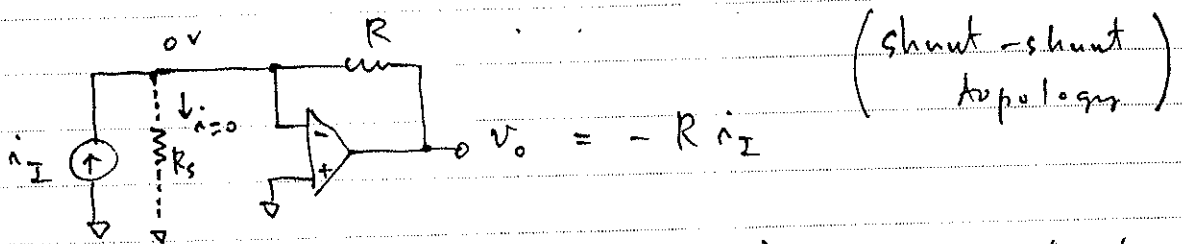


Chapter 2 Circuits with Resistive Feedback

2.1 Current-to-Voltage Converter (I-V converter)
 - Transresistance amplifier : $V_o = A i_I$



$A = -R$: gain or sensitivity
 (V/mA or V/ μ A)

no loading at input \leftarrow

In general, $V_o(s) = -Z(s) I_i(s)$
 \Rightarrow transimpedance amplifier

* closed-loop parameters

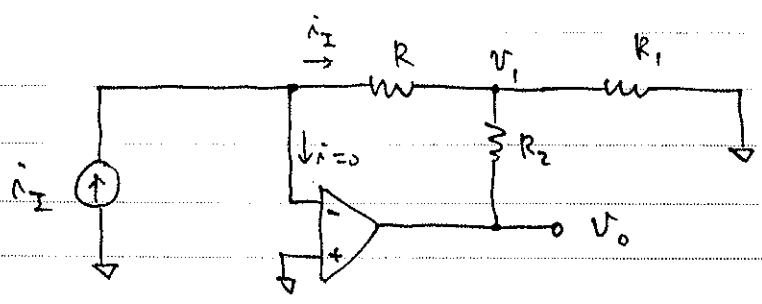
$$T = \frac{a r_d}{r_d + R_{\text{out}} + r_o}$$

$$A = -R \frac{1}{1 + 1/T}$$

$$R_i = \frac{r_d \parallel (R + r_o)}{1 + T}$$

$$R_o \approx \frac{r_o}{1 + T}$$

* High-Sensitivity I-V Converter



KCL at node 1,

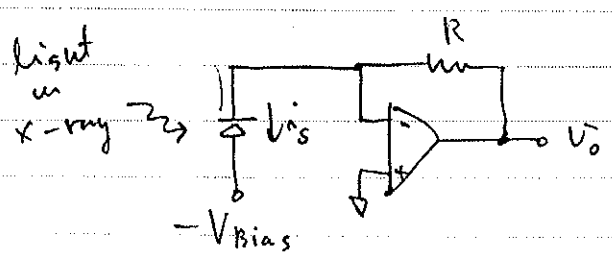
$$\frac{v_i}{R} + \frac{v_i}{R_1} + \frac{(v_i - v_o)}{R_2} = 0$$

$$v_i = -R i_I$$

$$\therefore v_o = -\left(1 + \frac{R_2}{R_1} + \frac{R_2}{R}\right) R i_I = -k R i_I$$

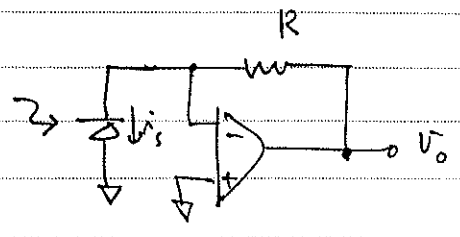
⇒ Use a low input bias current type op amp.
(JFET-input or MOSFET-input op amp.)

Si photodiode * Photodetector Amplifier



< photoconductive >

- high speed
- detection



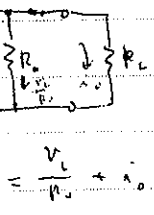
< photovoltaic >

- low noise
- measurement

2.2 Voltage-to-Current Converter

- V-I converter
- Transconductance amplifier : $i_o = A v_i$

In general, $i_o = A v_i - \frac{1}{R_o} v_L$



R_o : output resistance
 v_L : load voltage

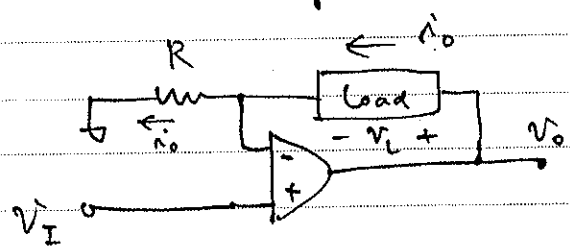
\Rightarrow We want $R_o \rightarrow \infty$

Voltage compliance : range of permissible v_L (before saturation)

Types of load $\left\{ \begin{array}{l} \text{floating} \\ \text{grounded} \end{array} \right.$

* Floating-Load Converter

- lower voltage compliance
- i_o supplied from power supply

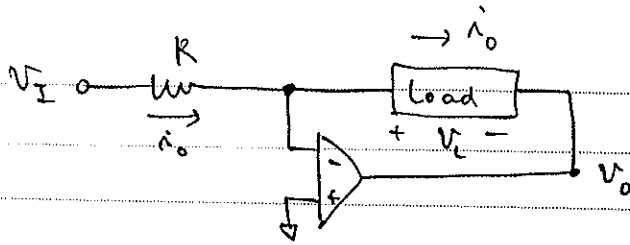


$$v_I = R i_o \Rightarrow i_o = \frac{v_I}{R}$$

$$v_o = v_I + v_L, \quad V_{OL} < v_o < V_{OH}$$

$\Rightarrow (V_{OL} - v_I) < v_L < (V_{OH} - v_I)$: voltage compliance

Ex) $v_I = 5V, R = 5k\Omega \Rightarrow i_o = 1mA$
 $(V_{OL} - 5) < v_L < (V_{OH} - 5)$
 \Rightarrow limits the value of load resistance



$$i_o = \frac{V_I}{R}, \quad V_o = -V_L \Rightarrow V_{oL} < V_L < V_{oH}$$

- Wider voltage compliance
- i_o supplied by V_I (source)

③ Maximum current is always limited by the op amp.

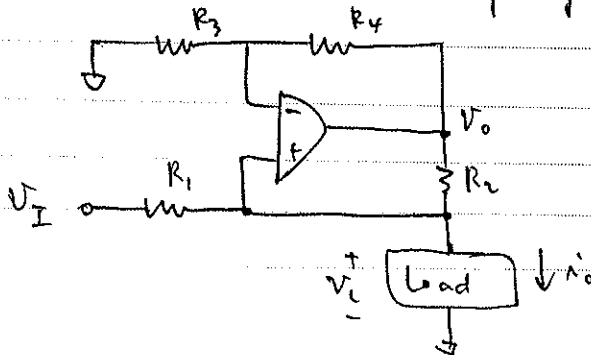
(Ex. 741 \Rightarrow max $i_o = 25 \text{ mA}$)

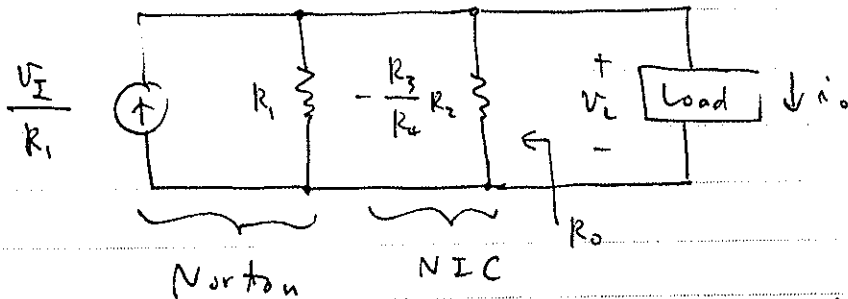
power of amp } \Rightarrow larger i_o
 Output current booster

④ Practical op amp limitation \Rightarrow see Eq. 2.7

* Grounded-Load converter

< How load current pump >





$$R_0 = R_1 \parallel \left(-\frac{R_3}{R_4} R_2 \right) = \frac{R_2}{R_2/R_1 - R_4/R_3}$$

If $\frac{R_4}{R_3} = \frac{R_2}{R_1}$, $R_0 \rightarrow \infty$ and $i_o = \frac{V_I}{R_1}$

$$V_L = V_P = V_N = \frac{R_3}{R_3 + R_4} V_o = \frac{R_1}{R_1 + R_2} V_o$$

$$\Rightarrow |V_L| \leq \frac{R_1}{R_1 + R_2} V_{sat}$$

Usually, $R_2 \approx 0.1 R_1$ for wider voltage compliance.

Example 2.4

Fig 2.8

⊙ Effect of Resistance Mismatch

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} (1 - \epsilon)$$

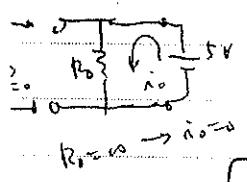
where ϵ is the imbalance factor.

$$\Rightarrow R_0 = \frac{R_2}{R_2/R_1 - R_4/R_3} = \frac{R_1}{\epsilon}$$

Example 2.5 : $\approx \infty$

- ⇒ Use highly precise resistors as resistance trimming.
- ⇒ Trimming resistor must be avoided whenever possible.

⊙ Howland circuit calibration: Fig. 2.9



⇒ Adjust \$R_{pot}\$ so that \$i_o = 0\$ for both cases.

Example 2.6

\$\frac{2}{3} R_L : R_{pot} = 12 \text{ k}\Omega\$

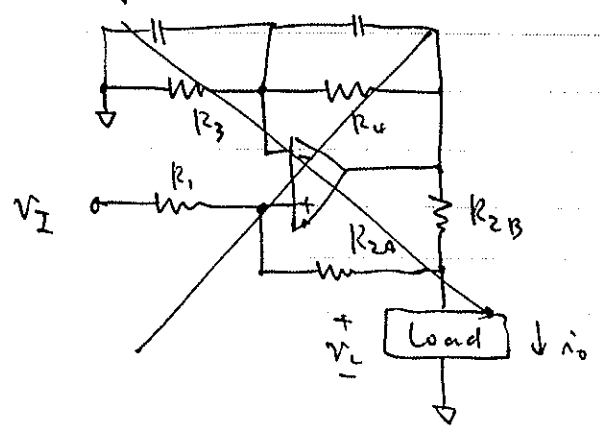
⊙ Effect of finite Open-loop Gain

Assuming \$R_2/R_1 = R_4/R_3\$,

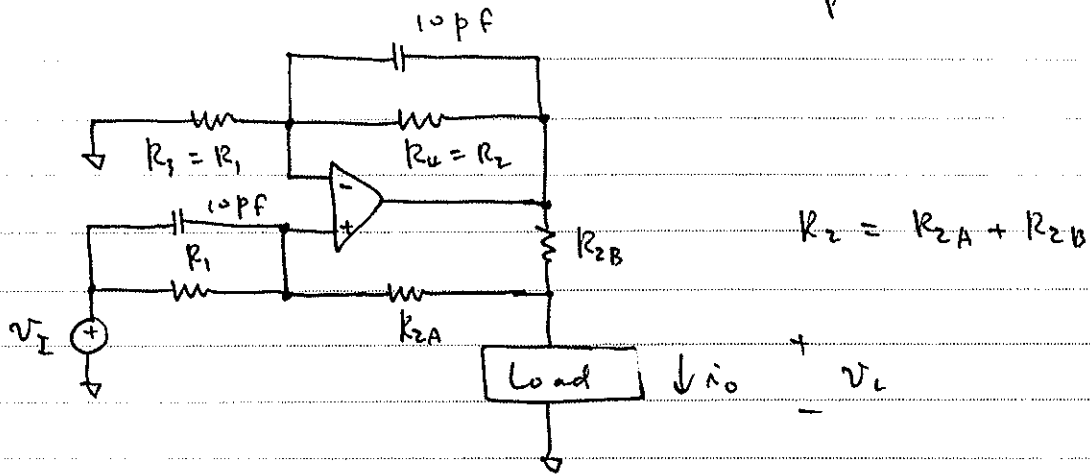
$$R_o = (R_1 \parallel R_2) \left(1 + \frac{a}{1 + R_2/R_1} \right)$$

$$i_o = \frac{V_I}{R_1} - \frac{V_L}{R_o}$$

⊙ Improved Howland Current Pump



$z_{in} \approx 10 \text{ pF} \Rightarrow$ at high freq, negative feedback is dominant compared to positive feedback.



Balance condition:

$$\frac{R_4}{R_3} = \frac{R_{2A} + R_{2B}}{R_1} \Rightarrow R_o = \infty$$

$$i_o = \frac{R_2/R_1}{R_{2B}} v_I$$

voltage compliance: $|v_o| \leq |V_{sat}| - R_{2B} |i_o|$

or $|v_o| \leq |V_{sat}| - (R_2/R_1) |v_I|$

Use large value for R_1, R_3, R_4, R_{2A}

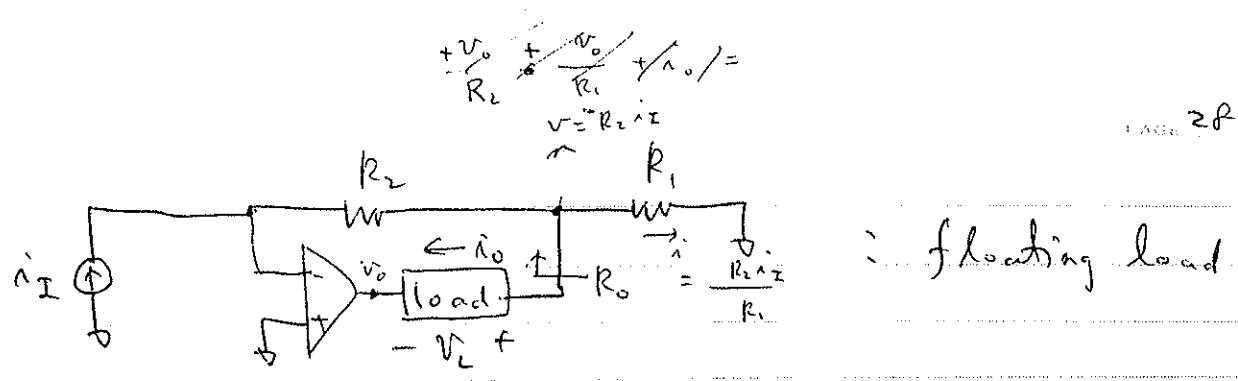
↓
Smaller power consumption.

2.3 Current Amplifiers

$$i_o = A_{it} i_i - \frac{1}{R_o} v_o$$

We need $R_o \rightarrow \infty$ so that

$$i_o = A_{it} i_i$$



$$\hat{I}_o = \hat{I}_I + \frac{R_2 \hat{I}_I}{R_1} = \left(1 + \frac{R_2}{R_1}\right) \hat{I}_I + \text{O} \parallel \frac{v_L}{R_0}$$

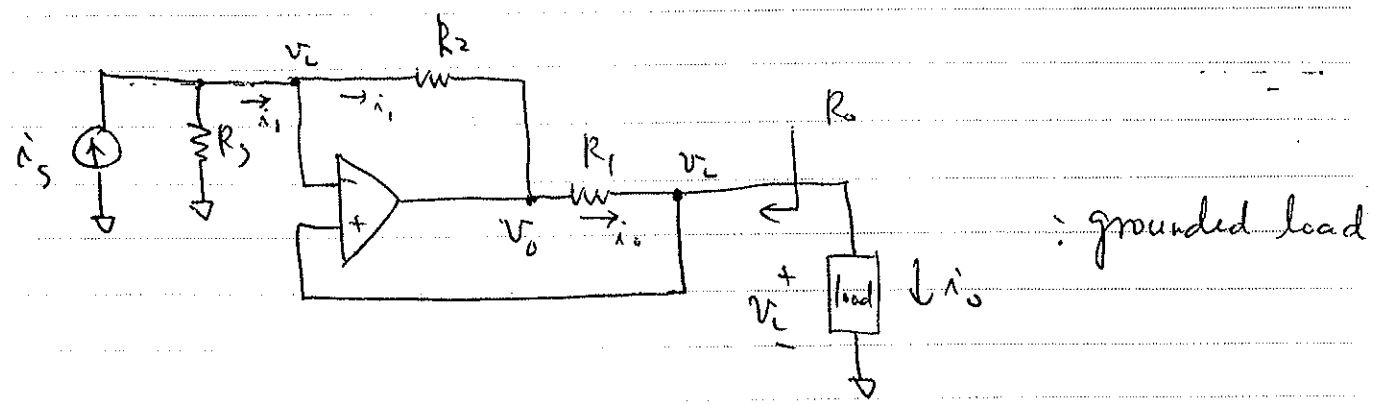
$R_0 \rightarrow \infty$

$$-(V_{OH} + R_2 \hat{I}_I) \leq v_L \leq -(V_{OL} + R_2 \hat{I}_I)$$

$(\because v_L + R_2 \hat{I}_I = v_o)$

For a finite a ,

$$A = 1 + \frac{R_2/R_1}{1 + 1/a}, \quad R_0 = R_1(1+a)$$



$$\hat{I}_1 = \hat{I}_s - \frac{v_L}{R_3}$$

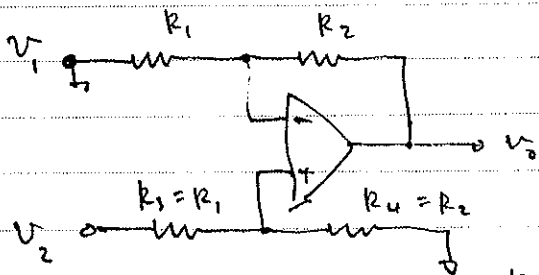
$$v_o = v_L - R_2 \hat{I}_1 = v_L - R_2 \left(\hat{I}_s - \frac{v_L}{R_3} \right)$$

$$\hat{I}_o = \frac{v_o - v_L}{R_1} = \left(-\frac{R_2}{R_1} \right) \hat{I}_s - \frac{1}{R_0} v_L$$

"A"

$$A = -\frac{R_2}{R_1}, \quad R_o = -\frac{R_1}{R_2} R_3$$

2.4 Difference Amplifier



$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

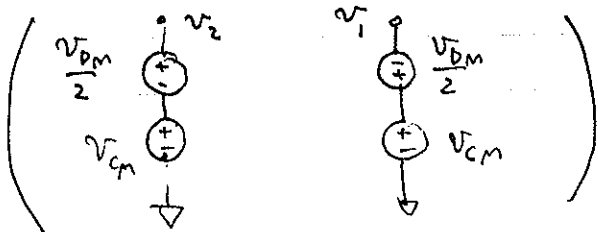
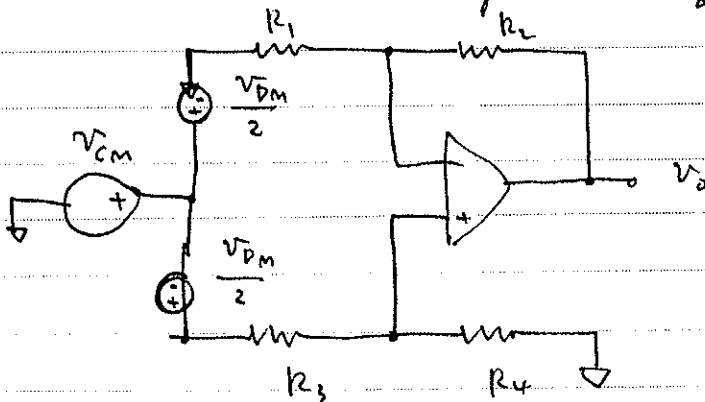
$$v_o = \frac{R_2}{R_1} (v_2 - v_1)$$

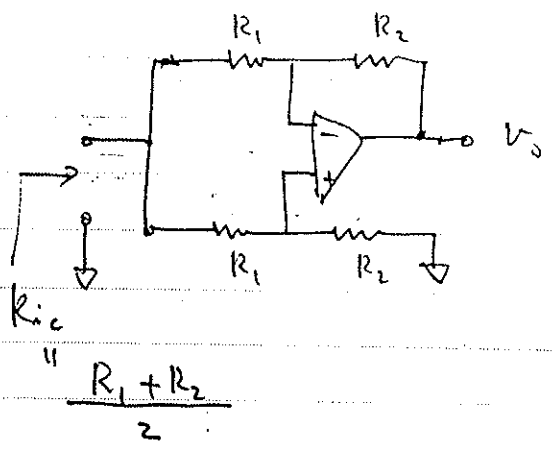
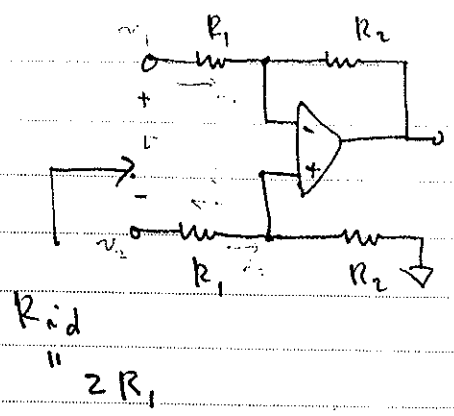
$v_{DM} = v_2 - v_1$: differential-mode input

$v_{CM} = \frac{1}{2} (v_2 + v_1)$: common-mode input

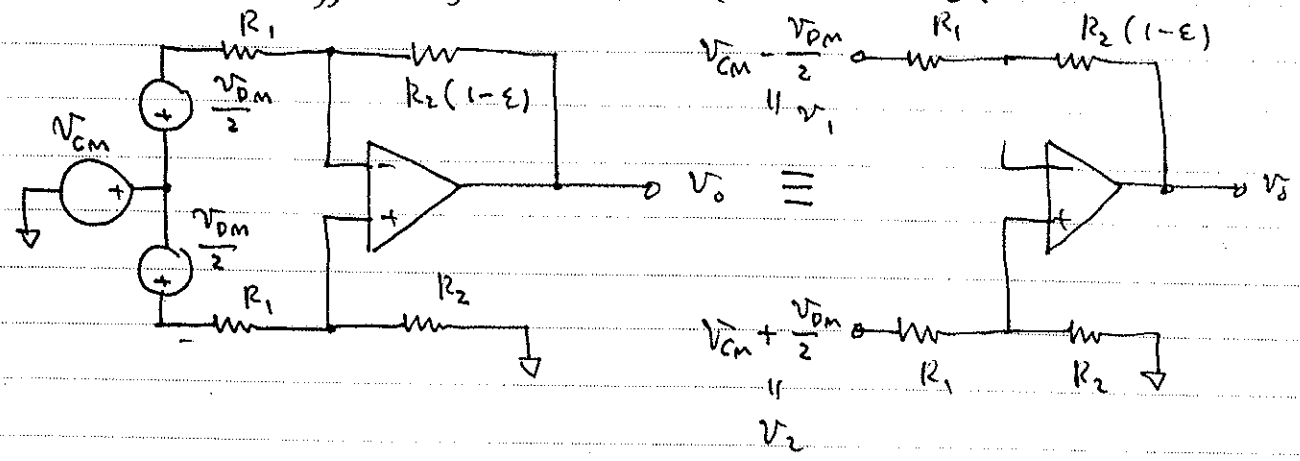
$$\Rightarrow v_1 = v_{CM} - \frac{v_{DM}}{2}, \quad v_2 = v_{CM} + \frac{v_{DM}}{2}$$

see Fig. 2.13 b





* Effect of Resistance Mismatch



Using Superposition,

$$v_o = - \frac{R_2(1-\epsilon)}{R_1} v_1 + \left(1 + \frac{R_2(1-\epsilon)}{R_1} \right) \cdot \frac{R_2}{R_1 + R_2} v_2$$

$$= - \frac{R_2(1-\epsilon)}{R_1} \left(v_{cm} - \frac{v_{dm}}{2} \right) + \left\{ 1 + \frac{R_2(1-\epsilon)}{R_1} \right\} \frac{R_2}{R_1 + R_2} \times \left(v_{cm} + \frac{v_{dm}}{2} \right)$$

$$\therefore v_o = A_{dm} v_{DM} + A_{cm} v_{CM}$$

where

$$A_{dm} = \frac{R_2}{R_1} \left(1 - \frac{R_1 + 2R_2}{R_1 + R_2} \frac{\epsilon}{2} \right)$$

$$A_{cm} = \frac{R_2}{R_1 + R_2} \epsilon \quad \left(\frac{R_1 + 2R_2}{R_1 + R_2} \right) (\frac{\epsilon}{2})$$

$\left\{ \begin{array}{l} A_{dm} : \text{differential-mode gain} \\ A_{cm} : \text{common-mode gain} \end{array} \right.$

$$\lim_{\epsilon \rightarrow 0} A_{dm} = \frac{R_2}{R_1}, \quad \lim_{\epsilon \rightarrow 0} A_{cm} = 0$$

$$CMRR = 20 \log_{10} \left| \frac{A_{dm}}{A_{cm}} \right| \quad (\text{dB})$$

↳ common-mode rejection ratio

$$\lim_{\epsilon \rightarrow 0} CMRR = \infty$$

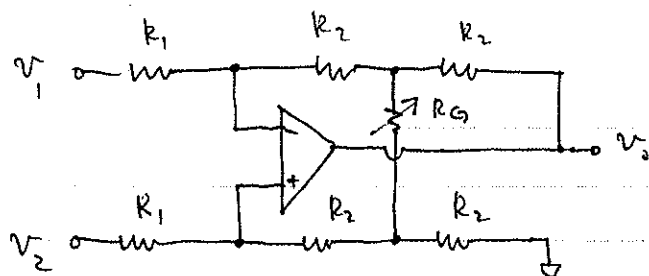
for a small ϵ ,

$$CMRR \approx 20 \log_{10} \left| \frac{1 + R_2/R_1}{\epsilon} \right|$$

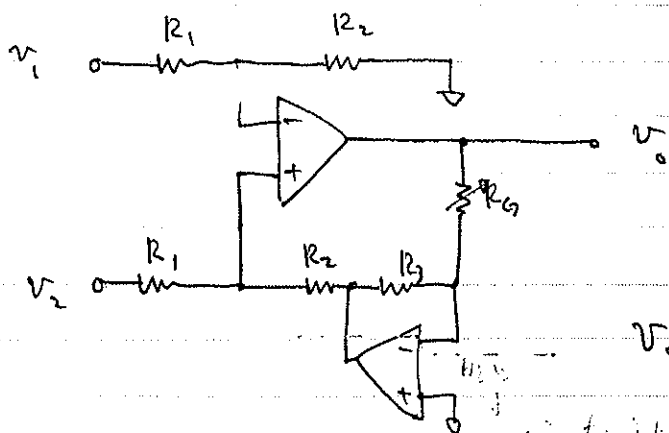
* Example 2.9 : $\frac{R_2}{R_1}$

$\left\{ \begin{array}{l} \text{Burr Brown INA 105 : } \epsilon < 0.002\%, \quad CMRR = 1 \text{ wdB} \\ \text{Calibration : Fig. 2.16} \\ \text{Resistor : metal film} \end{array} \right.$

* Variable Gain



$$v_o = \frac{2R_2}{R_1} \left(1 + \frac{R_2}{R_G} \right) (v_2 - v_1)$$



$$v_o = \frac{R_2 R_G}{R_1 R_3} (v_2 - v_1)$$

* Ground - Loop Interference Elimination

Fig 2.19 (a)

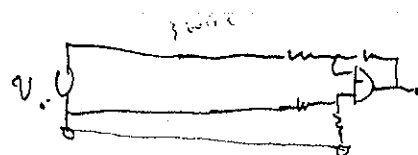
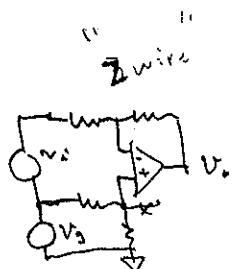
v_g : ground-loop interference or cross-talk for common return impedance

$$v_o = - \frac{R_2}{R_1} (v_i + v_g)$$

Fig 2.19 (b)

$$v_o = - \frac{R_2}{R_1} v_i$$

"double-ended transmission" or "balanced transmission"



2.5 Instrumentation Amplifier (IA)

- ① Extremely high R_{id} and R_{ic}
- ② Very low R_o
- ③ Accurate and stable gain (1 ~ 1000)
- ④ Extremely high CMRR

* Triple - Op - Amp IA

Fig 2.20 and 2.21

$$V_o = \left(1 + 2 \frac{R_3}{R_G} \right) \frac{R_2}{R_1} (v_2 - v_1)$$

Example 2.10 \rightarrow Fig 2.21

One-chip IA : AD522, AD620
INA101, INA212

(Gain control using external resistor R_G)

\Rightarrow Fig 2.22

\Rightarrow Note "Sense" and "Reference" pins

L

* Dual - Op - Amp IA

Fig. 2.23

$$V_o = \left(1 + \frac{R_2}{R_1} \right) (v_2 - v_1)$$

under $\frac{R_3}{R_4} = \frac{R_1}{R_2}$

Fig 2.24 : Variable gain

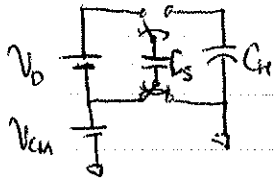
$$V_o = \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G} \right) (V_2 - V_1)$$

crit: delay for $v_1 \neq$ delay for v_2
 \Rightarrow do not use for high freq. signals

* Monolithic IA

Fig 2.25, Fig 2.26

* Flying-Capacitor Technique



2.6 Instrumentation Applications

* Active Guard

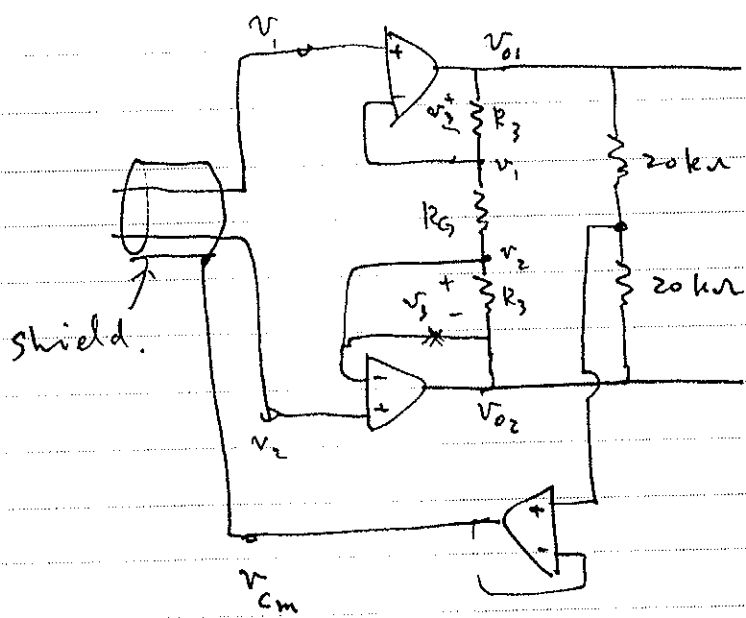
Double-ended transmission
 OR
 Balanced transmission } \Rightarrow Fig 2.28

"RC imbalance"

$$\rightarrow CMRR \approx 20 \log_{10} \frac{1}{2\pi f R_{dm} C_m}$$

$$\left\{ \begin{array}{l} R_{dm} = |R_{s1} - R_{s2}| \\ C_m = \frac{1}{2} (C_1 + C_2) \end{array} \right. \uparrow$$

Solution : active guard. (Fig. 2.29.)



$$v_{cm} = \frac{1}{2} (v_2 + v_1) = \frac{1}{2} (v_2 + v_3 + v_1 + v_3) = \frac{1}{2} (v_{01} + v_{02})$$

[Voltage across $R_s = v_3$ (\because same current through the same R)]

* Digitally Programmable Gain

Fig 2.30

* Output offsetting

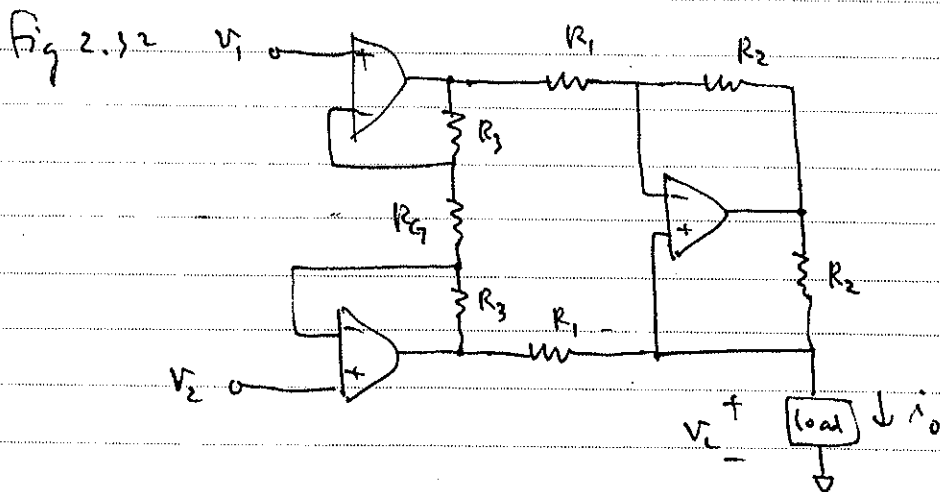
Fig 2.31

Using Superposition,

$$v_o = \left(1 + 2 \frac{R_3}{R_G}\right) \frac{R_2}{R_1} (v_2 - v_1) + \frac{R_1}{R_1 + R_2} \left(1 + \frac{R_2}{R_1}\right) V_{REF}$$

$$= A (v_2 - v_1) + V_{REF}$$

* Current - Output IA



$$i_o = \frac{1 + 2R_3/R_G}{R_1} (v_2 - v_1)$$

* Current - Input IA

Fig 2.34

$$v_o = -\frac{2R_2}{R_1} R_3 i_z$$

2.7 Transducer Bridge Amplifiers

Resistive transducer:

- Temp: thermistors, RTD
- Light: photoresistor
- Strain: strain gage
- Press: piezoresistive transducer

Transducer resistance = $R + \delta R = R(1 + \delta)$
reference deviation

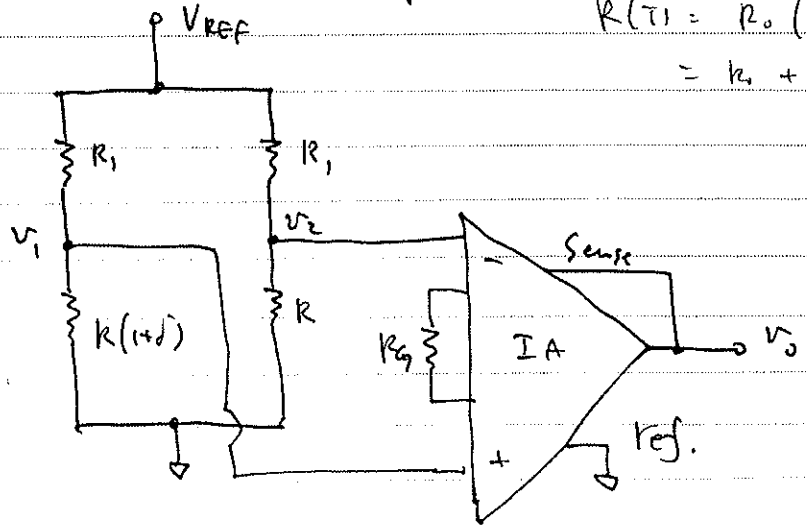
$\delta = \frac{\delta R}{R}$: fractional deviation

$\delta \times 100 (\%)$: percentage "

Example 2-11

$\alpha = 0.00392/^\circ\text{C}$, $T = 0^\circ\text{C} \rightarrow R = 100\Omega$
 $R(T) = 100(1 + 0.00392T) (\Omega)$
 $\Delta T = 10^\circ\text{C} \Rightarrow \delta R = 3.92\Omega$
 $\delta = \Delta R/R = 0.0392$
 $R(T) = R_0(1 + \alpha T)$
 $= R_0 + R_0\alpha T$

* Transducer Bridge



$$v_1 = V_{REF} \frac{R(1+\hat{d})}{R_1 + R(1+\hat{d})}$$

$$= \frac{R}{R_1 + R} V_{REF} + \frac{\hat{d} V_{REF}}{2 + R_1/R + R/R_1 + (1 + R/R_1)\hat{d}}$$

$$v_2 = \frac{R}{R_1 + R} V_{REF}$$

$$v_o = A(v_1 - v_2) = A V_{REF} \frac{\hat{d}}{2 + R_1/R + R/R_1 + (1 + R/R_1)\hat{d}}$$

for $\hat{d} \ll 1$,

$$v_o \approx \frac{A V_{REF} \hat{d}}{2 + \frac{R}{R} + \frac{R}{R_1}}$$

If $R = R_1$ and $\hat{d} \ll 1$,

$$v_o \approx \frac{A V_{REF} \hat{d}}{4}$$

$$\left(\text{If } R = R_1, \quad v_o = \frac{A V_{REF} \hat{d}}{4(1 + \hat{d}/2)} \right)$$

Example 2.12

$\frac{2}{3} A$

$V_{REF} = 15V$

(a)

$P_{TOT} = 10^{-3} \leq 10^{-3} \times 10^3 = 1W$

$\frac{1}{2} P_{TOT} = \frac{1}{2} \times 10^{-3} = 0.5 \times 10^{-3} = 0.5mW \Rightarrow R_1 = 15k\Omega$

ST: $\frac{1}{2} P_{TOT} = \frac{1}{2} \times 10^{-3} = 0.5 \times 10^{-3} = 0.5mW$

$\frac{1}{2} P_{TOT} = \frac{1}{2} \times 10^{-3} = 0.5 \times 10^{-3} = 0.5mW$

① $\epsilon = 1000 \Rightarrow \dots$
 $\Rightarrow v_0 = 1.224 \dots$
 $E_p = 2.47 \Rightarrow \dots$
 $i_{21} = 0.026 \dots$

* Bridge Calibration

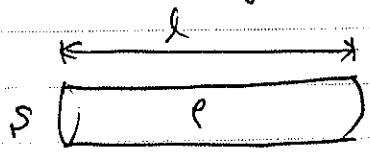
- ① $\Delta R = 0 \Rightarrow v_0 = 0$
- ② Sensitivity $(v_1 - v_2) / \Delta$

Fig 2.36

$\left\{ \begin{array}{l} R_2 : \text{bridge balancing} \\ R_3 : \text{sensitivity adjustment} \\ \quad \quad \quad (\text{by changing current}) \end{array} \right.$

Example 2.13 $\frac{3}{2} \mu$

* Strain Gauge Bridges $\left(\begin{array}{l} \bar{E} = -\sigma V \Rightarrow E = V/l \\ \bar{J} = \sigma \bar{E} \Rightarrow J = \sigma E = \sigma \frac{V}{l} \\ I = SJ \Rightarrow I = \sigma S \frac{V}{l} \end{array} \right)$



$$R = \rho \frac{l}{S}$$

strain $\Rightarrow l + \Delta l, S - \Delta S$

$$\frac{\rho l^2}{l^2 S} + \frac{2\rho l \Delta l}{S^2} + \frac{\rho \Delta l^2}{S^2}$$

$$\therefore R + \Delta R = \rho (l + \Delta l) / (S - \Delta S) = \frac{\rho (l + \Delta l)(l + \Delta l)}{l S}$$

$$\text{Volume} = (l + \Delta l)(S - \Delta S) = l S \Rightarrow S - \Delta S = \frac{l S}{l + \Delta l}$$

$$\Delta R = R \frac{\Delta l}{l} \left(2 + \frac{\Delta l}{l} \right)$$

Since $\Delta l/l \ll 2$,

$$\Delta R = 2R \frac{\Delta l}{l}$$

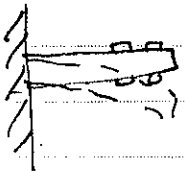
$\frac{\Delta l}{l}$: fractional elongation

Four active arm bridge \rightarrow Load cell : fig 2.37

$$V_1 = \frac{R + \Delta R}{2R} V_B, \quad V_2 = \frac{R - \Delta R}{2R} V_B$$

$$V_1 - V_2 = \frac{\Delta R}{R} V_B = V_B \delta$$

$$V_o = A V_B \delta$$



\Rightarrow $\left\{ \begin{array}{l} \text{Linearity} \\ \text{Higher sensitivity} \\ \text{Temperature compensation} \end{array} \right.$

Calibration in fig 2.37

$\left\{ \begin{array}{l} R_2 \ \& \ R_1 \\ R_3 \ \& \ R_4 \end{array} \right. : \begin{array}{l} \text{balancing} \\ \text{sensitivity} \end{array}$

Example 2.14

$\frac{2}{2R}$

* Single-Op-Amp Amplifier

Fig 2.38
$$V_o = \frac{R_2}{R} V_{REF} \frac{\delta}{R_1/R + (1 + R_1/R_2)(1 + \delta)}$$

for $\delta \ll 1$,

$$V_o \approx \frac{R_2}{R} V_{REF} \frac{\delta}{1 + R_1/R + R_1/R_2}$$

* Bridge Linearization ($\delta \ll 1$ linear approx)

Fig 2.39 floating-load V-I conversion

$$V_o = \frac{AR V_{REF}}{2R_1} \delta$$

Fig. 2.40
$$V_o = \frac{R_2 V_{REF}}{R_1} \delta$$

H/w 2.5, 2.9, 2.21, 2.25, 2.27, 2.28,
2.29, 2.35, 2.37, 2.47
2.41