a buffer. Repeat example 9.1.1 with the buffer.*

9.4 Summing Amplifier or Adder

9.5 Difference Amplifier or Subtractor

A difference amplifier is also a differential amplifier, which amplifies a differential input signal to produce a single-ended output signal. For the difference amplifier or subtractor in figure **??**, the output voltage v_o is

$$v_o = \frac{R_4}{R_3 + R_4}\frac{R_1 + R_2}{R_1}v_2 - \frac{R_2}{R_1}v_1 = \frac{R_4}{R_3}\frac{1}{1+\frac{R_4}{R_3}}\left(1+\frac{R_2}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1. \tag{9.4}$$

If $R_4/R_3 = R_2/R_1$,

$$v_o = \frac{R_2}{R_1}\left(v_2 - v_1\right) = \frac{R_2}{R_1}v_{dm}. \tag{9.5}$$

In general, for a differential amplifier,

$$v_o = H_{dm}v_{dm} + H_{cm}v_{cm} = \left(H_{dm} + \frac{1}{2}H_{cm}\right)v_2 - \left(H_{dm} - \frac{1}{2}H_{cm}\right)v_1 \tag{9.6}$$

where H_{dm} and H_{cm} are the differential-mode and common-mode gains, respectively. For an ideal differential amplifier, $H_{cm} = 0$. Since $H_{cm} \neq 0$ in practice, we define the common-mode rejection ratio (CMRR) as

$$CMRR = \frac{H_{dm}}{H_{cm}} \quad \text{or} \quad CMRR = 20\log_{10}\frac{H_{dm}}{H_{cm}} \quad \text{dB}. \tag{9.7}$$

Exercise 9.5.1 *For the difference amplifier in figure* **??**, *find the CMRR for the following cases:*

1. $\frac{R_4}{R_3} = \frac{R_2}{R_1}$.
2. $\frac{R_4}{R_3} = 1.01\frac{R_2}{R_1}$.
3. $\frac{R_4}{R_3} = 0.9999\frac{R_2}{R_1}$.

Exercise 9.5.2 *For three cases above, find the required tolerances of resistors.*

Exercise 9.5.3 *Using 1% resistors and a potentiometer, design a difference amplifier which you can adjust the CMRR. Explain the procedure to tune the circuit and measure its CMRR after tuning.*

Exercise 9.5.4 *Find expressions for input impedances of the difference amplifier in figure* **??**. *Discuss possible loading effects at the input. Will it affect the CMRR?*

9.6 Instrumentation Amplifier

The instrumentation amplifier is constructed by adding two buffers at the input of the difference amplifier to prevent loading effects from occurring. Comparing two designs in figure **??**(a) and (b), we find that the design in (b) is advantageous for more voltage gain.

The first stage is a differential amplifier with a differential output. Since

$$v_4 - v_3 = \left(1 + 2\frac{R_0}{R_G}\right)(v_2 - v_1), \tag{9.8}$$

its differential-mode voltage gain is

$$H_{dm1} = \frac{v_4 - v_3}{v_2 - v_1} = 1 + 2\frac{R_0}{R_G}. \tag{9.9}$$

The second stage is a difference amplifier with

$$v_o = \frac{R_4}{R_3}\frac{1}{1 + \frac{R_4}{R_3}}\left(1 + \frac{R_2}{R_1}\right)v_4 - \frac{R_2}{R_1}v_3. \tag{9.10}$$

If $R_4/R_3 = R_2/R_1$,

$$v_o = \frac{R_4}{R_3}(v_4 - v_3) = H_{dm2}(v_4 - v_3) \tag{9.11}$$

and

$$v_o = \left(1 + 2\frac{R_0}{R_G}\right)\frac{R_4}{R_3}(v_2 - v_1) = H_{dm1}H_{dm2}(v_2 - v_1) = H_{dm}(v_2 - v_1). \tag{9.12}$$

9.7 Voltage-to-Current Converter

A voltage-to-current converter is implemented as a trans-admittance or trans-conductance amplifier.

9.8 Current-to-Voltage Converter

A current-to-voltage converter is implemented as a trans-impedance or trans-resistance amplifier.

9.9 Filter

A filter has a gain that changes with frequency.

11

Noise and Interference

11.1 Gaussian Random Noise

All signals introduced so far are deterministic signals for which its value at time t is predetermined. Random signals are different from deterministic signals in the sense that its value at time t is not predetermined but the probability of a certain value at time t is given instead.

To represent a random signal, we introduce a random variable. A random variable X is characterized by its probability density function $f(x)$ where $f(x = a) := Pr\{x = a\}$ where $Pr\{\cdot\}$ is a probability of the event of an occurrence $\{x = a\}$. Since the sum of all probability is defined as 1,

$$\int_{-\infty}^{\infty} f(x)dx = 1.$$

The mean or average value μ is expressed as an expectation $E\{x\}$ of

$$\mu = E\{x\} = \int_{-\infty}^{\infty} xf(x)dx$$

and the variance σ^2 is

$$\sigma^2 = E\{(x-\mu)^2\} = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx.$$

If $\mu = 0$,

$$\sigma^2 = E\{x^2\} = \int_{-\infty}^{\infty} x^2 f(x)dx$$

is equivalent to the power.

We consider a Gaussian random variable X with its probability density function $f(x; \mu, \sigma^2)$:

$$f\left(x; \mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{11.1}$$

We often call it the normal distribution and denote the Gaussian random variable X by

$$X \sim \mathcal{N}\left(\mu, \sigma^2\right).$$

If $\mu = 0$,

$$f\left(x; 0, \sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}. \tag{11.2}$$

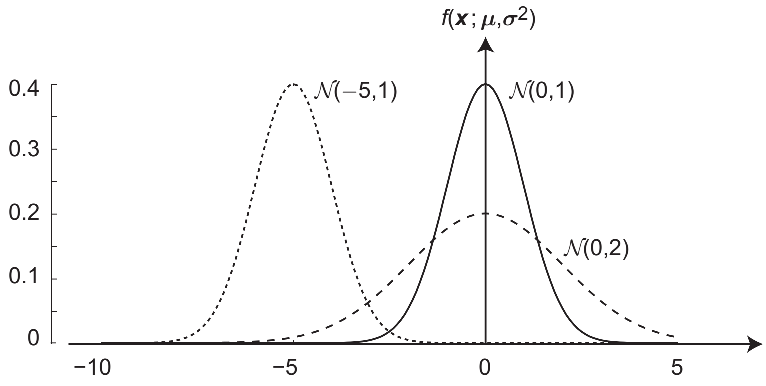

Figure 11.1 Three Gaussian probability density functions of $\mathcal{N}(0,1), \mathcal{N}(0,2)$ and $\mathcal{N}(-5,0)$.

If $\sigma^2 = 0$,

$$f(x;\mu,0) = \delta(x-\mu) \tag{11.3}$$

where $\delta(\cdot)$ is the Dirac delta function. Figure 11.1 shows three Gaussian probability density functions.

We deal with a zero-mean Gaussian random noise, $X \sim \mathcal{N}\left(0,\sigma^2\right)$. Figure 11.2 shows an occurrence or a realization of the random noise with $\sigma^2 = 1$. Noting that

$$\int_{-4\sigma}^{4\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \approx 0.99994,$$

we set the maximum or peak noise value of the noise as $x_p = 4\sigma$ with its peak-to-peak value of $x_{pp} = 8\sigma$.

xxxxxx Insert power spectral density, power spectrum, white noise, colored noise or pink noise, etc. xxxxxx

11.2 Thermal Noise

11.3 Shot Noise

11.4 Flicker Noise

11.5 Interference

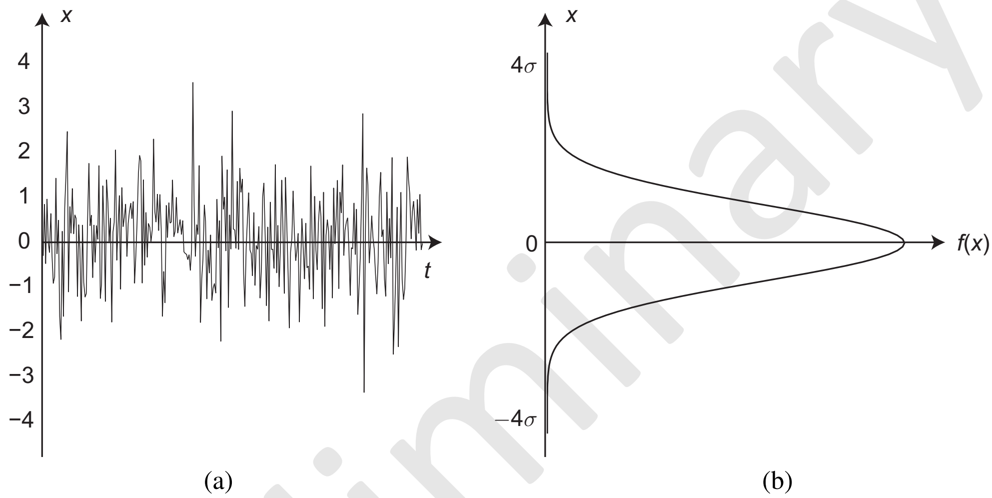

Figure 11.2 (a) An occurrence of a zero-mean Gaussian noise with variance of 1 and (b) its probability density function.

12

Signal-to-Noise Ratio (SNR)

12.1 Definition of SNR

We consider a measured signal $m(t)$ which includes a signal $s(t)$ and an additive noise $n(t)$:

$$m(t) = s(t) + n(t). \tag{12.1}$$

We may find the magnitude of the signal M_s using one of the followings:

$$M_s = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)dt, \tag{12.2}$$

$$M_s = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s^2(t)dt}, \tag{12.3}$$

$$M_s = \max\{|s(t)| \quad \text{for} \quad t_0 \le t \le t_0 + T\}. \tag{12.4}$$

Similarly, the magnitude of the noise M_n is determined as one of the followings:

$$M_n = \frac{1}{T}\int_{t_0}^{t_0+T} n(t)dt, \tag{12.5}$$

$$M_n = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} n^2(t)dt}, \tag{12.6}$$

$$M_n = \max\{|n(t)| \quad \text{for} \quad t_0 \le t \le t_0 + T\}. \tag{12.7}$$

Signal and noise powers are P_s and P_n, respectively, with

$$P_s = M_s^2 \quad \text{and} \quad P_n = M_n^2. \tag{12.8}$$

The signal-to-noise ratio (SNR) is a measure of relative strength of signal $s(t)$ with respect to noise $n(t)$. It is a measure of the quality of the measurement $m(t)$ and also a measure of the performance of the instrument. In most instrumentation, the SNR is the key factor to determine the resolution of a measurement. We define the SNR as

$$SNR = \frac{M_s}{M_n} \quad \text{or} \quad SNR = 20\log_{10}\frac{M_s}{M_n} \quad \text{dB}. \tag{12.9}$$

Selected Chapters of Biomedical Instrumentation Eung Je Woo

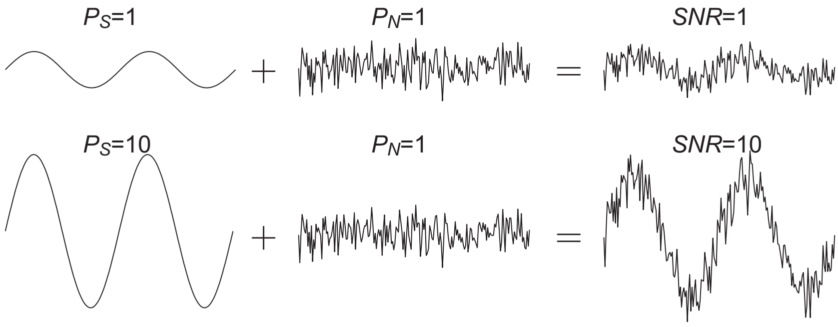

Figure 12.1 Two sinusoidal signals contaminated by a zero-mean Gaussian random noise with a variance of 1. SNRs of the top and bottom signals are 1 and 10, respectively, using (12.10).

Alternatively, we may define it using power as

$$SNR = \frac{P_s}{P_n} \quad \text{or} \quad SNR = 10\log_{10}\frac{P_s}{P_n} \quad \text{dB}. \tag{12.10}$$

When we mention the SNR, it is necessary to specify which definitions are used.

Figure 12.1 shows two examples of sinusoidal signals contaminated by a zero-mean Gaussian random noise with a variance of 1. Using the definition of the SNR in (12.10), one has the SNR of 1 and the other has 10. It is desirable to have intuitive feeing about a signal waveform with a certain SNR.

12.2 Measurement

12.3 Statistical Estimation

14

Digital Signal

14.1 Continuous-time Discrete-amplitude Signal

14.2 Discrete-time Discrete-amplitude Signal

14.3 Power and Energy

We consider a digital signal $s[n]$. Its instantaneous power $p[n]$ is

$$p[n] = s^2[n]. \tag{14.1}$$

The mean power P_s is either

$$P_s = \frac{1}{N_2 - N_1 + 1} \sum_{N_1}^{N_2} s^2[n] \tag{14.2}$$

or

$$P_s^\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} s^2[n]. \tag{14.3}$$

The energy of the signal $s[n]$ is either

$$E_s = \sum_{N_1}^{N_2} s^2[n] \tag{14.4}$$

or

$$E_s^\infty = \lim_{N \to \infty} \sum_{-N}^{N} s^2[n]. \tag{14.5}$$

Exercise 14.3.1 *Suggest examples of a signal with a finite energy, that is,* $E_s^\infty < \infty$. *In this case,* $P_s^\infty = 0$.

Exercise 14.3.2 *Suggest examples of a signal with a finite power, that is,* $P_s^\infty < \infty$. *In this case,* $E_s^\infty = 0$.

Selected Chapters of Biomedical Instrumentation Eung Je Woo

14.4 Constant Signal

A constant or dc signal is

$$s[n] = c \tag{14.6}$$

where c is a constant.

14.5 Unit Step Signal

The unit step signal is denoted as $u[n]$:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0. \end{cases} \tag{14.7}$$

14.6 Unit Impulse Signal

The unit impulse is denoted as $\delta[n]$:

$$\delta[n] = u[n+1] - u[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases} \tag{14.8}$$

Note that

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{m=0}^{\infty} \delta[n-m].$$

Exercise 14.6.1 *Discuss the property of* $s[n]\delta[n-n_0] = s[n_0]\delta[n-n_0]$.

14.7 Exponential Signal

A real exponential signal is

$$s[n] = Ae^{\tau n} = A\alpha^n \tag{14.9}$$

where both A and τ are real and $\alpha = e^{\tau}$. A complex exponential is

$$s[n] = |A|e^{\mathrm{i}\theta}|\alpha|^n e^{\mathrm{i}\omega n} = |A||\alpha|^n \left\{\cos(\omega n + \theta) + \mathrm{i}\sin(\omega n + \theta)\right\}. \tag{14.10}$$

The complex exponential signal is

$$s[n] = e^{\mathrm{i}\omega n} = \cos\omega n + \mathrm{i}\sin\omega n. \tag{14.11}$$

Note that

$$e^{\mathrm{i}\omega(n+N)} = e^{\mathrm{i}\omega N}e^{\mathrm{i}\omega n} = e^{\mathrm{i}\omega n}$$

when we choose the fundamental period N as

$$N = \frac{2\pi}{|\omega|}.$$

The fundamental frequency f is

$$f = \frac{1}{N}.$$

14.8 Sinusoidal Signal

A sinusoid is expressed as

$$s[n] = A\cos(\omega n + \theta) \quad \text{or} \quad s[n] = A\sin(\omega n + \theta) \tag{14.12}$$

where A is the amplitude, ω is the angular frequency in radian per second (rad/s) with $\omega = 2\pi f$ where f is the frequency in Hertz (Hz or 1/s), and θ is the phase angle in radian (rad). Figure **??** plots two sinusoidal signals. The sinusoid is periodic with a period

$$N = \frac{2\pi}{|\omega|} \quad \left(f = \frac{1}{N} \quad \text{or} \quad Nf = 1\right) \tag{14.13}$$

since for any integer m,

$$s[n+mN] = A\cos(\omega n + 2\pi mN + \theta) = A\cos(\omega n + \theta) = s[n].$$

The peak value of the sinusoid in (14.12) is $S_p = A$ and its peak-to-peak value is

$$S_{pp} = 2A. \tag{14.14}$$

We evaluate the mean value as

$$S_{mean} = \frac{1}{N}\sum_{k=n_0}^{n_0+N-1} s[k] = \frac{1}{N}\sum_{k=n_0}^{n_0+N-1} A\cos(\omega k + \theta) = 0. \tag{14.15}$$

Its root-mean-square (rms) value is

$$S_{rms} = \sqrt{\frac{1}{N}\sum_{k=n_0}^{n_0+N-1} s^2[k]} = \sqrt{\frac{1}{N}\sum_{k=n_0}^{n_0+N-1} A^2\cos^2(\omega k + \theta)} = \frac{A}{\sqrt{2}}. \tag{14.16}$$

The power is

$$p[n] = s^2[n] = A^2\cos^2(\omega n + \theta) \tag{14.17}$$

and we may evaluate the peak power as $P_p = A^2$. Its mean power is

$$P_{mean} = \frac{1}{N}\sum_{k=n_0}^{n_0+N-1} s^2[k]dt = \frac{1}{N}\sum_{k=n_0}^{n_0+N-1} A^2\cos^2(\omega k + \theta)dt = \frac{A^2}{2}. \tag{14.18}$$

The energy is computed as the product of the mean power by the time duration.

Exercise 14.8.1 *Prove (14.15) and (14.16).*

Exercise 14.8.2 *For a sinusoid $s[n] = A\cos(\omega n + \theta) + B$, find its peak, peak-to-peak, mean and rms values. Find its peak and mean power.*

14.9 Rectangular Signal

14.10 Ramp Signal

14.11 Triangular Signal

14.12 Hybrid Signal

17

Digital Signal Processing (DSP)

Programs shown in this chapter should be understood as examples. In real implementations, one should be careful about initialization of variable, data overflow and so on.

17.1 Implementation of DSP

Once data are in a digital system, we regard it as a sequence of numbers. Digital signal processing is numerical computation of those numbers using a processor. Its implementation is done by developing a program including multiple functions or subroutines.

There are special purpose microprocessors called a digital signal processor (also abbreviated as DSP). They perform a multiplication as fast as addition due to its hardware implementation.

17.1.1 Programming Language

The most commonly used programming language for DSP is the C language.

17.1.2 Digital Processing Block: Embedded Program

17.1.3 Firmware Skeleton for Real-time Processing

17.2 Differentiation

There are several schemes to implement differentiation. Following three methods are most common:

1. $y[n] = x[n] - x[n-1]$ (backward difference),
2. $y[n] = x[n+1] - x[n]$ (forward difference),
3. $y[n] = (x[n+1] - x[n-1])/2$ (central difference).

We may implement the backward difference using the C language as follows. Note the use of the `static int` variable to keep the value of $x[n-1]$ between function calls.

```
int Differentiator(int x)
{
    static int x1:
```

```
    int y;

    y=x-x1;
    x1=x;
    return(y);
}
```

17.3 Integration

We consider a moving window integration, which computes a sum most recent N data as

$$y[n] = \sum_{j=0}^{N-1} x[n-j].$$

We study two different implementations for $M = 10$. The second uses a ring buffer.

```
int MWI(int x)
{
    static int x1,x2,x3,x4,x5,x6,x7,x8,x9;
    int y;

    y=x+x1+x2+x3+x4+x5+x6+x7+x8+x9;
    x9=x8;x8=x7;x7=x6;x6=x5;x5=x4;x4=x3;x3=x2;x2=x1;x1=x;
    return(y);
}

int MWI(int x)
{
    static int x[9],p,y;

    y-=x[p];
    y+=x;
    x[p]=x;
    if(++p==9) p=0;
    return(y);
}
```

17.4 Filtering

A digital filter is in general expressed as the following difference equation:

$$y[n] + a_1 y[n-1] + \cdots + a_M y[n-M] = b_0 x[n] + b_1 x[n-1] + \cdots + a_N x[n-N] \tag{17.1}$$

or

$$y[n] = \sum_{j=0}^{N} b_j x[n-j] - \sum_{j=1}^{M} a_j y[n-j].$$

If $a_j = 0$ for all $j = 1, 2, \cdots, M$, the filter is called the finite impulse response (FIR) filter. Otherwise, it is an infinite impulse response (IIR) filter. FIR filters are always stable and have the linear phase property. One may use IIR filters for better selectivity in the frequency domain with careful analysis on its stability and phase distortion. Implementation of a general digital filter in (17.1) is shown for the case of $M = 2$ and $N = 3$ as an example.

```
#define a1  1
#define a2  -0.5
#define b0  2
#define b1  -4
#define b2  2
#define b3  0.25

int IIR_Filter(int x}
{
    static int y1,y2,x1,x2,x3;
    int y;

    y=b0*x+b1*x1+b2*x2+b3*x3-a1*y1-a2*y2;
    x3=x2;x2=x1;x1=x;
    y2=y1;y1=y;
    return(y);
}
```

The above implementation requires floating point multiplications and will be slow unless a floating point digital signal processor is used. For a real-time processing using an integer microprocessor, a better implementation is shown below. Note that the idea of using shift operators has a limitation since some digital filters are not integer filters.

```
int IIR_Filter(int x}
{
    static int y1,y2,x1,x2,x3;
    int y;

    y=x<<1-x1<<2+x2<<1+x3>>2-y1-y2>>1;
    x3=x2;x2=x1;x1=x;
    y2=y1;y1=y;
    return(y);
}
```

Exercise 17.4.1 *Discuss pros and cons of using shift operators for multiplication and division. Suggest an expression for* $2.525 \times x$ *using shift operators.*

17.5 Nonlinear Digital Signal Processing

We may easily implement nonlinear processing similarly. As examples, we consider rectification, thresholding, erosion and dilation.

```
int Rectifier(int x)
{
    if(x<0) return(-x);
    else return(x);
```

```
}

#define TRUE    1
#define FALSE   0
#define TH      0x0010

Boolean Threshold(int x)
{
    if (x>TH) return(TRUE);
    else return(FALSE);
}

#define ER_WIDTH    10
extern int FindMax(int *)

int Erosion(int x}
{
    static int x[ER_WIDTH],p;

    x[p]=x;
    if(++p==ER_WIDTH) p=ER_WIDTH;

    return(FindMax(x));
}

#define DL_WIDTH    32
extern int FindMin(int *)

int Dilation(int x}
{
    static int x[DL_WIDTH],p;

    x[p]=x;
    if(++p==DL_WIDTH) p=DL_WIDTH;

    return(FindMin(x));
}
```

Exercise 17.5.1 *Implement a median filter with its data window size of 16.*

17.6 Multi-channel Digital Signal Processing

Care must be given to multi-channel digital signal processing. Assume that we get two channels of input data from an ADC and use the same lowpass filter to both channels. The implementation below is wrong.

```
void main{void}
{
    ...
```

```
    x1=GetChannel1Data();
    x2=GetChannel2Data();
    y1=LPF(x1);
    y2=LPF(x2);

    ...
}

int LPF(int x)
{
    static int x1,x2;
    int y;

    y=(x+x1<<1+x2)>>2;
    x2=x1;x1=x;
    return(y);
}
```

Exercise 17.6.1 *The example above is implemented in a wrong way. Correct the bug.*

Exercise 17.6.2 *In terms of structure, compare digital filtering of multi-channel signals with their analog filtering methods.*

19

Distortionless Measurement

19.1 Transfer Function for Distortionless Measurement

We measure an input signal denoted as $x(t)$ using a measurement system characterized by its transfer function $H(\cdot)$. When there is no distortion, the output $y(t)$ should be expressed as

$$y(t) = Kx(t - \tau)$$

where $K$and τ are the gain and time delay of the system. The frequency transfer function is

$$H(\mathrm{i}\omega) = \frac{\mathcal{F}\{y(t)\}}{\mathcal{F}\{x(t)\}} = \frac{Ke^{-\mathrm{i}\omega\tau}X(\mathrm{i}\omega)}{X(\mathrm{i}\omega)} = Ke^{-\mathrm{i}\omega\tau}. \tag{19.1}$$

This means that for all ω,

$$|H(\mathrm{i}\omega)| = K \quad \text{(flat magnitude response)}, \tag{19.2}$$

$$\theta(\mathrm{i}\omega) = \angle H(\mathrm{i}\omega) = -\tau\omega \quad \text{(linear phase response)}. \tag{19.3}$$

The linear phase response implies that the the amount of phase change must be proportional to the frequency $\omega = 2\pi f$. In practice, two requirements in (19.2) and (19.3) need to be satisfied within a frequency range of a signal bandwidth.

For a sinusoidal input

$$x(t) = A\cos(\omega t + \theta_0),$$

a distortionless output is

$$y(t) = KA\cos(\omega t + \theta_0 - \tau\omega) = KA\cos\{\omega(t - \tau) + \theta_0\}.$$

This means that the elapsed time for a sinusoid to pass the system must be the same fixed value of τ for all frequencies ω.

Exercise 19.1.1 *Describe a frequency response of an ideal voltage amplifier for ECG signals with* ± 5 *V range and* $[0.05, 100]$ *Hz bandwidth.*

19.2 Amplitude Distortion

19.3 Phase Distortion

21

Performance Measure

21.1 Dynamic Range

The dynamic range of an instrument is the range of its input for which output values are given. For example, [-5,5] mV, [0,10] Ω, [0,100]°C, and [-50,300] mmHg are dynamic ranges. It is often expressed as a full scale (FS).

21.2 Accuracy

The accuracy of an instrument describes how close is a reading to the true value. It is often expressed as the error E:

$$E := \frac{|X_m - X_t|}{|X_t|} \quad (\times 100\%) \tag{21.1}$$

where X_t and X_m are the true and measured value. Two commonly used expressions are % of reading and % of FS.

Exercise 21.2.1 *Using a voltmeter with a measurement range of ±100 V and an accuracy of ±1% of reading, you measure 10 V across a resistor. This means that the true voltage is between 9.9 and 10.1 V.*

Exercise 21.2.2 *Using a voltmeter with a measurement range of ±100 V and an accuracy of ±1% of FS, you measure 10 V across a resistor. This means that the true voltage is between 9 and 11 V.*

21.3 Precision

Though the precision is defined as repeatability in some literature, we describe the precision of an instrument as the smallest possible change in a reading. In our definition, the precision has nothing to do with the accuracy.

Exercise 21.3.1 *You are provided with two digital thermometers with a measurement range of 0 to 10°C. One has the accuracy of ±0.1% of FS and displays its output with 4 digits. One is assigned to the first digit below the decimal point. The other has the accuracy of ±1% of FS and displays its output with 5 digits. Two are assigned to the first and second digits below the decimal point. Discuss the accuracy and precision of these two digital thermometers.*

Selected Chapters of Biomedical Instrumentation Eung Je Woo

21.4 Resolution

The resolution, Δ is the smallest change of input which an instrument can measure within its accuracy. It is determined by both the accuracy and precision. We may evaluate the SNR of the instrument as

$$SNR = \frac{DR}{\Delta} \quad \text{or} \quad 20\log_{10}\frac{DR}{\Delta}\ \text{dB}$$

where DR is the dynamic range.

Exercise 21.4.1 *For the exercise 21.3.1, find the resolution and SNR of each thermometer. Which one is better?*

21.5 Bandwidth

The bandwidth of an instrument is a range of frequency for which its output is determined within a given resolution. It is expressed by low and high cutoff frequencies, f_L and f_H, respectively. When the frequency of an input signal f is outside the bandwidth, the accuracy is not supported. In order to measure slowly varying input signals, f_L must be lower than the smallest frequency of the input. When input signals change fast, f_H must be higher than the largest frequency if the input.

21.6 Response Time

21.6.1 Time Constant

21.6.2 Group Delay

22

Sensor and Signal Conditioning

22.1 Stain Gage

The stress σ in a solid material is defined as a force per unit area. It has the same dimension as the pressure. We assume a rod of length L. Applying a longitudinal force, we stretch it to a length of $L+\delta$. The strain ϵ is the ratio of δ/L. We consider an elastic material whose stress-strain curve is depicted in figure **??**. Within the elastic region, we apply a force f or stress σ to produce a strain ϵ which is proportional to f or σ.

The material is made of a conductor with resistivity ρ. Without any force, the cross-sectional area of the rod is A. The resistance R is

$$R=\rho\frac{L}{A}.$$

For changes in ρ, L and A,

$$dR=\frac{\rho}{A}dL-\frac{\rho L}{A^2}dA+\frac{L}{A}d\rho \quad \text{or} \quad \frac{dR}{R}=\frac{dL}{L}-\frac{dA}{A}+\frac{d\rho}{\rho}.$$

Using the Poisson ratio μ as

$$\mu=\frac{dD}{D}=-\frac{dL}{L} \quad \text{and} \quad -\frac{dA}{A}=2\mu\frac{dL}{L}$$

where D is the diameter, we get

$$\frac{dR}{R}=(1+2\mu)\frac{dL}{L}+\frac{d\rho}{\rho}.$$

The first and second terms are the dimensional and piezoresistive effects on the change in resistance. The gage factor G of a strain gage is

$$G=\frac{dR/R}{dL/L}=1+2\mu+\frac{d\rho/\rho}{dL/L} \tag{22.1}$$

where $\epsilon=dL/L$ is the strain. The gage factor is between 1 and 2 for metal stain gages. For semiconductor strain gages, it is between 100 and 200. Depending on the type (p- or n-type semiconductor), the gage factor is either positive of negative. Though semiconductor strain gages have a high gage factor, they have a high temperature coefficient requiring a temperature compensation.

Selected Chapters of Biomedical Instrumentation Eung Je Woo

22.2 Force Sensing Resistor

22.3 Electric Sensor

22.4 Electronic Sensor

22.5 Optical Sensor

22.6 Mechanical Sensor

22.7 Chemical Sensor

23

Sampling and Quantization

23.1 Sampling and Sampling Frequency

23.2 Effects of Sampling

23.3 Quantization

23.4 Quantization Noise

We consider an n-bit ADC with its input range of $[-A, A]$. Since the ADC divides the input range by 2^n discrete values, the quantization level Δ is

$$\Delta = \frac{2A}{2^n}. \tag{23.1}$$

Since we cannot control the sampling moment, the amplitude error The quantization noise q is a random variable and follows a uniform distribution in $[\Delta/2, -\Delta/2]$ with a probability density of $1/\Delta$. Since its mean

$$E\{q\} = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} q dq = 0,$$

the variance or power is

$$P_Q = \sigma^2 = E\{q^2\} = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta} q^2 dq = \frac{\Delta^2}{12}.$$

We assume that the analog signal to be quantized is a full-scale sinusoid:

$$s(t) = A \cos \omega t$$

with its signal power $P_S = A^2/2$. The SNR using the signal power and the quantization noise power is

$$SNR_Q = 10 \log_{10} \frac{P_S}{P_Q} = 10 \log_{10} \frac{\frac{A^2}{2}}{\frac{\Delta^2}{12}} = 10 \log_{10} 3 \times 2^{2n-1} = 6.02n + 1.761 \quad \text{dB}. \tag{23.2}$$

We may take a different view on this problem. We evaluate the dynamic range DR of the ADC as

$$DR = \frac{2A}{\Delta} = 2^n \quad \text{or} \quad DR = 20 \log_{10} 2^n = 6.02n \quad \text{dB}. \tag{23.3}$$

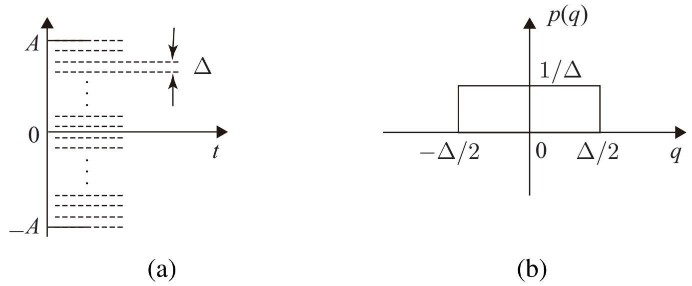

Figure 23.1 (a) Quantization level and (b) probability density of the quantization error or nosie.

Note that SNR_Q in (23.2) is approximately the dynamic range of a signal which can be represented by using an n-bit ADC. Using a 16-bit ADC, the dynamic range is 98 dB, which may cover sound signals we can perceive with the smallest to the largest amplitudes.

We assume that there exists an analog random noise at the input of the ADC with zero mean and variance or power of $P_N = \sigma_N^2$. We can express the SNR of the analog sinusoidal signal SNR_A as

$$SNR_A = 10 \log_{10} \frac{P_S}{P_N} = 10 \log_{10} \frac{\frac{A^2}{2}}{\sigma_N^2} \quad \text{dB}.$$

In order for the n-bit ADC to have a meaning LSB, we need

$$SNR_A > SNR_Q \tag{23.4}$$

or

$$P_N = \sigma_N^2 < P_Q = \frac{\Delta^2}{12}.$$

If the input signal is not full scale, that is, $s(t) = B \cos \omega t$ with $B < A$, then SNR_Q is smaller than its maximum value in (23.2). When we adopt an automatic gain control to maximize SNR_Q, we should be careful about the condition in (23.4) for all gains. If SNR_A is deteriorated for a large gain for some reason, we may not use such a large gain and accept the condition of $B < A$. Instead, we may increase n.

24

Signal and Data Processing

24.1 Real-time Digital Signal Processing

24.2 Statistical Method

24.3 Least Squares Method

We introduce the least squares method using the following data set:

$$\mathbf{y} = [y_1\ y_2\ \cdots\ y_M]^T \quad \text{output data,}$$
$$\mathbf{x} = [x_1\ x_2\ \cdots\ x_M]^T \quad \text{input data.}$$

We wish to find a best polynomial

$$y(x) = a_N x^N + a_{N-1}x^{N-1} + \cdots + a_1 x + a_0$$

which minimizes the following sum of squared errors:

$$\Phi(\mathbf{a}) = \sum_{j=1}^{M} \{y_i - y(x_i)\}^2 \tag{24.1}$$

where $\mathbf{a} = [a_N\ a_{N-1}\ \cdots\ a_1\ a_0]^T$. We may rewrite (24.1) as

$$\Phi(\mathbf{a}) = \|\mathbf{y} - \mathbf{X}\mathbf{a}\|^2 = (\mathbf{y} - \mathbf{X}\mathbf{a})^T(\mathbf{y} - \mathbf{X}\mathbf{a}) = \mathbf{y}^T\mathbf{y} - \mathbf{y}^T\mathbf{X}\mathbf{a} - \mathbf{a}^T\mathbf{X}^T\mathbf{y} + \mathbf{a}^T\mathbf{X}^T\mathbf{X}\mathbf{a} \tag{24.2}$$

where

$$\mathbf{X} = \begin{bmatrix} x_1^N & x_1^{N-1} & \cdots & x_1 & 1 \\ x_2^N & x_2^{N-1} & \cdots & x_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \\ x_M^N & x_M^{N-1} & \cdots & x_M & 1 \end{bmatrix}. \tag{24.3}$$

From $= 0$, we get

$$\mathbf{a} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} \tag{24.4}$$

for $N < M$.

Exercise 24.3.1 *Derive (24.4).*

Exercise 24.3.2 *For $N = 1$, find expressions for a_1 and a_0 using (24.4).*

Exercise 24.3.3 *For $N = 1$, find expressions for a_1 and a_0 using*

$$\frac{\partial \Phi}{\partial a_1} = 0 \quad \textit{and} \quad \frac{\partial \Phi}{\partial a_0} = 0.$$

Compare the results with those of exercise 24.3.3.

24.4 Feature Extraction

24.5 Classification

24.6 Interpretation

26

Electrical Safety

Electrical safety is an important issue in medical instrumentation. Figure 26.1 shows a structure of a medical electrical equipment. The device is isolated from the earth ground for electrical safety of a patient. The patient leakage current and patient auxiliary current illustrated in figure 26.2 are commonly used to check electrical safety. These are measured at normal and various fault conditions. Figure 26.3 shows safety limits of most medical electrical equipments under normal condition according to an international standard. The standard defines numerous testing conditions and also types of medical equipments. One must thoroughly understand details of local and also international electrical safety regulations that will be applied to an equipment being developed. Two methods of isolation and insulation are commonly implemented in the device to satisfy an established design goal.

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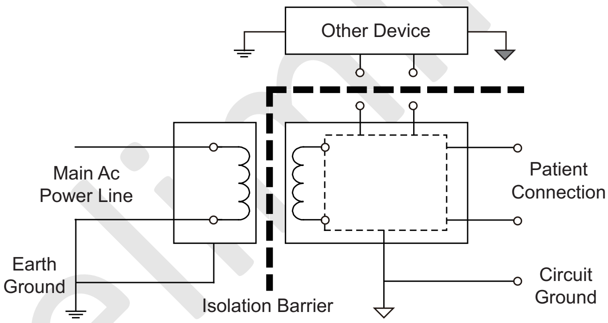

Figure 26.1 Structure of a medical electrical equipment.

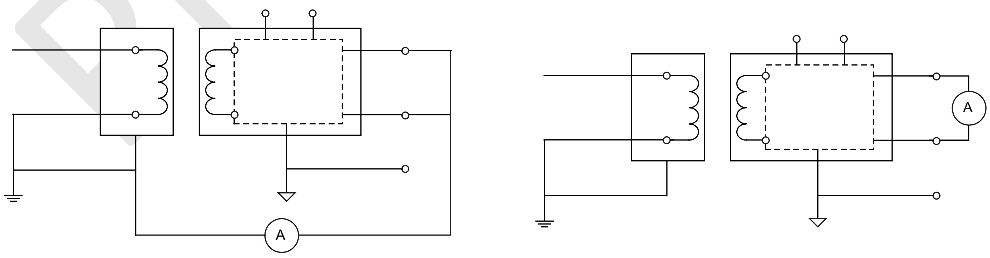

Figure 26.2 (a) Patient leakage current and (b) patient auxiliary current.

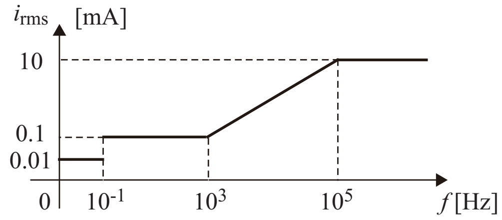

Figure 26.3 Safety limits of the patient leakage current and patient auxiliary current of most medical electrical equipments under normal condition.

26.1 Isolation

As shown in figure 26.1 the device is isolated from the earth ground, that is, it is floating. This requires isolated dc power supplies and non-conductive signal transmission between the earth-grounded and isolated parts. The core device for power isolation is an isolation transformer, which has isolated secondary coils. Isolation voltage and leakage current between the primary and secondary coils should meed the requirements. Since the mechanical structure of the equipment also affect amounts of of leakage currents, safety tests must be performed with all parts assembled into the mechanical structure. When an isolated part consumes a small amount of power, a dc-dc converter may suffice to provide the power. Similar testing must be performed to the device with dc-dc converters. Choosing a dc-dc converter, one must be careful about switching noise.

26.1.1 Totally Isolated Device

One may design a totally isolated device with battery power. Patient leakage currents are not a major concern in this case though patient auxiliary currents must be checked. It could be a complete stand-alone device equipped with all input and output means. The stand-alone device is connected to a base station for data exchange or charging. It may also has a wireless communication channel for real-time data communications.

Alternative design is a device without an output display, for example. In this case, the portable device acquires and sends data through wireless communication channels.

26.1.2 Partially Isolated Device

Non-conductive signal transmission requires one of the following coupling methods:

Transformer coupling:

Capacitive coupling:

Optical coupling:

26.2 Insulation

26.3 Safety Test

29

Biopotential

29.1 Basics of Biopotential

29.1.1 Excitable Cell and Resting Membrane Potential

Excitable cells are abundant in nerves and muscles. The cell is enclosed by its membrane dividing intra- and extra-cellular spaces. Ion concentrations in intra- and extra-cellular fluids are different causing concentration gradients across the thin membrane. Since the membrane is semi-permeable, some ions pass through the membrane and others can not.

In figure 29.1(a), an impermeable membrane separates a bucket into two parts. We fill one part with a saline of 0.1% NaCl solution and the other with 1% NaCl solution. Note that the charge neutrality holds in the bulk solutions at both sides. We assume that a pair of *ideal* electrodes without any contact potential and contact impedance at the electrode-electrolyte interface are available. Since no ion crosses the impermeable membrane, no current flows and we find zero voltage difference between the electrodes immersed in the salines.

We replace the impermeable membrane with a permeable membrane through which both Na^+ and Cl^- can freely pass as in figure 29.1(b). Assuming that mobilities are same, positive and negative ions move as a pair thereby no net ion movement occurs. This diffusion process stops when the equilibrium is reached where both parts have the same concentration of NaCl. Since there is no net charge movement, no current flows and the voltage difference is zero.

From figure 29.1(a) and (b), we can see that concentration gradients across a membrane does not suffice to induce a voltage difference across the membrane. We now consider the

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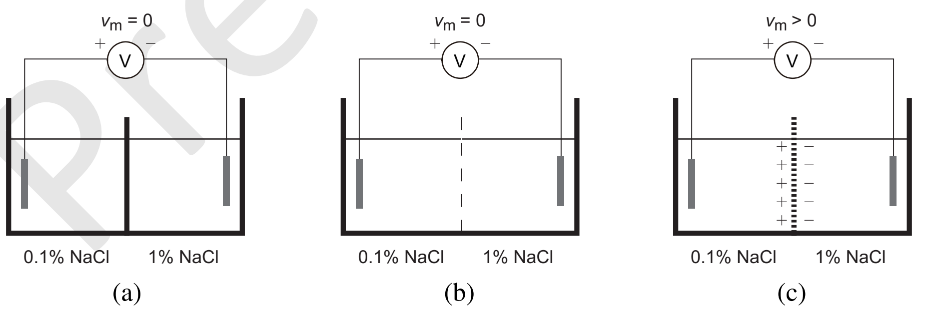

Figure 29.1 (a) Impermeable, (b) permeable and (c) semi-permeable membranes. A nonzero voltage difference appears across the semi-permeable membrane separating two salines with different concentrations.

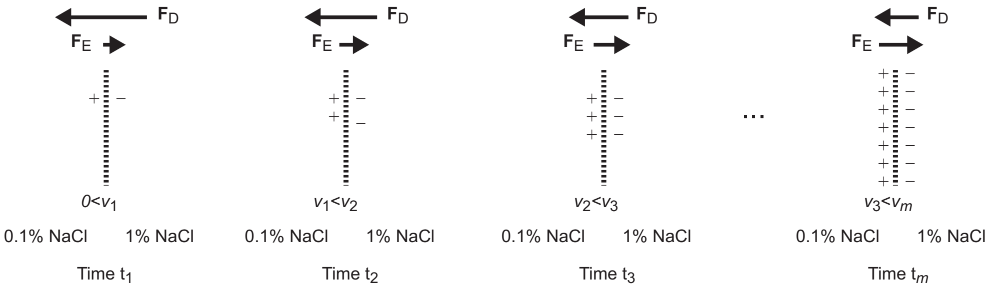

Figure 29.2 Formation of a resting membrane potential v_m at the dynamic equilibrium of the diffusion force F_D and electric Coulomb force F_E.

case of a semi-permeable membrane in figure 29.1(c), where we assume that only Na^+ is permeable. Figure 29.2 illustrates the diffusion process at different times. As soon as one Na^+ migrates into the other side by diffusion, there occurs a charge separation by the membrane; Na^+ in the left and Cl^- in the right sides. This charge separation forms a charge double layer between the membrane. The Coulomb force between separated charges produce an electric field across the membrane. Note that the electric field intensity increases as more Na^+ ions move from the right to the left side.

The first Na^+ ion moves to the left side by diffusion without any impeding force. After the first one has been positioned in the left side, there exists an electric field generating an impeding Coulomb force, which hinders the second Na^+ ion from being diffused into the left side. Until the diffusion force is larger than the Coulomb force, Na^+ ions keep migrating to the left side. When two forces are equal, a dynamic equilibrium is reached and there is no net charge crossing the membrane. At the dynamic equilibrium, the formed charge double layer results in a voltage difference between the membrane. When this occurs in an excitable cell, we call it the resting membrane potential (RMP).

We derive the RMP for a simple case where variables change along x-axis only. We assume a positive ion P^{n+} with its valence of n and charge of nq where $q = 1.602 \times 10^{-19}$ C. We denote the molar concentration of a positive ion by $c_P(x)$. We assume that there exists a dc voltage $v(x)$ due to an electric field $\mathbf{E} = E\mathbf{a}_x = -\frac{dv(x)}{dx}\mathbf{a}_x$ where $\mathbf{a}_x$ is the unit vector in the x-direction. The ion experiences a Coulomb force and moves with a mean velocity of

$$u_P(x) = -\mu_P nq \frac{dv(x)}{dx} \tag{29.1}$$

where μ_P is the mobility of P^{n+}. We assume a rectangular cuboid with its surface area S on the yz-plane. Its thickness along the x-axis is $dt\ u_P$ where dt is a short time interval. The number of P^{n+} ions inside the cuboid is

$$N_P(x) = (\text{volume}) \times (\text{concentration}) = (S v_P(x)\ dt) \times (A_V c_P(x)) \tag{29.2}$$

where $A_V = 6.022 \times 10^{23}$ is the Avogardro constant. Since $N_P(x)$ ions pass through the surface S of the cuboid during dt, we define the ion flow density due to the electric Coulomb force as

$$F_P^E(x) = \frac{N_P(x)}{Sdt} = -\mu_P nq \frac{dv_P(x)}{dx} A_V c_P(x). \tag{29.3}$$

The ion flow density due to the diffusion is

$$F_P^D(x) = -D_P A_V \frac{dc_P(x)}{dx} \tag{29.4}$$

where D_P is the diffusion constant for the ion P^{n+}. Summing these two ion flows, we get the total ion flow as

$$F_P = F_P^E + F_P^D = -\mu_P nq \frac{dv_P(x)}{dx} A_V c_P(x) - D_P A_V \frac{dc_P(x)}{dx}. \tag{29.5}$$

From the Einstein relation

$$\mu_P = \frac{D_P}{Tk_B}$$

where T is the temperature in K and $k_B = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant. Using the Faraday constant $F = A_V q = 96,485.3399$ C/mol and the universal gas constant $R = k_B A_V = 8.314472$ J/(K mol),

$$\mu_P nq = \frac{nF}{RT} D_P$$

and

$$F_P = -\frac{nF}{RT} A_V D_P \frac{dv_P(x)}{dx} c_P(x) - D_P A_V \frac{dc_P(x)}{dx}. \tag{29.6}$$

Since $F_P = 0$ at the dynamic equilibrium,

$$\frac{nF}{RT}\frac{dv_P(x)}{dx} = -\frac{1}{c_P(x)}\frac{dc_P(x)}{dx}. \tag{29.7}$$

We integrate (29.7) from a point x_{in} inside the cell membrane to a point x_{out} outside the membrane as

$$\frac{nF}{RT}\int_{x_{in}}^{x_{out}} \frac{dv_P(x)}{dx} dx = -\int_{x_{in}}^{x_{out}} c_P(x)\frac{dc_P(x)}{dx} dx$$

or

$$\frac{nF}{RT}\left\{v_P(x_{out}) - v_P(x_{in})\right\} = -\left\{\ln c_P(x_{out}) - \ln c_P(x_{in})\right\}.$$

If we measure the RMP in the intracellular fluid with respect to the extracellular fluid, that is, $v_P(x_{out}) = 0$, the RMP $V_P^{RMP} = v_P(x_{in})$ is

$$V_P^{RMP} = \frac{RT}{nF}\ln\frac{c_P(x_{out})}{c_P(x_{in})} = 61.5 \times \log_{10}\frac{c_P(x_{out})}{c_P(x_{in})} \text{ mV at } 37°\text{C}. \tag{29.8}$$

This is known as the Nernst equation of a membrane permeable to the positive ion P^{n+} only.

Exercise 29.1.1 *Derive a Nernst equation of a membrane permeable to both positive ion P^{n+} and negative ion N^{m-}. Assume that the membrane has permeability of P_P and P_N for positive and negative ions, respectively.*

Exercise 29.1.2 *Table 29.1 shows ion concentrations and permeability of a frog skeletal muscle. Show that the RMP is expressed as the following Goldman-Hodgkin-Katz equation:*

$$V^{RMP} = \frac{RT}{F}\ln\left\{\frac{P_K C_K^o + P_{Na}C_{Na}^o + P_{Cl}C_{Cl}^i}{P_K C_K^i + P_{Na}C_{Na}^i + P_{Cl}C_{Cl}^o}\right\}$$

where C_X^i and C_X^o P_X are molar concentrations of the ion X in intra- and extracellular fluids, respectively, and P_X is the permeability of the membrane for the ion X.

Table 29.1 Excitable cells of the frog skeletal muscle. C_X^i and C_X^o P_X are molar concentrations of the ion X in intra- and extracellular fluids, respectively, and P_X is the permeability of the membrane for the ion X.

Ion (X)	C_X^i, mmol/l	C_X^o, mmol/l	P_X, cm/s
Na^+	12	145	2×10^{-8}
K^+	155	4	2×2^{-6}
Cl^-	4	120	2×2^{-6}

29.1.2 Action Potential

29.1.3 Nerve Conduction

29.1.4 Volume Conductor Field

Generation of action potentials and their propagation produce endogenous current flows. We denote internal current density and voltage at position $\mathbf{r}$ as $\mathbf{J}(\mathbf{r})$ and $v(\mathbf{r})$, respectively. At low frequencies, we can consider only conductivity $\sigma(\mathbf{r})$ neglecting effects of permittivity. Note that $\sigma(\mathbf{r})$ changes depending on the position $\mathbf{r}$. Since $\mathbf{J}(\mathbf{r}) = -\sigma(\mathbf{r})\nabla v(\mathbf{r})$ and $\nabla \cdot \mathbf{J}(\mathbf{r}) =$ *source current*,

$$\begin{cases} -\nabla \cdot (\sigma(\mathbf{r})\nabla v(\mathbf{r})) = f_s(\mathbf{r}) & \text{in} \quad \mathcal{D} \\ -\sigma \nabla v \cdot \mathbf{n} = 0 & \text{on} \quad \partial\mathcal{D} \end{cases} \tag{29.9}$$

where f_s is the endogenous source current, $\mathcal{D}$ is the domain of the body with its boundary $\partial\mathcal{D}$ and $\mathbf{n}$ is the outward unit normal vector on $\partial\mathcal{D}$. The boundary condition known as the Neumann boundary condition indicates that there is no current flowing out of the body $\mathcal{D}$ into the surrounding air, which is an insulator.

29.2 Electrode

29.2.1 Electrode-Electrolyte Interface

29.2.2 Polarizable and Non-polarizable Electrode

29.2.3 Skin-Electrode Interface

29.2.4 Equivalent Circuit Model

29.3 Biopotential Amplifier

29.3.1 Specification of Biopotential

Commonly measured biopotentials include electrocardiogram (ECG), electromyogram (EMG), electroencephalogram (EEG), electroneurogram (ENG), electroretinogram (ERG), electrooculogram (EOG) and so on. Table 29.2 summarizes them.

29.3.2 Model of Biopotential Signal Source

Figure 29.3 illustrates a model of a biopotential amplifier.

Table 29.2 Specifications of commonly measured biopotentials.

Biopotential	Origin	Amplitude	Bandwidth
ECG	Heart	± 5 mV	0.05–100 Hz
EMG	Muscle	$\pm$xxx mV	10–10,000 Hz
EEG	Brain	± 50 μV	1–30 Hz
ENG	Nerve		
ERG	Retina		
EOG	Eye		

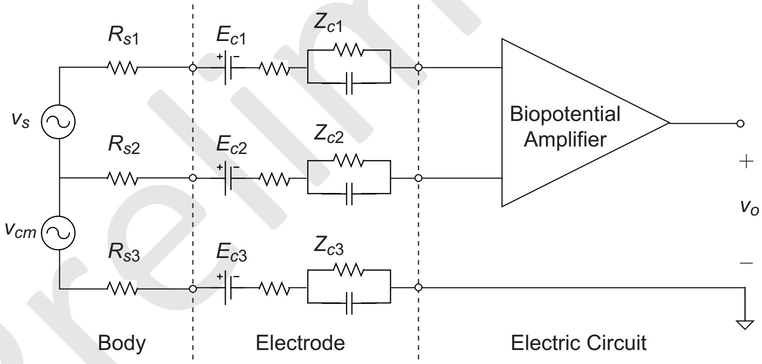

Figure 29.3 A model of a biopotential amplifier including source signal, body resistance, electrode and amplifier.

29.3.3 Differential Highpass Filter

29.3.4 Instrumentation Amplifier with Lowpass Filter

29.3.5 Gain Amplifier and Level Shifter

29.4 Biosignal Processing

30

Bioimpedance

30.1 Basics of Bioimpedance

30.1.1 Admittivity and Impedivity

30.1.2 Admittance and Impedance

30.2 Phase-sensitive Demodulation

We assume a bioimpedance of

$$\mathbf{Z} = R + \mathrm{i}X = Z\angle\theta\ \Omega. \tag{30.1}$$

Injection current $i(t)$ is

$$i(t) = I\cos\omega t \quad \text{or} \quad \mathbf{I} = I\angle 0\ \mathrm{A} \tag{30.2}$$

where its amplitude I and frequency ω are fixed and known. The injection current induces a voltage $v(t)$ across the bioimpedance as

$$v(t) = V\cos(\omega t + \theta) \quad \text{or} \quad \mathbf{V} = \mathbf{ZI} = ZI\angle\theta = V\angle\theta\ \mathrm{V}. \tag{30.3}$$

Figure 30.1 illustrates typical waveforms of injection current and induced voltage across $\mathbf{Z}$.

In (30.3), $i(t)$ is modulated by $\mathbf{Z}$ to produce $v(t)$. To recover $\mathbf{Z}$, we demodulate $v(t)$. Since we need to get both V and θ, we adopt the phase-sensitive demodulation or synchronous

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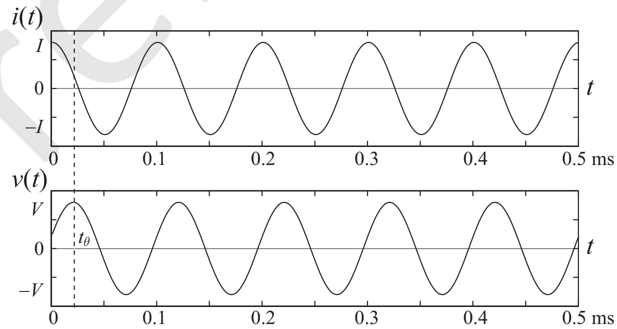

Figure 30.1 Typical waveforms of injection current and induced voltage across a bioimpedance.

detection method illustrated in figure 30.2. The phase-sensitive demodulation has two channels I and Q of in-phase and quadrature, respectively. For the in-phase channel,

$$\begin{aligned} V_I &= \frac{1}{nT}\int_{t_0}^{t_0+nT} v(t)\cos\omega t dt \\ &= \frac{1}{nT}\int_{t_0}^{t_0+nT} V\cos(\omega t+\theta)\cos\omega t dt \\ &= \frac{1}{nT}\int_{t_0}^{t_0+nT} \frac{V}{2}\left\{\cos(2\omega t+\theta)+\cos\theta\right\}dt \\ &= \frac{V}{2}\cos\theta. \end{aligned} \tag{30.4}$$

For the quadrature channel,

$$\begin{aligned} V_Q &= \frac{1}{nT}\int_{t_0}^{t_0+nT} v(t)\sin\omega t dt \\ &= \frac{1}{nT}\int_{t_0}^{t_0+nT} V\cos(\omega t+\theta)\sin\omega t dt \\ &= \frac{1}{nT}\int_{t_0}^{t_0+nT} \frac{V}{2}\left\{-\sin(2\omega t+\theta)+\sin\theta\right\}dt \\ &= \frac{V}{2}\sin\theta. \end{aligned} \tag{30.5}$$

We recover $\mathbf{Z} = Z\angle\theta$ as

$$Z = \frac{\sqrt{4V_I^2+4V_Q^2}}{I} \tag{30.6}$$

and

$$\theta = \tan^{-1}\frac{V_Q}{V_I}. \tag{30.7}$$

Figure 30.3 illustrates relations among $\mathbf{I}, \mathbf{Z}$ and $\mathbf{V}$ with their in-phase and quadrature components. Note that in-phase and quadrature components are in-phase and quadrature, respectively, with respect to the injection current $i(t) = I\cos\omega t$ or $\mathbf{I} = I\angle\theta$.

30.3 Two- and Four-electrode Method

30.4 Constant Current Source

30.5 Constant Voltage Source

30.6 Voltage Amplifier

30.7 Demodulator

30.8 Bioimpedance Spectroscopy

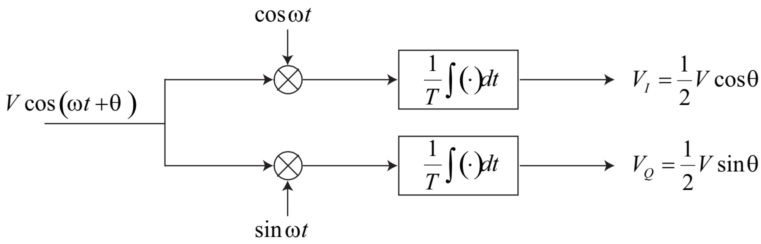

Figure 30.2 Diagram of phase-sensitive demodulation.

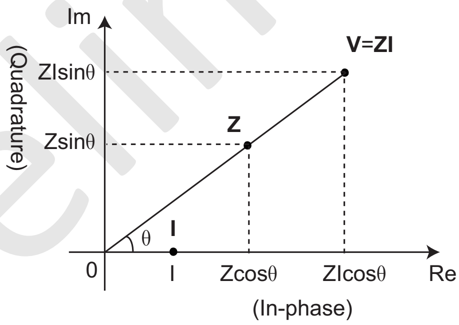

Figure 30.3 In-phase and quadrature components of the phase-sensitive demodulation.