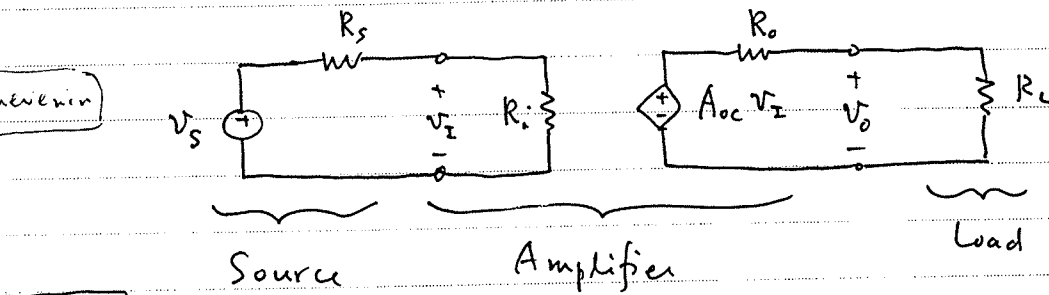
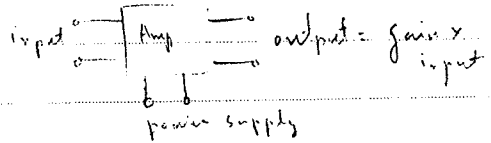


Chapter 1 Operational Amplifier Fundamentals

1.1 Amplifier Fundamentals

(Linear Amplifier)

* Voltage Amplifier



VCVS

- R_i : input resistance
- R_o : output resistance
- v_I : input voltage
- v_o : output voltage
- A_{oc} : Voltage gain (unloaded or open-circuit)

$$v_o = \frac{R_L}{R_o + R_L} A_{oc} v_I, \quad v_I = \frac{R_i}{R_s + R_i} v_s$$

dc transfer function (no load on output)

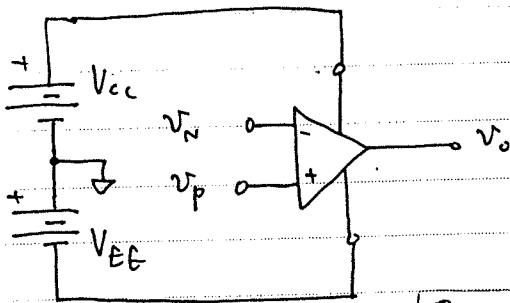
$$\frac{v_o}{v_s} = \underbrace{\frac{R_i}{R_s + R_i}}_{\text{loading}} A_{oc} \underbrace{\frac{R_L}{R_o + R_L}}_{\text{loading}} \quad \text{: Source-to-load gain}$$

$$\therefore \left| \frac{v_o}{v_s} \right| \leq |A_{oc}|$$

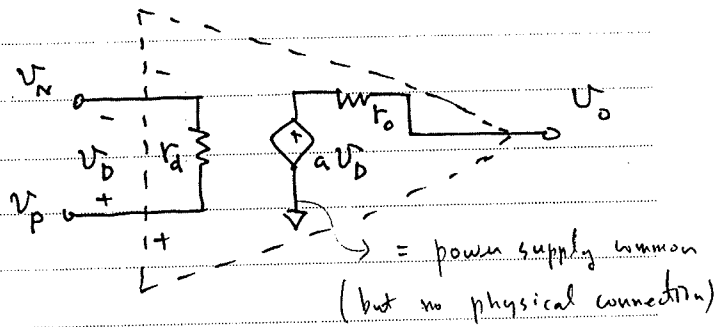
(loading effect only)

- $\left\{ \begin{array}{l} R_i = \infty \text{ and } R_o = 0 \Rightarrow \text{no loading (ideal amplifier)} \\ R_i \gg R_s \text{ and } R_o \ll R_L \Rightarrow \text{minimal loading} \end{array} \right.$

1.2 The Operational Amplifier : high-gain voltage amplifier (dc amp)



[Fig. 1.3]



- v_N : inverting input
 - v_p : non-inverting input
 - V_{cc} and V_{ee} : supply voltage
 - r_d : differential input resistance
 - a : voltage gain (unloaded)
 - r_o : output resistance
- } open-loop parameters

$v_D = v_p - v_N$: differential input voltage

$v_o = a v_D = a (v_p - v_N)$: open-circuit output voltage

- input : double-ended or differential
- output : single-ended

Op amp { differential amplifier
dc amplifier

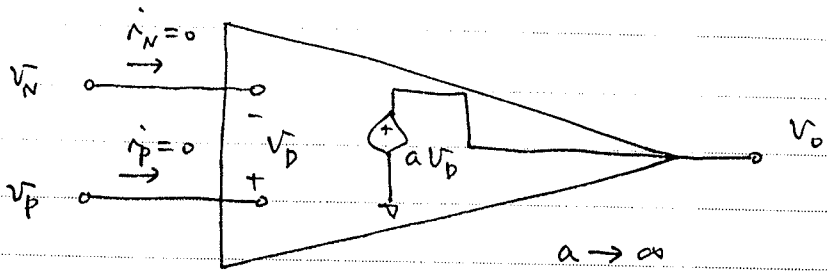
* Ideal Op Amp.

$a \rightarrow \infty$

$r_o = 0$

$r_d = \infty$

$i_p = i_N = 0$

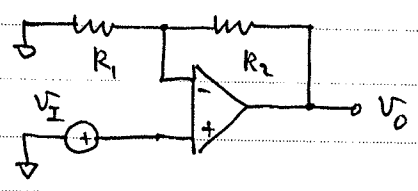


$$\lim_{a \rightarrow \infty} v_O = \lim_{a \rightarrow \infty} a v_D \quad (= \text{some constant})$$

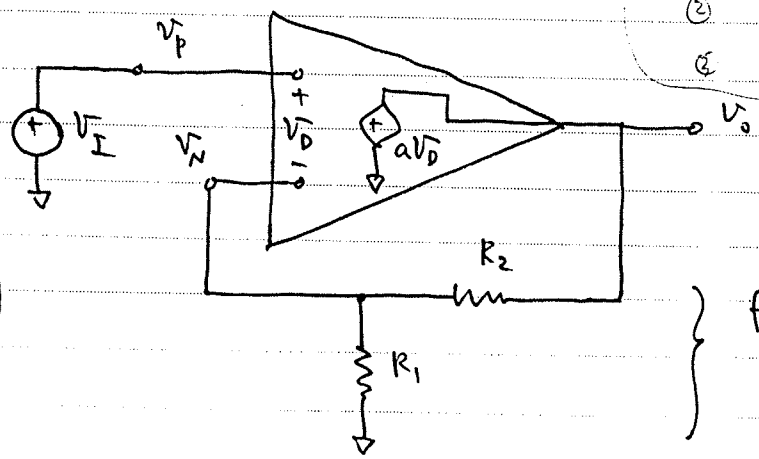
(as $a \rightarrow \infty, v_D \rightarrow 0$)

1.3 Basic Op Amp. Considerations [op amp op amp circuit]

* Noninverting Amplifier



- * Ideal Op amp. circuit with negative feedback for linear amplification.
- ① $v_N = v_P$
- ② $i_N = i_P = 0$
- ③ output saturation.



$R_d = \infty$
 $R_i = \infty$

feedback network

$$v_P = v_N = \frac{R_1}{R_1 + R_2} v_O = \frac{1}{1 + R_2/R_1} v_O$$

$$v_o = a(v_p - v_n) = a \left(v_I - \frac{1}{1 + R_2/R_1} v_o \right)$$

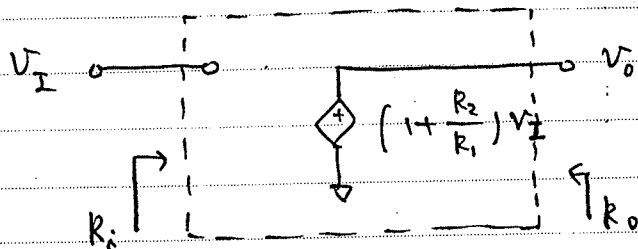
$$A = \frac{v_o}{v_I} = \left(1 + \frac{R_2}{R_1} \right) \frac{1}{1 + (1 + R_2/R_1)/a}$$

$\left\{ \begin{array}{l} a = \text{open-loop gain} \\ A = \text{closed-loop gain} \end{array} \right.$

Example 1.2 $\hat{=} 2.11$

$$A_{\text{ideal}} = \lim_{a \rightarrow \infty} A = 1 + \frac{R_2}{R_1}$$

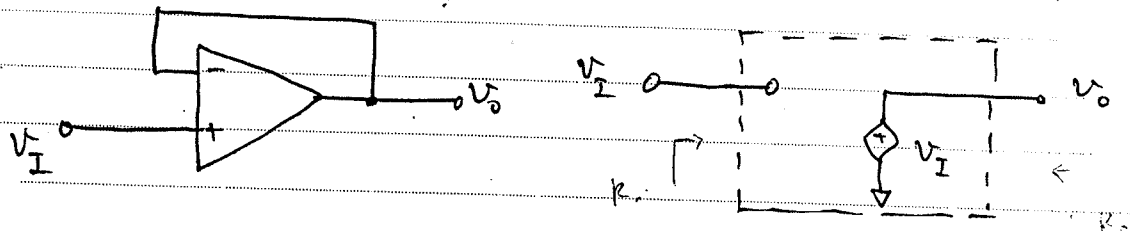
\Rightarrow "Accurate and stable gain by external components only."



$\left\{ \begin{array}{l} R_i = \infty : \text{closed-loop input resistance} \\ R_o = 0 : \text{" " output "} \end{array} \right.$

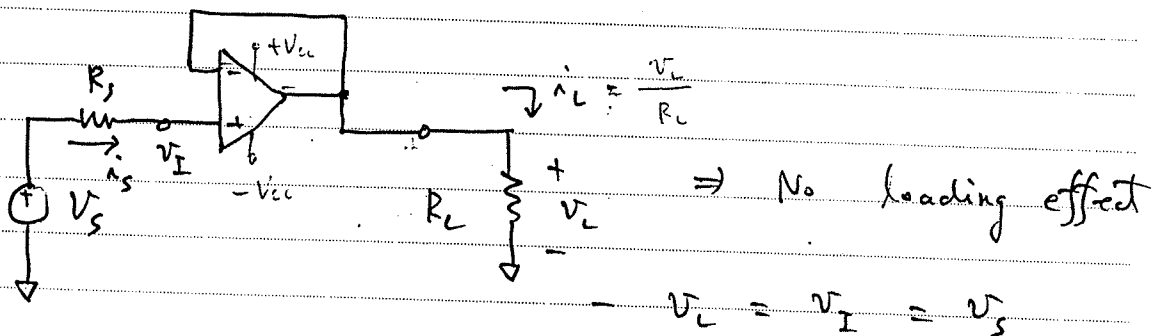
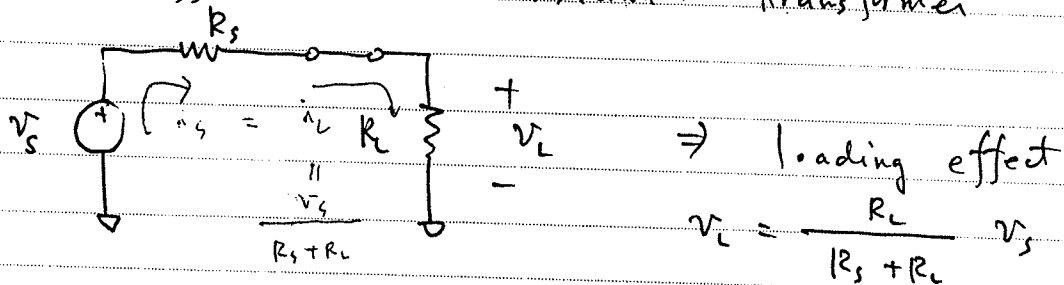
\Rightarrow We need $a \rightarrow \infty$ to get good performances of op amp circuit.

* Voltage follower (Unity-gain Amplifier)



$$R_i = \infty, R_o = 0, A = 1$$

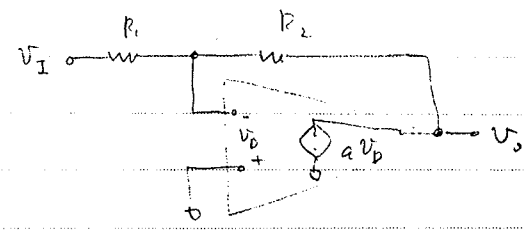
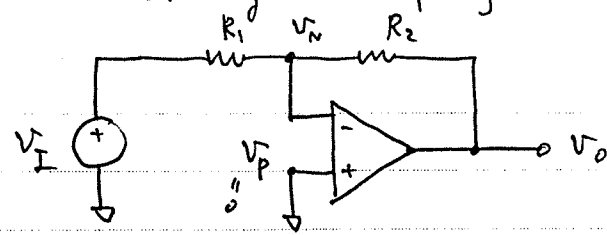
⇒ Buffer or resistance transformer



- $i_s = 0$

- i_L is supplied by power supply of op-amp circuit.

* Inverting Amplifier

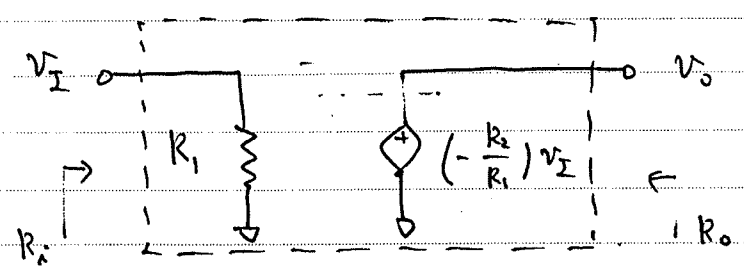


$$v_N = \frac{R_2}{R_1 + R_2} v_I + \frac{R_1}{R_1 + R_2} v_o \quad (\text{Superposition})$$

$$v_o = a(v_P - v_N) = a \left(-\frac{1}{1 + R_1/R_2} v_I - \frac{1}{1 + R_2/R_1} v_o \right)$$

$$A = \frac{v_o}{v_I} = -\frac{R_2}{R_1} \frac{1}{1 + (1 + R_2/R_1)/a}$$

$$A_{ideal} = \lim_{a \rightarrow \infty} A = -\frac{R_2}{R_1}$$

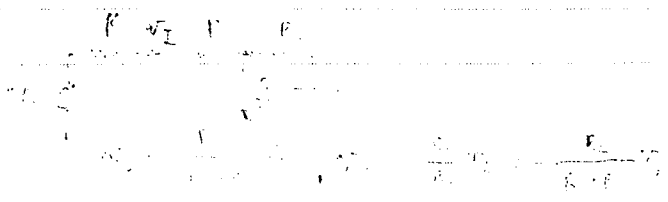


$$R_i = R_1, \quad R_o = 0$$

- $v_P = 0 \Rightarrow \lim_{a \rightarrow \infty} v_N = 0$: virtual ground ↗ otherwise $v_o \rightarrow \infty$

- Loading effect at input ($\because R_i = R_1$)

Fig 1.12



1.4 Ideal Op Amp Circuit Analysis

- Input voltage constraint : $V_p = V_n$
- Input current constraint : $i_p = i_n = 0$

↑
Negative feedback

3.7.21 3.7.22
3.7.23

→ $\left(\lim_{a \rightarrow \infty} V_n = V_p \quad ; \quad V_n \text{ follow } V_p \text{ by negative feedback} \right)$
↘ virtual short

* Summing Amplifier (Adder)

Fig 1.15

$R_{ik} = R_k, \quad R_o = 0$

Example 1.5

(Fig 1.16) ↙
↖

* Difference Amplifier (Subtractor)

Fig 1.17

$R_{i1} = R_1, \quad R_{i2} = R_3 + R_4$

(use Superposition)

$v_o = \frac{R_2}{R_1} \left(\frac{1 + R_1/R_2}{1 + R_3/R_4} v_2 - v_1 \right) \quad R_o = 0$

$v_{o1} = -\frac{R_2}{R_1} v_1$

$v_{o2} = \left(1 + \frac{R_2}{R_1} \right) \frac{v_2}{\frac{R_3 + R_4}{R_2}}$

$\frac{R_3}{R_4} = \frac{R_1}{R_2} \Rightarrow$

Balanced bridge

$v_o = \frac{R_2}{R_1} (v_2 - v_1)$

Example 1.6

* Differentiator

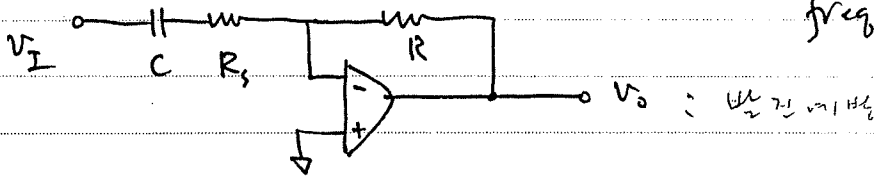
Fig 1.18

\Rightarrow may oscillate (unstable)

$v_o(t) = RC \frac{dv_i(t)}{dt}$

$v_o(s) = -RCs v_i(s)$

- Stabilized differentiator over a limited frequency range.



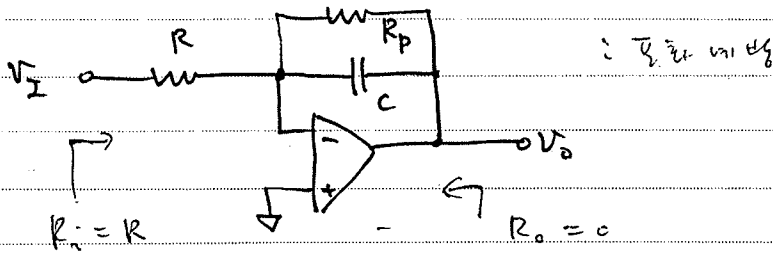
* Integrator

$$v_{out} = -\frac{1}{R_c} \int v_I(t) dt + v_o(0)$$

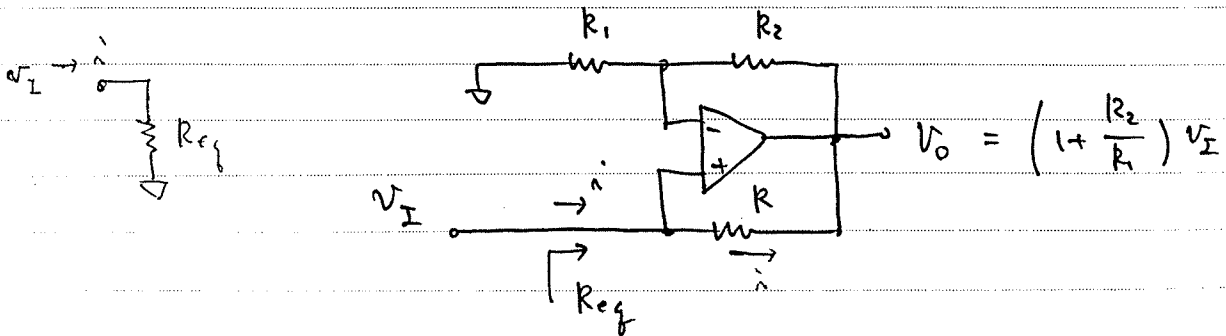
Fig 1.19

$R_i = R, R_o = 0$
 → may saturate (even with $v_I = 0$)

- Lossy integrator over a limited freq. range.



* Negative-Resistance Converter (NIC)



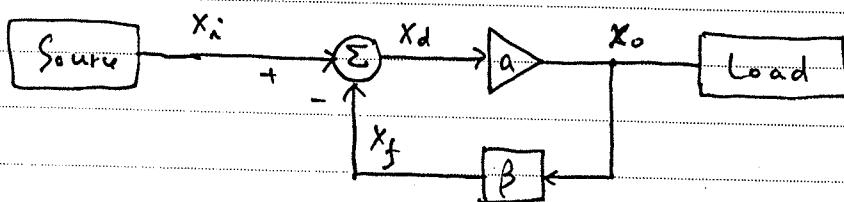
$$v_o = \left(1 + \frac{R_2}{R}\right) v_I$$

$$i = \frac{v_I - v_o}{R} = -\frac{R_2}{R_1 R} v_I$$

$$R_{eq} = \frac{v_I}{i} = -\frac{R_1}{R_2} R \quad \text{: negative resistance}$$

- Current flows into the source (v_I)
- Neutralize unwanted resistance
- Control pole location

1.5 Negative feedback : degenerative (\neq regenerative)



- Amplifier or error amplifier : $x_o = a x_d$
- feedback network : $x_f = \beta x_o$
- Summing network : $x_d = x_i - x_f$

$\left\{ \begin{array}{l} a : \text{open-loop gain, forward gain} \\ \beta : \text{feedback factor} \\ x_d : \text{error signal} \\ x_i : \text{input} \\ x_o : \text{output} \end{array} \right.$

$A =$ closed-loop gain

$$\begin{aligned}
 1_o = a x_d &= \frac{x_o}{x_i} \\
 &= a (x_i - x_f) \\
 &= a (x_i - \beta x_o) \\
 1 + a\beta x_o = ax_i &= \frac{ax_i}{1 + a\beta}
 \end{aligned}$$

$a\beta > 0 \Leftrightarrow$ negative feedback $\Leftrightarrow A < a$

$1 + a\beta$: amount of feedback
 $T = a\beta$ = return ratio or loop gain

$$A = \frac{1}{\beta} \frac{T}{1+T}$$

$$A_{ideal} = \lim_{T \rightarrow \infty} A = \frac{1}{\beta}$$

$$A = A_{ideal} \cdot \frac{1}{1 + 1/T} \quad \uparrow$$

$$\text{Error function} = \frac{1}{1 + 1/T} = 1 - \frac{1}{1+T} = 1 - \varepsilon$$

$$A = A_{ideal} (1 - \varepsilon)$$

$$\varepsilon = \frac{1}{1+T} \quad : \quad \text{fractional deviation of } A \text{ from } A_{ideal}$$

$$\text{Gain error (\%)} = 100 \times \frac{A - A_{ideal}}{A_{ideal}} \approx -\frac{100}{T} \quad \text{for } T \gg 1$$

Example 1.7 $\frac{A}{\beta} x_i$

$$\left\{ \begin{aligned} x_d &= \frac{x_o}{a} = \frac{Ax_i}{a} = \frac{A}{a} x_i = \frac{1}{1+T} x_i \Rightarrow x_d \rightarrow 0 \text{ as } T \rightarrow \infty \\ x_f &= \beta x_o = \beta Ax_i = \frac{1}{1 + \frac{1}{T}} x_i \Rightarrow x_f \rightarrow x_i \text{ as } T \rightarrow \infty \end{aligned} \right.$$

\Rightarrow Op amp in in $x_f = x_o$, $x_d \rightarrow 0$ as $T \rightarrow \infty$

* Gain Desensitivity

$$A = \frac{a}{1 + a\beta}$$

$$\frac{dA}{da} = \frac{1}{(1 + a\beta)^2}, \quad 1 + a\beta = \frac{a}{A}$$

$$\frac{dA}{A} = \frac{1}{1 + T} \frac{da}{a}$$

$$\Rightarrow 100 \frac{\Delta A}{A} \approx \frac{1}{1 + T} \left(100 \frac{\Delta a}{a} \right)$$

1 + T : desensitivity factor

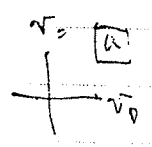
Similarity,

$$100 \frac{\Delta A}{A} \approx -100 \frac{\Delta \beta}{\beta}$$

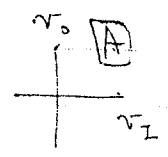
Example 1.8 $f_{2.211}$

* Nonlinear Distortion Reduction

for a linear amplifier, $x_o = a x_d$ or $x_o = Ax_i$
 In general, $a = \frac{dx_o}{dx_d}$ or $A = \frac{dx_o}{dx_i}$



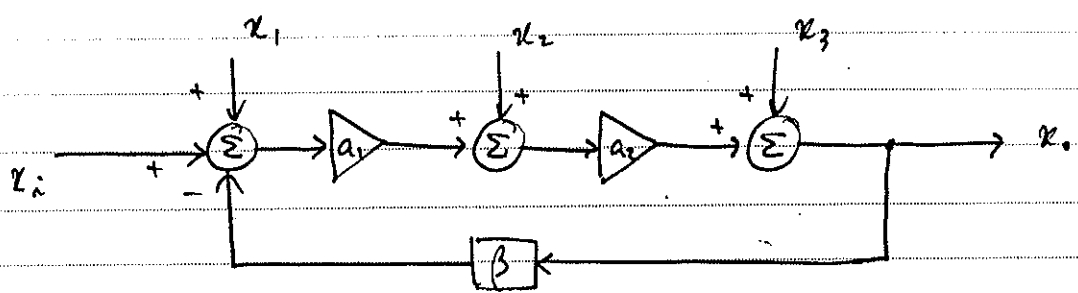
VTC : voltage transfer curve (Fig 1.23)



< a : non linear and saturation
 A : linear and "

Fig. 1.20 y_2 scale $y_{2.1}$

* Effect of Feedback on Disturbances and Noise



Examples :

- x_1 : input noise or input offset error
- x_2 : power-supply hum
- x_3 : output noise or load changes

$$x_0 = x_3 + a_2 \left[x_2 + a_1 (x_i - \beta x_0 + x_1) \right]$$

$$\Rightarrow x_0 = \frac{a_1 a_2}{1 + a_1 a_2 \beta} \left(x_i + x_1 + \frac{x_2}{a_1} + \frac{x_3}{a_1 a_2} \right)$$

noise reduction

For $a_1 a_2 \beta \gg 1$,

$$x_0 = \frac{1}{\beta} \left(x_i + x_1 + \frac{x_2}{a_1} + \frac{x_3}{a_1 a_2} \right)$$

(noise gain = $\frac{1}{\beta}$)
 → gain for input noise x_1

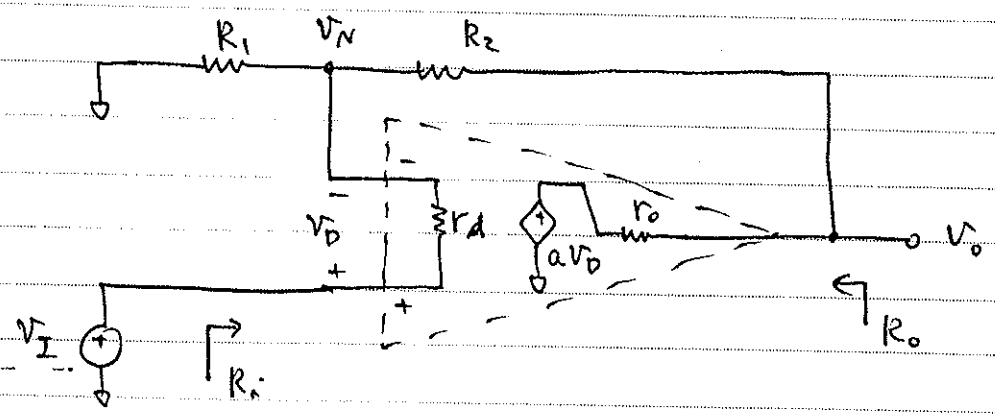
* $x_1 = \dots$

1.6 Feedback in Op Amp Circuits

input (Summing)	output (Sampling)
Series ($R_i \uparrow$)	Series ($R_o \uparrow$)
Shunt ($R_i \downarrow$)	Shunt ($R_o \downarrow$)

Fig 1.2.6

* Noninverting Configuration



Topology : Series - shunt

$$\frac{V_I - V_N}{r_d} - \frac{V_N}{R_1} + \frac{V_o - V_N}{R_2} = 0 \quad ; \text{KCL at } N$$

$$\frac{V_N - V_o}{R_2} + \frac{a(V_I - V_N) - V_o}{r_o} = 0 \quad ; \text{KCL at } o$$

$$\Rightarrow A = \frac{V_o}{V_I} = \frac{(1 + R_2/R_1)a + r_o/r_d}{1 + a + R_2/R_1 + (R_2 + r_o)/r_d + r_o/R_1}$$

$$\approx \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + 1/T}$$

with $\beta = \frac{R_1}{R_1 + R_2}$

Finding R_i : Fig 1.28 (a)

$$R_i = r_d \left(1 + \frac{a}{1 + (R_2 + r_o)/R_1} \right) + R_1 \parallel (R_2 + r_o)$$

for large a and $r_o \ll R_2$,

$$R_i \approx r_d (1 + a\beta) = r_d (1 + T)$$

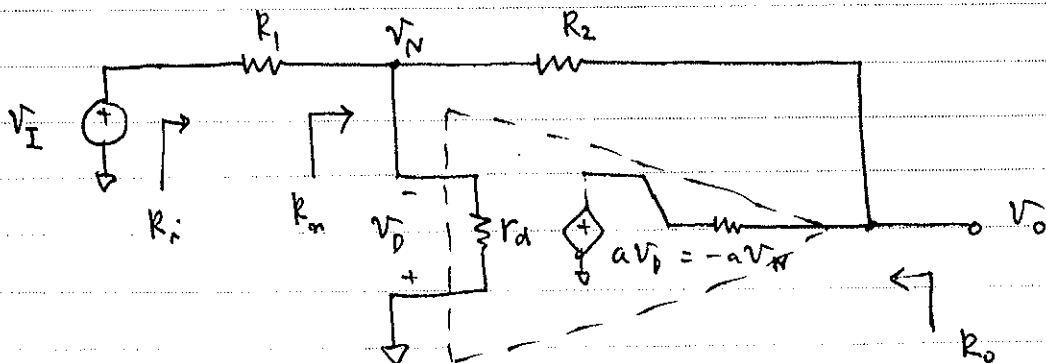
Finding R_o : Fig 1.28 (b)

$$R_o = \frac{r_o}{1 + (a + r_o/R_1 + r_o/r_d) / (1 + R_2/R_1 + R_2/r_d)}$$

for r_d in $\mu\Omega$, R_1 & R_2 in $k\Omega$
 r_o in Ω

$$R_o \approx \frac{r_o}{1 + T}$$

* Inverting Configuration



$$A = \frac{a R_2 - r_o}{(1+a)R_1 + (R_2 + r_o)(1 + R_1/r_d)}$$

for $r_o \ll R_2$ and $R_1/r_d \ll 1$,

$$A \approx \frac{R_2}{R_1} \frac{1}{1 + 1/T}$$

$$\text{with } \beta = \frac{R_1}{R_1 + R_2}$$

(Note that $A_{ideal} = \frac{1}{\beta}$)

Finding R_i : Fig 1.30

$$R_i = R_1 + R_m$$

$$= R_1 + \frac{R_2 + r_o}{1 + a + (R_2 + r_o)/r_d}$$

$$\approx R_1$$

Finding R_o : Fig 1.29 \rightarrow Fig 1.28(b)

$$R_o \approx \frac{r_o}{1 + T}$$

Example 1.10

\Rightarrow "Ideal op amp assumption is valid for most cases"

1.7 Loop Gain, $T = a\beta$

$$A = A_{ideal} \times \frac{1}{1 + 1/T}$$
$$R \approx r \times (1 + T)^{\pm 1}$$

$$A_{ideal} = \frac{1}{\beta} \quad ; \text{ not always true}$$

* Finding T Directly

- ① Suppress all input sources
- ② Break the loop
- ③ Inject a test signal V_T
- ④ Find the return signal $V_R = a\beta \times (-1) \times V_T$

$$\Rightarrow T = - \frac{V_R}{V_T} \Big|_{x_2=0}$$

See Fig 1.31 \Rightarrow Eq (1.73)

* Finding β

- ① Suppress all input sources
- ② Disconnect the op amp
- ③ Replace the op amp with r_d and r_o
- ④ Apply a test source V_T via r_o
- ⑤ Find V_D across r_d

$$\Rightarrow \beta = - \frac{V_D}{V_T} \Big|_{x_2=0}$$

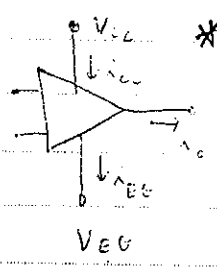
See fig 1.33 \Rightarrow Eq (1.25)

1.8 Op Amp Powering

or Decoupling Capacitor

Fig 1.36 Bypass capacitor : ac noise \approx shunt out
 { 0.1 μ F ceramic or low-inductance type
 10 μ F polarized capacitor : Board level
 * bypass

** i_o i_{cc} i_{EE} I_Q I_{CC} I_{EE}



* Current flow and Power Dissipation

$$\left\{ \begin{array}{l} i_o \\ i_{cc} \\ i_{EE} \end{array} \right\}$$

See fig 1.37
 $i_{cc} = i_{EE} + i_o$

$i_o = 0$ $i_{cc} = i_{EE} = I_Q \sim \text{mA}$ (10% of op amp)

- { op amp is sourcing $\Rightarrow i_{cc} = i_{EE} + i_o$
- { op amp is sinking $\Rightarrow i_{EE} = i_{cc} + i_o$

$$I_Q = i_{cc} |_{i_o=0} = i_{EE} |_{i_o=0}$$

\hookrightarrow quiescent supply current $\sim \text{mA}$

micropower op amp : $I_Q \sim \text{in } \mu\text{A}$

power dissipation due to i_o and I_Q
 = internal power dissipation

\hookrightarrow check against max. rating in datasheet

⑨ Example 1.14 : needs to be explained.

Example
* Output Saturation

- See Fig 1.39

$$\begin{cases} +V_{sat} = V_{OH} \approx V_{CC} - 2 \text{ [V]} \\ -V_{sat} = V_{OL} \approx V_{EE} + 2 \text{ [V]} \end{cases}$$

- rail-to-rail op amp

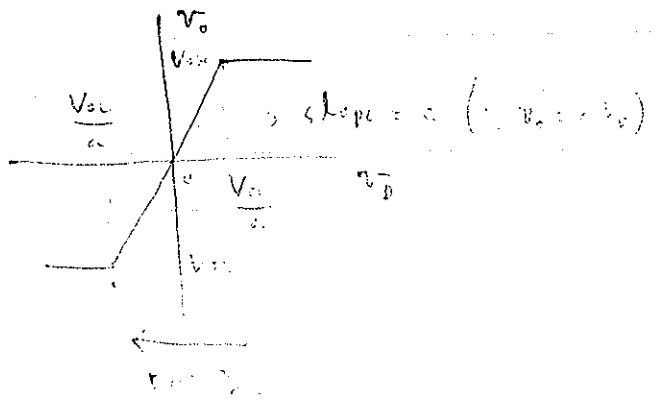
$$\begin{cases} V_{OH} = V_{CC} \\ V_{OL} = V_{EE} \end{cases}$$

- $V_{OL} < v_o < V_{OH} \Rightarrow$ op amp is in linear region

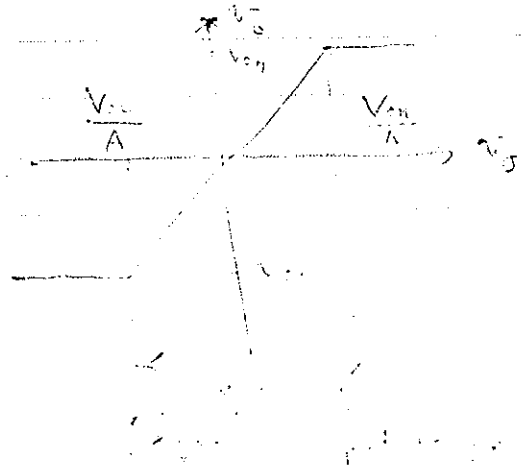
- Clipping : Fig 1.40

* Input range, A , V_{OH} , $V_{OL} \Rightarrow$ output dynamic range (output swing)

* open loop gain, a



* closed loop gain, A



Example 1.16 \hat{z}_{2011}

or \hat{z}_{2011}

Prob; Ideal op amp. with negative feedback for linear operation

- ① $v_p = v_n$
- ② $i_p = i_n = 0$
- ③ Saturation.