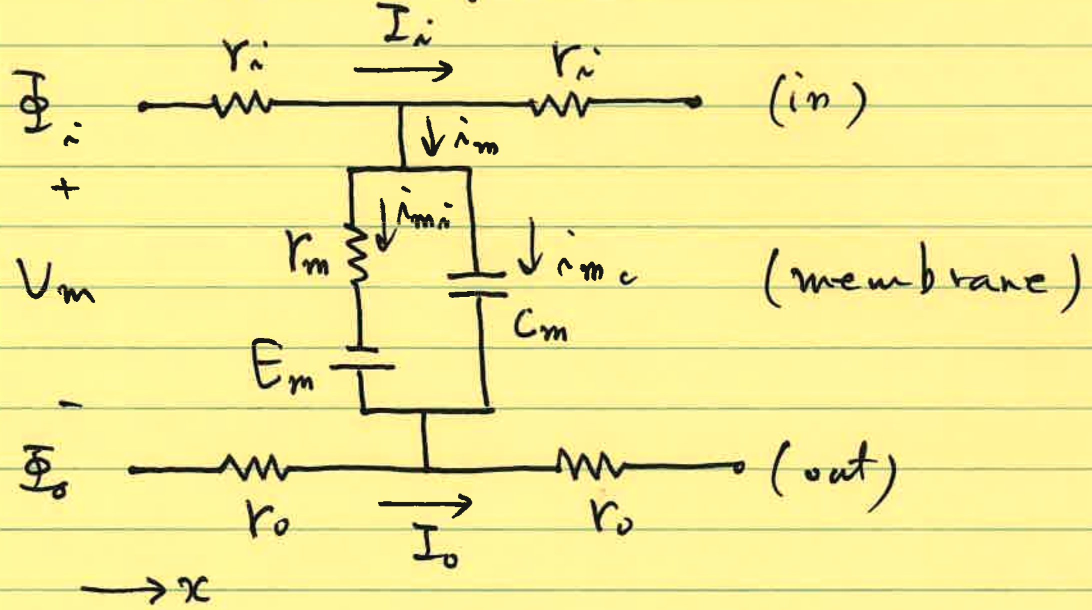
* Cable Equation



r_i : intracellular axial resistance per unit length [Ω/m]

r_o : extracellular " [Ω/m]

r_m : membrane resistance times unit length [$\Omega \cdot$m] in radial direction

c_m : membrane capacitance per unit length [F/m]

I_i : total longitudinal intracellular current [A]

I_o : " " extracellular " [A]

i_m : total transmembrane current per unit length [A/m]

i_{mc} : capacitive component of i_m

i_{mi} : ionic "

Φ_i : potential inside membrane [V]

Φ_o : " outside " [V]

$V_m = \Phi_i - \Phi_o$: membrane voltage [V]

V_r : resting membrane voltage [V]

$V' = V_m - V_r$: deviation of membrane voltage from V_r

Without stimulating current, $I_i + I_o = 0$

At resting state, $I_i = I_o = I_m = 0$, $V_m = V_r$, $V' = 0$

During activation, $I_i + I_o = 0$, $V' \neq 0$

Since V_r is constant for all x,

$$\frac{\partial V'}{\partial x} = \frac{\partial V_m}{\partial x}, \quad \frac{\partial V'}{\partial t} = \frac{\partial V_m}{\partial t}$$

(a) In steady state, $\frac{\partial}{\partial t} = 0$.

$$\frac{\partial \Phi_i}{\partial x} = -I_i r_i, \quad \frac{\partial \Phi_o}{\partial x} = -I_o r_o$$

$$i_m = -\frac{\partial I_i}{\partial x} = \frac{\partial I_o}{\partial x}$$

(loss of I_i) (gain of I_o)

$$\frac{\partial V'}{\partial x} = \frac{\partial \Phi_i}{\partial x} - \frac{\partial \Phi_o}{\partial x} = -I_i r_i + I_o r_o$$

$$\frac{\partial^2 V'}{\partial x^2} = -r_i \underbrace{\frac{\partial I_i}{\partial x}}_{-i_m} + r_o \underbrace{\frac{\partial I_o}{\partial x}}_{i_m} = (r_i + r_o)\, i_m$$

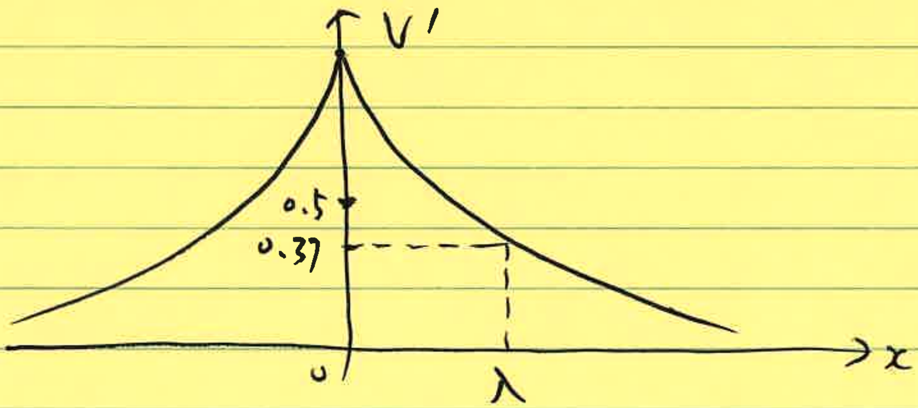
Since $C_m \frac{dV'}{dt} = 0$, $i_m = \frac{V'}{r_m}$

$$\frac{\partial^2 V'}{\partial x^2} = \frac{r_i + r_o}{r_m} V'$$

$$\frac{\partial^2 V'}{\partial x^2} - \lambda^{-2} V' = 0 , \quad \lambda = \sqrt{\frac{r_m}{r_i + r_o}} \simeq \sqrt{\frac{r_m}{r_i}}$$

$$V'_{x=0} = V'(0) , \quad V'_{x=\infty} = 0$$

$$V'(x) = V'(0)\, e^{-\frac{x}{\lambda}}$$



(a) Step-current stimulation

$$i_m = i_{mi} + i_{mc}$$

$$\frac{1}{r_i + r_o} \frac{\partial^2 V'}{\partial x^2} = \frac{V'}{r_m} + c_m \frac{\partial V'}{\partial t}$$

$$\frac{r_m}{r_i + r_o} \frac{\partial^2 V'}{\partial x^2} - V' - r_m c_m \frac{\partial V'}{\partial t} = 0$$

$$-\lambda^2 \frac{\partial^2 V'}{\partial x^2} + \tau \frac{\partial V'}{\partial t} + V' = 0$$