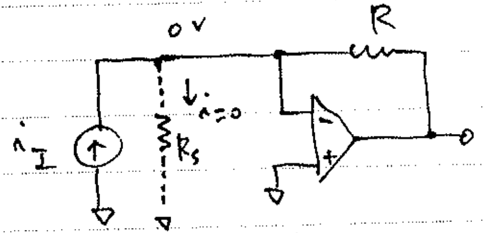
Chapter 2 Circuits with Resistive Feedback

2.1 Current-to-Voltage Converter (I-V Converter)

- Transresistance amplifier : $v_O = A\, i_I$



$v_o = -R\, i_I$

(shunt-shunt topology)

no loading at input

$A = -R$: gain or sensitivity (V/mA or V/μA)

In general, $V_o(s) = -Z(s)\, I_i(s)$
⇒ transimpedance amplifier

* closed-loop parameters

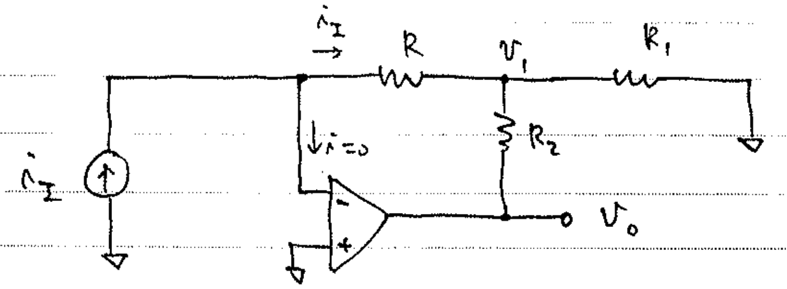
$$T = \frac{a r_d}{r_d + R + r_o}$$

$$A = -R \frac{1}{1 + 1/T}$$

$$R_i = \frac{r_d \parallel (R + r_o)}{1 + T}$$

$$R_o \simeq \frac{r_o}{1 + T}$$

* High-Sensitivity I-V Converter



KCL at node 1,

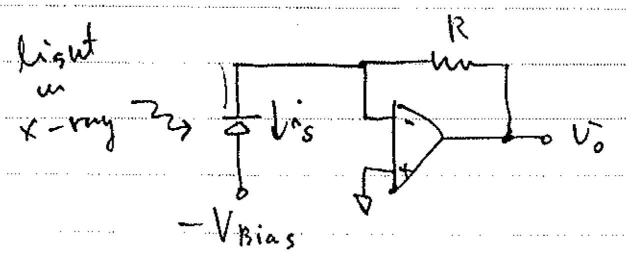
$$\frac{v_1}{R} + \frac{v_1}{R_1} + \frac{(v_1 - v_o)}{R_2} = 0$$

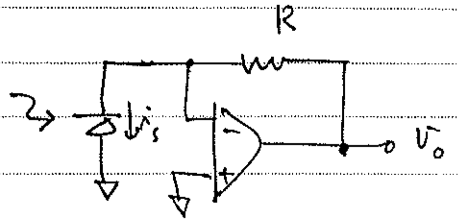
$$v_1 = -R i_I$$

$$\therefore\ v_o = -\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R}\right) R i_I = -kR i_I$$

⇒ Use a low input bias current type Op amp.
(JFET-input or MOSFET-input Op amp)

Si photodiode * Photodetector Amplifier





< photoconductive >
- high speed
- detection

< photovoltaic >
- low noise
- measurement.

2.2 Voltage-to-Current Converter

- V-I converter
- Transconductance amplifier : $i_o = A v_I$

In general, $i_o = A v_I - \frac{1}{R_o} v_L$

R_o : output resistance
v_L : load voltage

$= \frac{v_L}{R_o} + i_o$

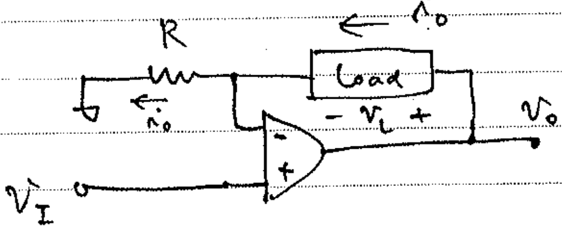
$\Rightarrow$ We want $R_o \to \infty$

Voltage compliance : range of permissible v_L (before saturation)

Types of load: floating, grounded

* Floating-Load Converter

- lower voltage compliance
- i_o supplied from power supply



$v_I = R i_o \Rightarrow i_o = \frac{v_I}{R}$

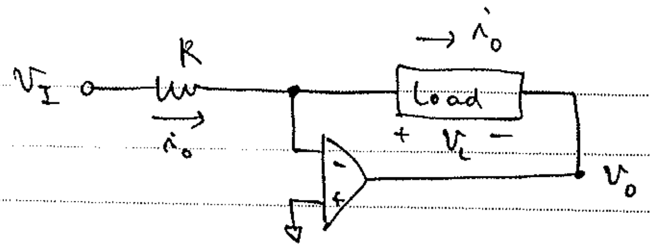
$v_o = v_I + v_L$, $V_{OL} < v_o < V_{OH}$

$\Rightarrow (V_{OL} - v_I) < v_L < (V_{OH} - v_I)$: voltage compliance

Ex) $v_I = 5$ V , $R = 5$ kΩ $\Rightarrow i_o = 1$ mA

$(V_{OL} - 5) < v_L < (V_{OH} - 5)$

$\Rightarrow$ limits the value of load resistance



$i_o = \frac{V_I}{R}$, $V_o = -V_L \Rightarrow V_{OL} < V_L < V_{OH}$

- wider voltage compliance
- i_o supplied by V_I (source)

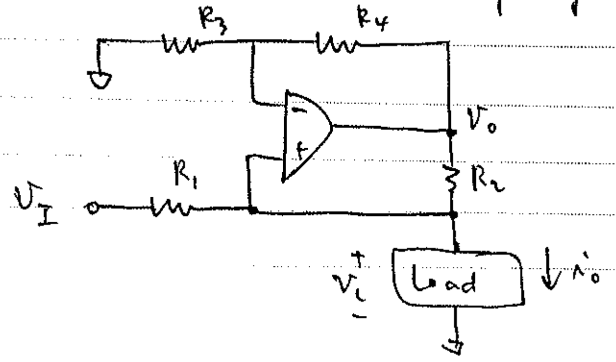
⑨ Maximum current is always limited by the op amp.
(Ex. 741 ⇒ max i_o = 25 mA)

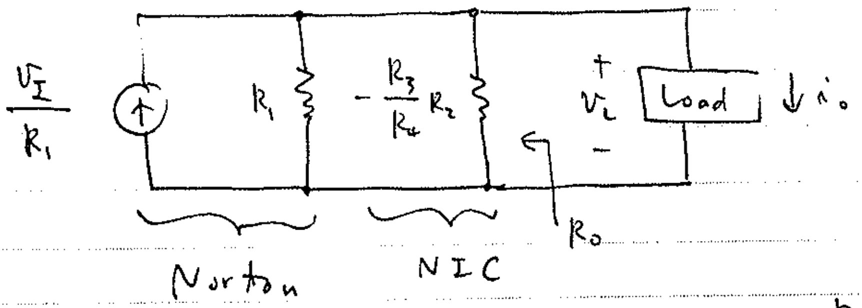
power op amp
Output current booster } ⇒ larger i_o

θ Practical op amp limitation ⇒ see Eq. 2.7

* Grounded-Load Converter

< Howland current pump >





$$R_o = R_1 \,//\, \left(-\frac{R_3}{R_4}R_2\right) = \frac{R_2}{R_2/R_1 - R_4/R_3}$$

If $\frac{R_4}{R_3} = \frac{R_2}{R_1}$, $R_o \to \infty$ and $i_o = \frac{V_I}{R_1}$

$$v_L = v_P = V_N = \frac{R_3}{R_3 + R_4} v_o = \frac{R_1}{R_1 + R_2} v_o$$

$$\Rightarrow \quad |v_L| \le \frac{R_1}{R_1 + R_2} V_{sat}$$

Usually, $R_2 \simeq 0.1 R_1$ for wider voltage compliance ↑

$\boxed{\text{Example 2.4}}$ Fig 2.8

◎ Effect of Resistance Mismatch

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}(1 - \varepsilon)$$

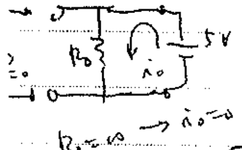
where ε is the imbalance factor.

$$\Rightarrow \quad R_o = \frac{R_2}{R_2/R_1 - R_4/R_3} = \frac{R_1}{\varepsilon}$$

$\boxed{\text{Example 2.5}}$: 공부.

→ Use highly precise resistors or resistance trimming.
→ Trimming resistor must be avoided whenever possible.

◎ Howland circuit calibration : Fig. 2.9



→ Adjust R_{pot} so that $i_o=0$ for both cases.

Example 2.6 참고 : R_{pot} 의 결정 방법.

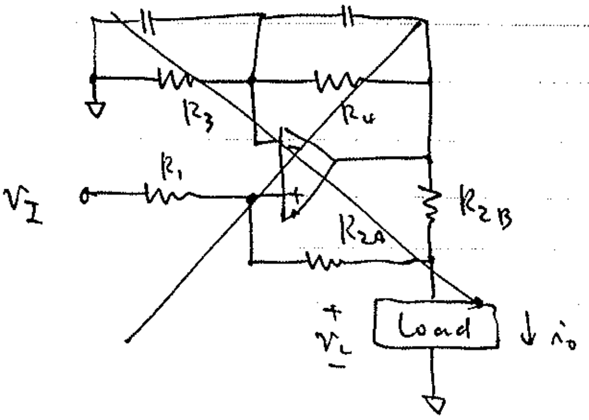
◎ Effect of Finite Open-Loop Gain

Assuming $R_2/R_1 = R_4/R_3$,

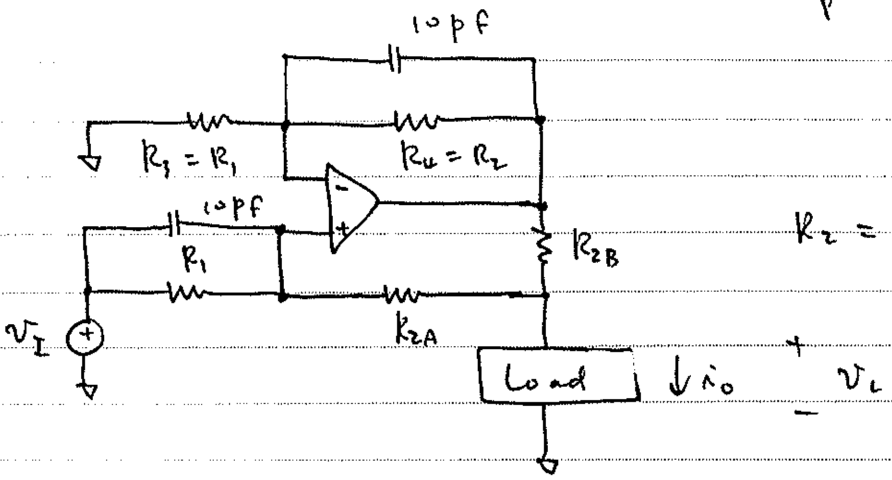
$$R_o = (R_1 \parallel R_2)\left(1 + \frac{a}{1+R_2/R_1}\right) \quad \cdots$$

$$i_o = \frac{v_I}{R_1} - \frac{v_L}{R_o}$$

◎ Improved Howland Current Pump



2]n1 10 pf $\Longrightarrow$ at high freq, negative feedback is dominant compared to positive feedback



$R_2 = R_{2A} + R_{2B}$

Balance condition :

$$\frac{R_4}{R_3} = \frac{R_{2A} + R_{2B}}{R_1} \Longrightarrow R_o = \infty$$

$$i_o = \frac{R_2/R_1}{R_{2B}} v_I$$

voltage compliance : $|v_L| \le |V_{sat}| - R_{2B}|i_o|$

or $|v_L| \le |V_{sat}| - (R_2/R_1)|v_I|$

Use large value for R_1, R_3, R_4, R_{2A}

$\Downarrow$

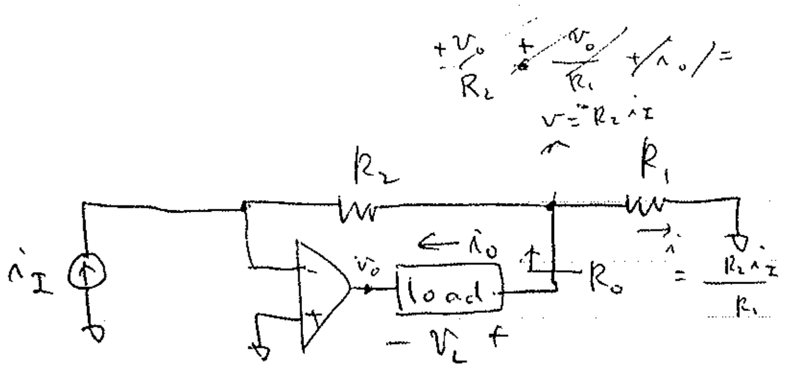
Smaller power consumption.

2.3 Current Amplifiers

$$i_o = A i_I - \frac{1}{R_o} v_L$$

We need $R_o \to \infty$ so that

$$i_o = A i_I$$



: floating load

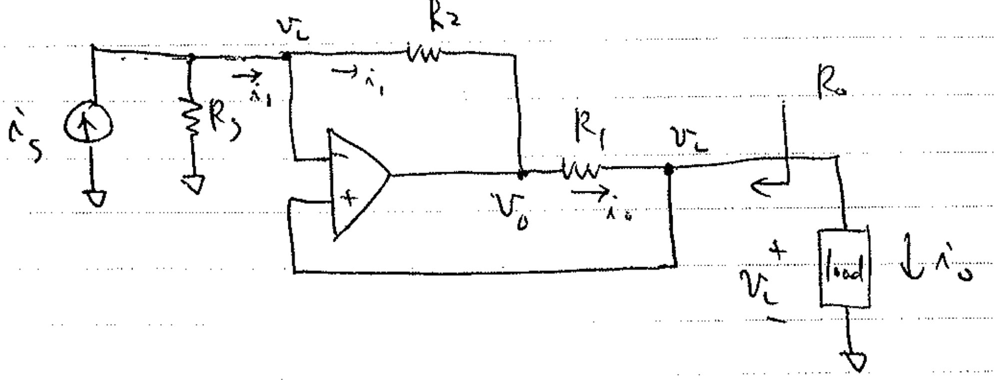
$$\left(\begin{array}{l} i_o = i_I + \frac{R_2 i_I}{R_1} = \left(1+\frac{R_2}{R_1}\right) i_I + \underset{\frac{v_L}{R_o}}{\underset{\shortparallel}{0}} \\ R_o \to \infty \end{array}\right.$$

$$-(V_{OH} + R_2 i_I) \le v_L \le -(V_{OL} + R_2 i_I)$$

$$(\because \quad v_L + R_2 i_I = v_o)$$

For a finite a,

$$A = 1 + \frac{R_2/R_1}{1 + 1/a}, \quad R_o = R_1(1+a)$$



: grounded load

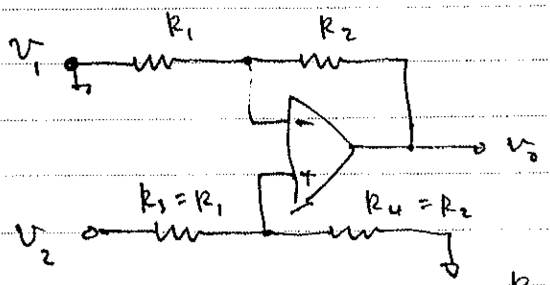
$$i_1 = i_S - \frac{v_L}{R_S}$$

$$v_o = v_L - R_2 i_1 = v_L - R_2\left(i_S - \frac{v_L}{R_S}\right)$$

$$i_o = \frac{v_o - v_L}{R_1} = \underset{A}{\underset{\shortparallel}{\left(-\frac{R_2}{R_1}\right)}} i_S - \frac{1}{R_o} v_L$$

$$A = -\frac{R_2}{R_1} \quad , \quad R_o = -\frac{R_1}{R_2} R_3$$

2.4 Difference Amplifier



$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

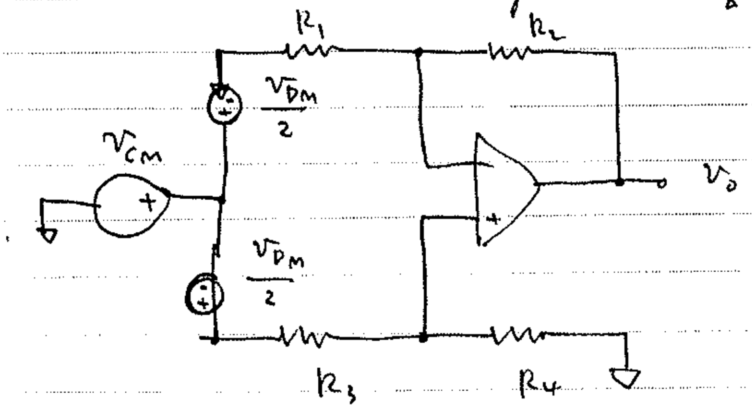
$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

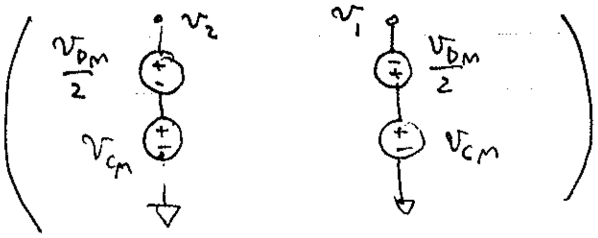
$v_{DM} = v_2 - v_1$: differential-mode input

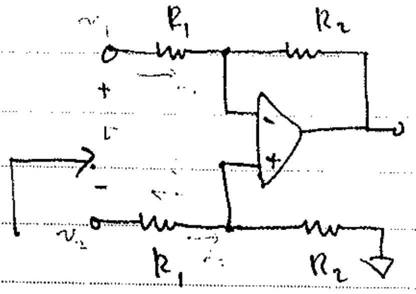
$v_{CM} = \frac{1}{2}(v_2 + v_1)$: common-mode input

$$\Rightarrow \quad v_1 = v_{CM} - \frac{v_{DM}}{2} \quad , \quad v_2 = v_{CM} + \frac{v_{DM}}{2}$$

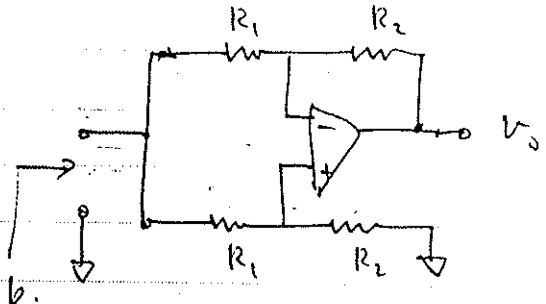
see fig 2.13 b





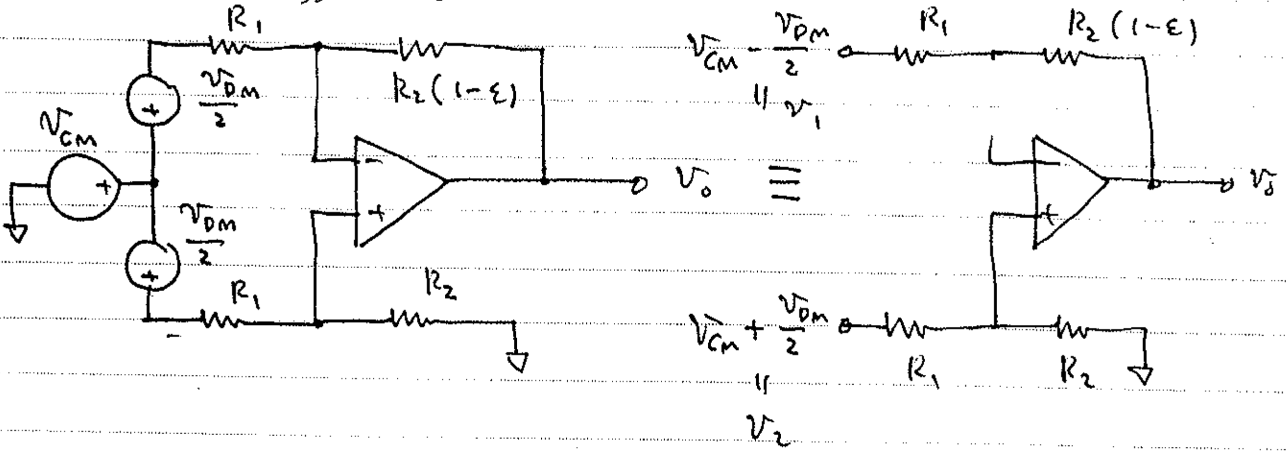


$R_{id} = 2R_1$



$R_{ic} = \frac{R_1+R_2}{2}$

* Effect of Resistance Mismatch



Using superposition,

$$v_o = -\frac{R_2(1-\varepsilon)}{R_1} v_1 + \left(1+\frac{R_2(1-\varepsilon)}{R_1}\right)\frac{R_2}{R_1+R_2} v_2$$

$$= -\frac{R_2(1-\varepsilon)}{R_1}\left(v_{CM}-\frac{v_{DM}}{2}\right) + \left\{1+\frac{R_2(1-\varepsilon)}{R_1}\right\}\frac{R_2}{R_1+R_2} \times \left(v_{CM}+\frac{v_{DM}}{2}\right)$$

$$\therefore \quad v_o = A_{dm} v_{DM} + A_{cm} v_{CM}$$

where
$$A_{dm} = \frac{R_2}{R_1}\left(1 - \underbrace{\frac{R_1 + 2R_2}{R_1 + R_2}\,\frac{\varepsilon}{2}}_{\hookrightarrow \text{ 무시가능 } (\varepsilon\text{이 작을 때})}\right)$$

$$A_{cm} = \frac{R_2}{R_1 + R_2}\,\varepsilon$$

{ A_{dm} : differential-mode gain
{ A_{cm} : common-mode gain

$$\lim_{\varepsilon \to 0} A_{dm} = \frac{R_2}{R_1}, \qquad \lim_{\varepsilon \to 0} A_{cm} = 0$$

$$\text{CMRR} = 20 \log_{10} \left| \frac{A_{dm}}{A_{cm}} \right| \quad (\text{dB})$$

↳ common-mode rejection ratio

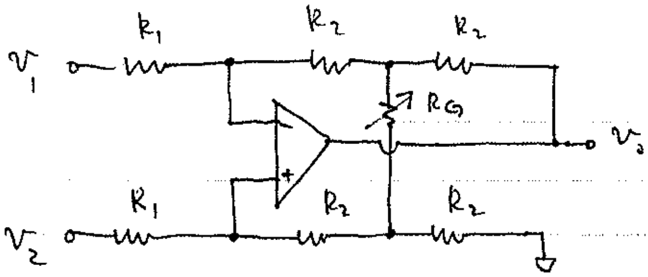
$$\lim_{\varepsilon \to 0} \text{CMRR} = \infty$$

for a small ε,
$$\text{CMRR} \simeq 20 \log_{10} \left| \frac{1 + R_2/R_1}{\varepsilon} \right|$$

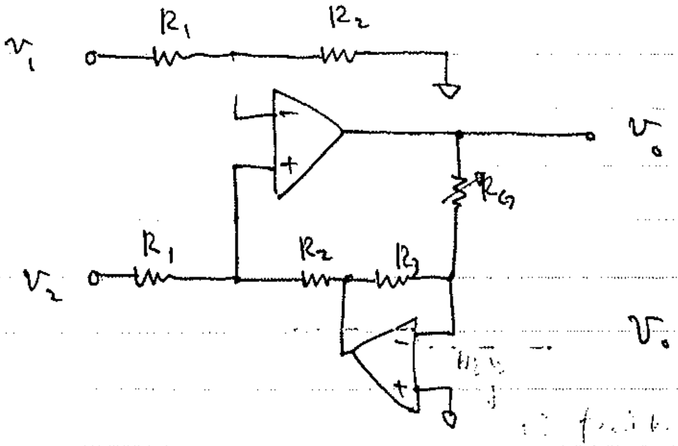
* Example 2.9 : 설명 필요.

{ Burr Brown INA105 : $\varepsilon < 0.002\%$, CMRR = 100dB
{ Calibration : Fig. 2.16
{ Resistor : metal film

* Variable Gain



$$v_o = \frac{2R_2}{R_1}\left(1+\frac{R_2}{R_G}\right)(v_2 - v_1)$$



$$v_o = \frac{R_2 R_G}{R_1 R_3}(v_2 - v_1)$$

* Ground-Loop Interference Elimination

Fig 2.19 (a)

v_g : ground-loop interference or
cross-talk for common return impedance

$$v_o = -\frac{R_2}{R_1}(v_i + v_g)$$

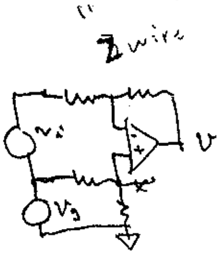
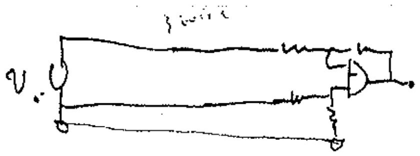


Fig 2.19 (b)

$$v_o = -\frac{R_2}{R_1} v_i$$



"double-ended transmission" or "balanced transmission"

2.5 Instrumentation Amplifier (IA)

① Extremely high R_{id} and R_{ic}
② Very low R_o
③ Accurate and stable gain (1 ~ 1000)
④ Extremely high CMRR

* Triple-Op-Amp IA

Fig 2.20 and 2.21

$$v_o = \left(1+2\frac{R_3}{R_G}\right)\frac{R_2}{R_1}(v_2 - v_1)$$

Example 2.10 → fig 2.21

One-chip IA : AD522, AD620
INA101, INA2128
(Gain control using external resistor R_G)
⇒ fig 2.22
⇒ Note "Sense" and "Reference" pins

* Dual-Op-Amp IA

Fig. 2.23

$$v_o = \left(1+\frac{R_2}{R_1}\right)(v_2 - v_1)$$

under $\frac{R_3}{R_4} = \frac{R_1}{R_2}$

Fig 2.24 : variable gain

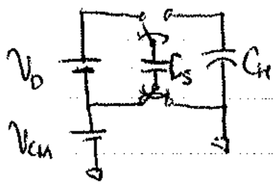
$$v_o = \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right)(v_2 - v_1)$$

[illegible] : delay for v_1 ≠ delay for v_2
⇒ do not use for high freq. signals

* Monolithic IA

fig 2.25 , fig 2.26

* flying - Capacitor Technique



2.6 Instrumentation Applications

* Active Guard

Double-ended transmission
OR
Balanced transmission

} ⇒ fig 2.28

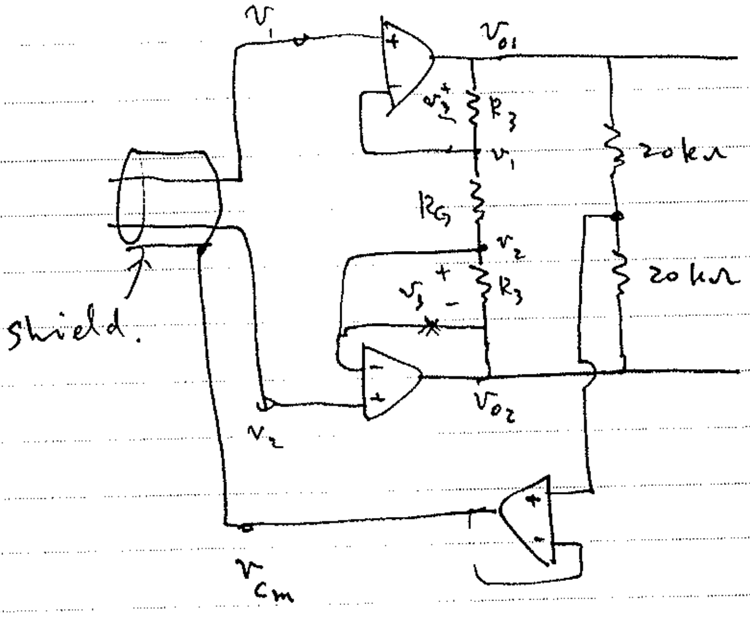
"RC imbalance" ⟶

$$CMRR \doteq 20 \log_{10} \frac{1}{2\pi f R_{dm} C_{cm}}$$

$$\begin{cases} R_{dm} = |R_{s1} - R_{s2}| \\ C_{cm} = \frac{1}{2}(C_1 + C_2) \end{cases}$$

↑

Solution : active guard. (Fig. 2.29)



[illegible] +, −

$$\therefore V_{cm} = \frac{1}{2}(v_2 + v_1) = \frac{1}{2}(\underbrace{V_2 - V_3}_{} + \underbrace{V_1 + V_3}_{}) = \frac{1}{2}(V_{01} + V_{02})$$

[Voltage across R_3 = V_3 ($\because$ same current throught the same R)]

* Digitally Programmable Gain

Fig 2.30

* Output offsetting

Fig 2.31

Using superposition,

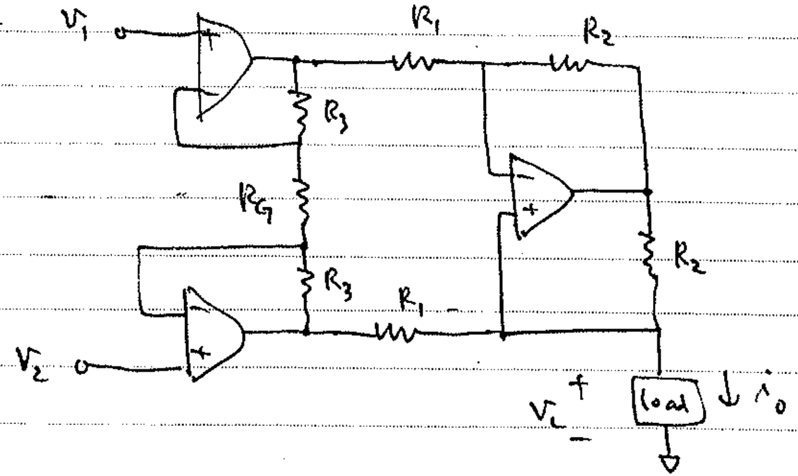
$$v_o = \left(1 + 2\frac{R_3}{R_G}\right)\frac{R_2}{R_1}(v_2 - v_1)$$

$$+ \frac{R_1}{R_1 + R_2}\left(1 + \frac{R_2}{R_1}\right) V_{REF}$$

$$= A(v_2 - v_1) + V_{REF}$$

* Current - Output IA

Fig 2.32



$$i_o = \frac{1 + 2R_3/R_G}{R_1}(v_2 - v_1)$$

* Current - Input IA

Fig 2.34 , $v_o = -\frac{2R_2}{R_1} R_3 i_I$

2.7 Transducer Bridge Amplifiers

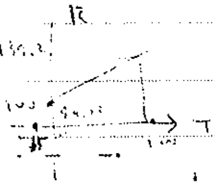
Resistive transducer :

- Temp : thermistors , RTD
- Light : photoresistor
- Strain : strain gage
- Press : piezoresistive transducer

Transducer resistance $= \underset{\text{reference}}{R} + \underset{\text{deviation}}{\Delta R} = R(1+\delta)$

$\delta = \dfrac{\Delta R}{R}$: fractional deviation

$\delta \times 100$ (%) : percentage "



Example 2.11 풀이.

$\alpha = 0.00392/°C$, $T = 0°C \rightarrow R = 100\Omega$

$R(T) = 100(1 + 0.00392T)$ (Ω)

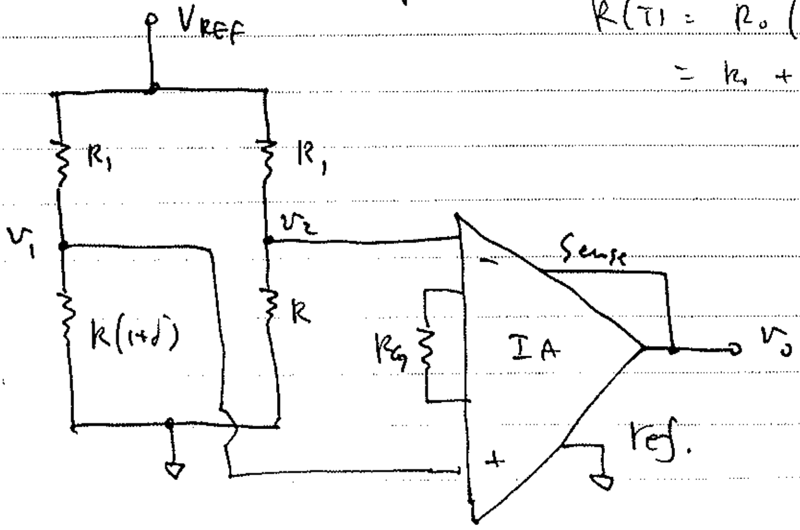
$\Delta T = 10°C \Rightarrow \Delta R = 3.92\,\Omega$

$\delta = \alpha\Delta T = 0.0392$

$R(T) = R_0(1+\alpha T)$,

$= R_0 + R_0\alpha T$

* Transducer Bridge



$$v_1 = V_{REF} \frac{R(1+\delta)}{R_1 + R(1+\delta)}$$

$$= \frac{R}{R_1+R} V_{REF} + \frac{\delta \; V_{REF}}{2 + R_1/R + R/R_1 + (1+R/R_1)\delta}$$

$$v_2 = \frac{R}{R_1+R} V_{REF}$$

$$v_o = A(v_1 - v_2) = A V_{REF} \frac{\delta}{2 + R_1/R + R/R_1 + (1+R/R_1)\delta}$$

For $\delta \ll 1$,

$$v_o \doteq \frac{A V_{REF} \; \delta}{2 + \frac{R_1}{R} + \frac{R}{R_1}}$$

If $R = R_1$ and $\delta \ll 1$,

$$v_o \doteq \frac{A V_{REF}}{4} \delta$$

$$\left(\text{If} \quad R = R_1, \qquad v_o = \frac{A V_{REF}}{4} \frac{\delta}{1+\delta/2} \right)$$

$\boxed{\text{Example 2.12}}$ $\frac{2}{3}$ a

$V_{REF} = 15$ V

(a) $P_{R1} = R\,i^2 \le$ [illegible]

[illegible] $\Rightarrow$ [illegible] $\Rightarrow R_1 = 15\,k\Omega$

[illegible]

[illegible]

① [illegible] ⇒ [illegible]

⇒ v_2 = [illegible]

Eq. 2.42 ⇒ [illegible]

[illegible] ⇒ [illegible]

* Bridge Calibration

① $\Delta R = 0 \Rightarrow v_0 = 0$

② Sensitivity $(v_1 - v_2)/\sigma$

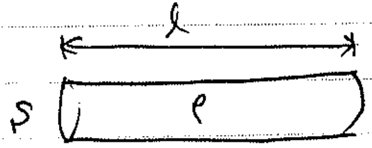
Fig 2.36

$\begin{cases} R_2 : & \text{bridge balancing} \\ R_3 : & \text{sensitivity adjustment} \\ & \text{(by changing current)} \end{cases}$

$\boxed{\text{Example 2.13}}$ [illegible].

* Strain Gauge Bridges

$$\left(\begin{aligned} \bar{E} &= -\nabla V \Rightarrow E = V/l \\ \bar{J} &= \sigma\bar{E} \Rightarrow J = \sigma E = \sigma\frac{V}{l} \\ I &= SJ \Rightarrow I = \sigma S\frac{V}{l} \end{aligned}\right)$$



$$R = \rho\frac{l}{S} \quad \sigma$$

strain ⇒ $l + \Delta l$, $S - \Delta S$

$$\frac{\rho l^2}{lS} + \frac{2\rho l\Delta l}{lS} + \frac{\rho\Delta l^2}{lS}$$

$$\therefore R + \Delta R = \rho(l+\Delta l)/(S - \Delta S) = \frac{\rho(l+\Delta l)(l+\Delta l)}{lS}$$

$$\text{Volume} = (l+\Delta l)(S - \Delta S) = lS \Rightarrow S - \Delta S = \frac{lS}{l+\Delta l}$$

$(lS - l\Delta S + \Delta l S - \Delta l\Delta S - lS)$

$$\Delta R = R\frac{\Delta l}{l}\left(2 + \frac{\Delta l}{l}\right)$$

Since $\Delta l / l \ll 2$,

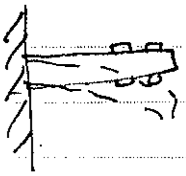
$$\Delta R = 2R\frac{\Delta l}{l}$$

$\frac{\Delta l}{l}$: fractional elongation

Four active arm bridge → @ Load cell : fig 2.37

$$v_1 = \frac{R+\Delta R}{2R}V_B, \quad v_2 = \frac{R-\Delta R}{2R}V_B$$

$$v_1 - v_2 = \frac{\Delta R}{R}V_B = V_B\,\delta$$



$$v_0 = A\,V_B\,\delta$$

⇒ { Linearity
Higher sensitivity
Temperature compensation

Calibration in fig 2.37

{ R_2 & R_1 : balancing
R_3 & R_4 : Sensitivity

$\boxed{\text{Example 2.14}}$ $\frac{7}{3}\Omega$

* Single - Op - Amp Amplifier

Fig 2.38 $$v_o = \frac{R_2}{R} V_{REF} \frac{\delta}{R_1/R + (1 + R_1/R_2)(1+\delta)}$$

for $\delta \ll 1$,

$$v_o \simeq \frac{R_2}{R} V_{REF} \frac{\delta}{1 + R_1/R + R_1/R_2}$$

* Bridge Linearization (지금까지 $\delta \ll 1$ 일때 linear 임)

Fig 2.39 Floating - load V - I converter

$$v_o = \frac{A R V_{REF}}{2R_1} \delta$$

Fig. 2.40 $$v_o = \frac{R_2 V_{REF}}{R_1} \delta$$

H/W 2.5, 2.9, 2.21, 2.25, 2.27, 2.28,
2.29, 2.35, 2.37, ∧ 2.47
2.41