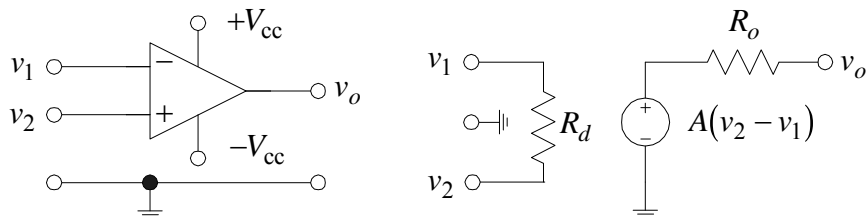


Chapter 3. Amplifiers and Signal Processing

3.1 Ideal OP Amps

- Op amp
 - High-gain dc differential amplifier
 - Dc power supplies are required
 - Usually used with external negative feedback
- Assume ideal op amp \Rightarrow design circuit \Rightarrow check nonideal characteristics are important \Rightarrow modify if necessary



Ideal Characteristics

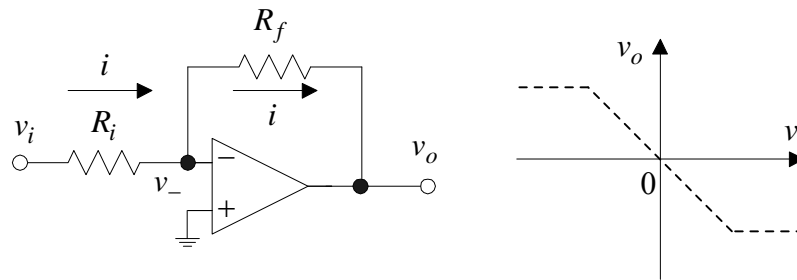
- $A = \infty$ (gain is infinite)
- $v_o = 0$ when $v_1 = v_2$ (no offset voltage)
- $R_d = \infty$ (input impedance is infinite)
- $R_o = 0$ (output impedance is zero)
- Bandwidth = ∞ (no frequency response limitation) and no phase shift

Two Basic Rules

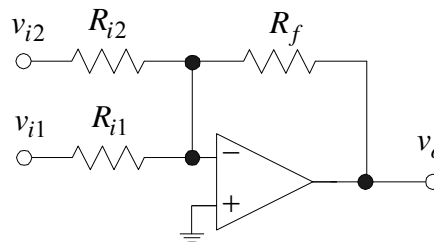
- Rule 1 When the op amp output is in its linear region, the two input terminals are at the same voltage.
- Rule 2 No current flows into either input terminal of the op amp.
- Saturation of output at slightly lower than power supply voltages

3.2 Inverting Amplifiers

- Inverting amplifier and input-output characteristic

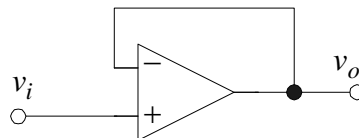


- Virtual ground: $v_- = 0$
 - Analysis: $v_o = -i R_f = -v_i \frac{R_f}{R_i}$ or $\frac{v_o}{v_i} = -\frac{R_f}{R_i}$
 - $R_{in} = R_i$ and $R_{out} = 0$
- Summing amplifier: $v_o = -R_f \left(\frac{v_{i1}}{R_{i1}} + \frac{v_{i2}}{R_{i2}} \right)$

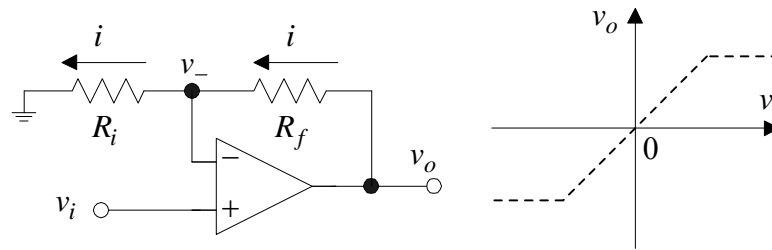


3.3 Noninverting Amplifiers

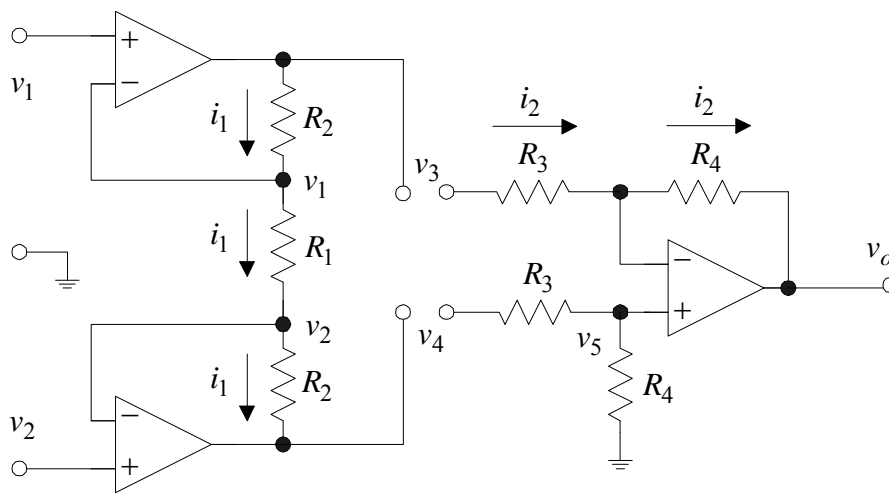
- Unity-gain follower or buffer: $v_o = v_i$, $R_{in} = \infty$, and $R_{out} = 0$



- Noninverting amplifier and input-output characteristic
- $v_- = v_i$
 - Analysis: $\frac{v_o}{v_i} = \frac{i(R_f + R_i)}{i R_i} = 1 + \frac{R_f}{R_i}$
 - $R_{in} = \infty$ and $R_{out} = 0$



3.4 Differential Amplifiers



○ One-op-amp differential amplifier

- Analysis: $v_5 = \frac{R_4}{R_3 + R_4} v_4$, $i_2 = \frac{v_3 - v_5}{R_3} = \frac{v_5 - v_o}{R_4}$, $v_o = \frac{R_4}{R_3} (v_4 - v_3)$
- Common-mode rejection ratio (CMRR): $CMRR = \frac{G_d}{G_c}$ or $20 \log \frac{G_d}{G_c}$ dB
 - Differential mode gain, $G_d = \frac{R_4}{R_3}$
 - Common mode gain, G_c : gain for $v_3 = v_4$
- $R_{in} < \infty$ (could be small) and $R_{out} = 0$

○ Three-op-amp differential amplifier (instrumentation amplifier)

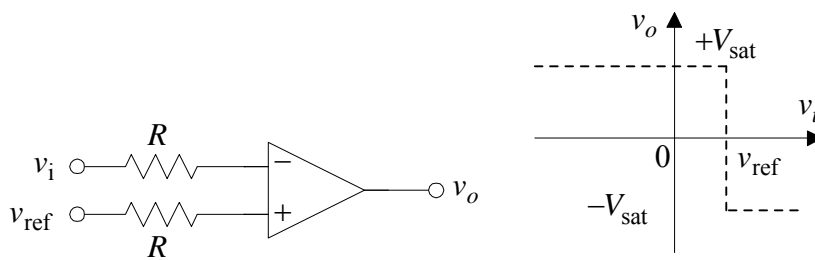
- Analysis: $v_3 - v_4 = i_1 (2R_2 + R_1)$, $v_1 - v_2 = i_1 R_1$, $v_3 - v_4 = \left(1 + 2 \frac{R_2}{R_1}\right) (v_1 - v_2)$,

$$v_o = \left(1 + 2 \frac{R_2}{R_1}\right) \frac{R_4}{R_3} (v_2 - v_1)$$

- Common-mode rejection ratio (CMRR): $CMRR = \frac{G_d}{G_c}$ or $20 \log \frac{G_d}{G_c}$ dB
 - Differential mode gain, $G_d = \left(1 + 2 \frac{R_2}{R_1}\right) \frac{R_4}{R_3}$
 - Common model gain, G_c : gain for $v_1 = v_2$
- $R_{in} = \infty$ and $R_{out} = 0$

3.5 Comparators

- Simple comparator or Schmitt trigger



- $v_i > v_{ref} \Rightarrow v_o = -V_{sat}$ and $v_i < v_{ref} \Rightarrow v_o = +V_{sat}$
- R minimizes overdriving op amp input.
- Sensitive to noise at input
- v_i and v_{ref} can be interchanged.

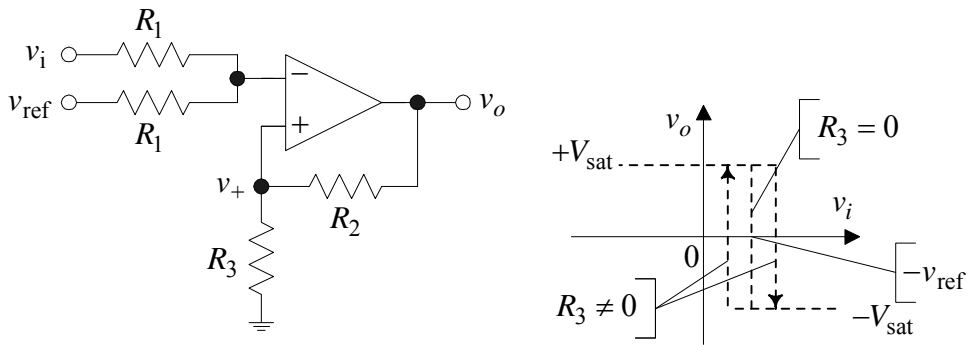
- Comparator with hysteresis

- Positive feedback with R_2 and $R_3 \Rightarrow$ hysteresis \Rightarrow insensitive to input noise
- Analysis

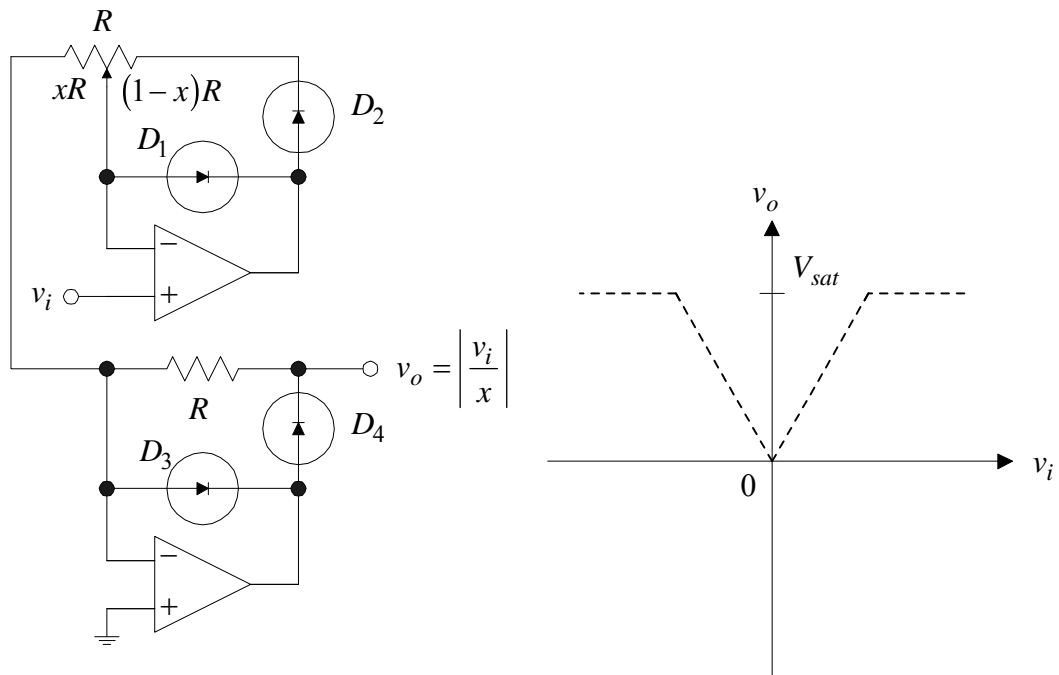
$$\begin{aligned} \square \quad v_o = +V_{sat} &\Rightarrow v_+ = \frac{R_3}{R_2 + R_3} V_{sat} \Rightarrow v_i \text{ must be greater than} \\ & -v_{ref} + \frac{R_3}{R_2 + R_3} V_{sat} \text{ to produce } v_o = -V_{sat} \end{aligned}$$

□ $v_o = -V_{sat} \Rightarrow v_+ = -\frac{R_3}{R_2 + R_3}V_{sat} \Rightarrow v_i$ must be smaller than $-v_{ref} - \frac{R_3}{R_2 + R_3}V_{sat}$ to produce $v_o = +V_{sat}$

- R_3 controls the width of the hysteresis.



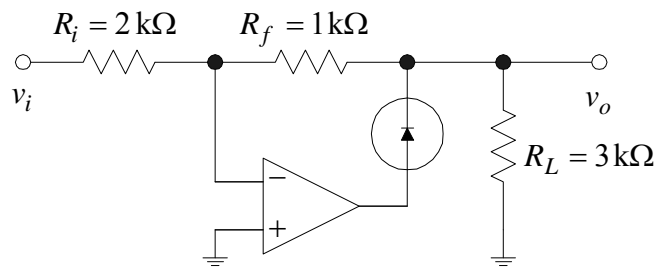
3.6 Rectifiers



○ Full-wave precision rectifier

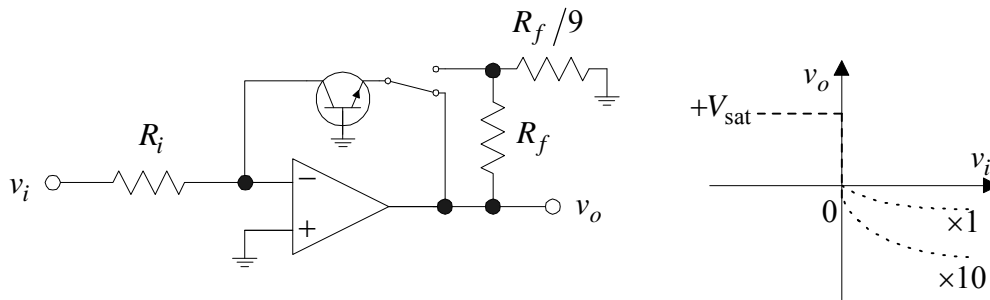
- $v_i > 0 \Rightarrow D_2$ and D_3 ON, D_1 and D_4 OFF \Rightarrow upper op amp circuit becomes a noninverting amplifier with gain of $1/x$, lower op amp circuit has no effect on output

- $v_i < 0 \Rightarrow D_2$ and D_3 OFF, D_1 and D_4 ON \Rightarrow upper op amp circuit has no effect on output becomes a noninverting amplifier with gain of $1/x$, lower op amp circuit becomes an inverting amplifier with gain of $-1/x$
- Variable gain and high input impedance
- Half-wave precision rectifier
 - Upper or lower op amp circuit



- One op-amp full-wave rectifier
 - Gain is a function of load \Rightarrow constant load is required

3.7 Logarithmic Amplifiers



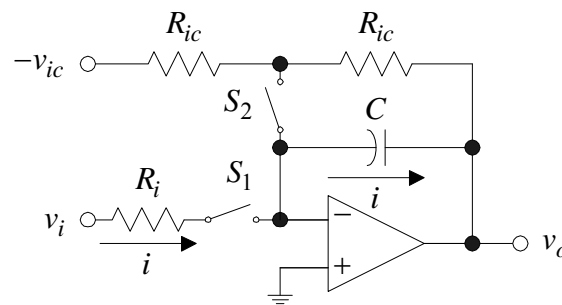
- Without boosting
 - For a transistor, $V_{BE} = 0.060 \log\left(\frac{I_C}{I_S}\right)$ with reverse saturation current

$$I_S = 10^{-13} \text{ A at } 27^\circ \text{ C}$$

- Transdiode configuration: $I_C = \frac{v_i}{R_i}$ and $v_o = V_{BE} = 0.060 \log\left(\frac{v_i}{I_S R_i}\right)$

- For $10^{-7} \text{ A} < I_C < 10^{-2} \text{ A}$, $-0.66 \text{ V} < v_o < -0.36 \text{ V}$
- With boosting: same as noninverting amplifier
- Temperature compensation for accuracy
- Antilog (exponential) circuit: interchange resistor with transistor
- Applications
 - Multiplication, division, power
 - Dynamic range compression
 - Linealization

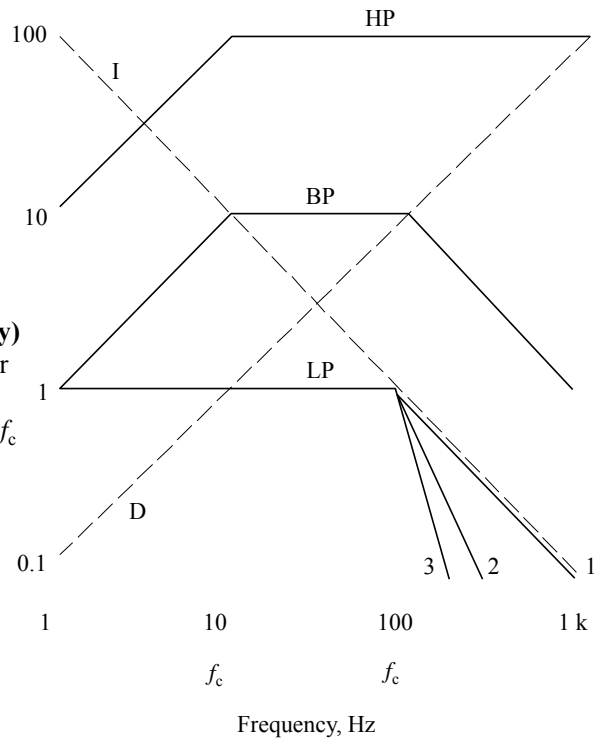
3.8 Integrators



- Integrator
 - Initial condition setting: S_1 open and S_2 closed $\Rightarrow v_o = v_{ic}$ (inverting amplifier) and $v_c(0) = -v_{ic}$
 - Integration: S_1 closed and S_2 open $\Rightarrow v_c = \frac{1}{C} \int_0^{t_1} i dt - v_{ic}$ and $i = \frac{v_i}{R} \Rightarrow$

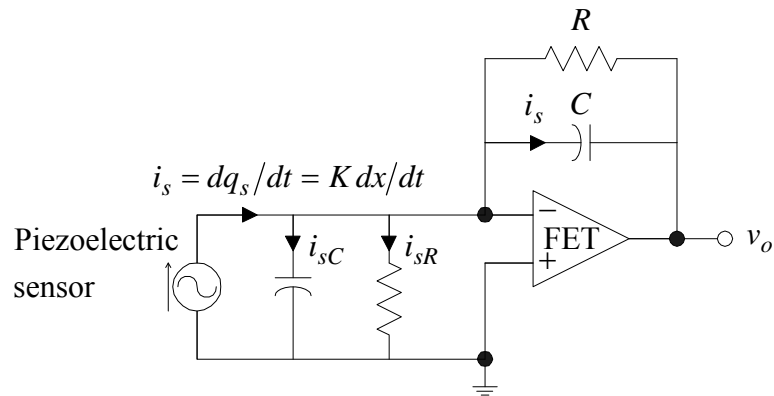
$$v_o = -\frac{1}{RC} \int_0^{t_1} v_i dt + v_{ic}$$
 - Hold: S_1 open and S_2 open $\Rightarrow v_o$ is hold
 - Frequency response: $\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega RC} = -\frac{1}{j\omega\tau}$
 - Drift and saturation problem

Figure 3.10 Bode plot (gain versus frequency) for various filters. Integrator (I); differentiator (D); low pass (LP), 1, 2, 3 section (pole); high pass (HP); bandpass (BP). Corner frequencies f_c for high-pass, low-pass, and bandpass filters.

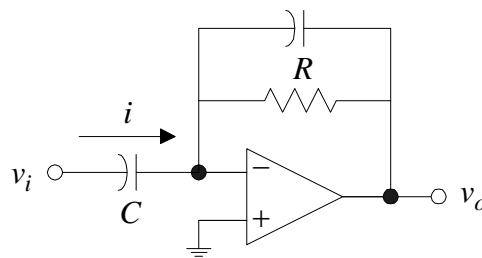


○ Charge amplifier

- Virtual ground $\Rightarrow i_{sC} = i_{sR} = 0 \Rightarrow$ long cable can be used
- $i_s = K \frac{dx}{dt} \Rightarrow v_o = -\frac{1}{C} \int_0^{t_1} K \frac{dx}{dt} dt = -\frac{Kx}{C}$
- Drift and saturation problem
- Large feedback resistor
 - Prevents saturation
 - Highpass filter with $f_c = \frac{1}{2\pi RC} \Rightarrow$ no frequency response improvement over voltage amplifier



3.9 Differentiators



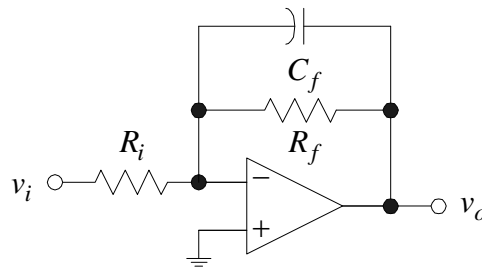
○ Differentiator

- $i = C \frac{dv_i}{dt}$ and $v_o = -Ri = -RC \frac{dv_i}{dt}$
- Frequency response: $\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R}{1/j\omega C} = -j\omega RC = -j\omega\tau$

- Differentiator output: tends to oscillate and noisy due to amplification of high frequency components

3.10 Active Filters

Low-Pass Filter



- Frequency response:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{\frac{R_f / j\omega C_f}{\left[\frac{1}{j\omega C_f} + R_f\right]}}{R_i} = -\frac{R_f}{(1 + j\omega R_f C_f)R_i} = -\frac{R_f}{R_i} \frac{1}{(1 + j\omega\tau)}$$

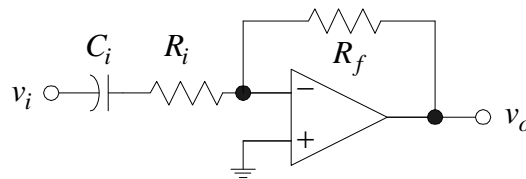
- If $\omega \ll 1/\tau$ or $f \ll f_c$ with $f_c = 1/2\pi R_f C_f$ circuit becomes inverting amplifier

with gain $-R_f/R_i$

- If $\omega \gg 1/\tau$ or $f \gg f_c$, circuit becomes integrator

- Cutoff frequency or corner frequency: $f_c = 1/2\pi R_f C_f$

High-Pass Filter



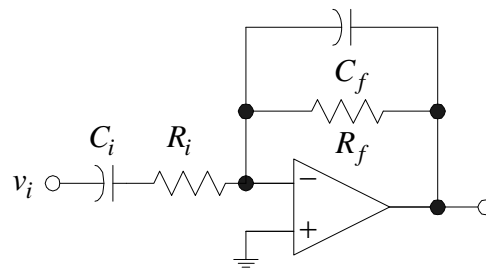
- Frequency response:

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{\left(\frac{1}{j\omega C_i} + R_i\right)} = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i} = -\frac{R_f}{R_i} \frac{j\omega\tau}{1 + j\omega\tau}$$

- If $\omega \ll 1/\tau$ or $f \ll f_c$ with $f_c = 1/2\pi R_i C_i$, circuit becomes differentiator

- If $\omega \gg 1/\tau$ or $f \gg f_c$, circuit becomes inverting amplifier with gain $-R_f/R_i$

- Cutoff frequency or corner frequency: $f_c = 1/2\pi R_i C_i$

Band-Pass Filter

- Series combination of lowpass and highpass filter
- Two cutoff frequencies or corner frequencies: $f_{c1} = 1/2\pi R_i C_i$ and $f_{c2} = 1/2\pi R_f C_f$
with $f_{c2} > f_{c1}$
 - If $f \ll f_{c1}$, circuit becomes differentiator
 - If $f \gg f_{c2}$, circuit becomes integrator

3.11 Frequency Response

- For a real op amp, bandwidth is not infinite.

Open-Loop Gain

- Op amp is multi-stage dc differential amplifier with high gain
- Stray or junction capacitances in each stage \Rightarrow gain attenuation (-1 slope on log-log plot and -90° phase shift per stage) \Rightarrow slope changes with frequency
- Real op amp has a limited open-loop bandwidth
- Possible oscillation (gain greater than 1 at -180° phase shift)

Compensation

- Add a capacitor (external or internal) \Rightarrow fixed slope of -1 and maximal phase shift of -90° , open-loop cutoff frequency of about 40 Hz
- No oscillation

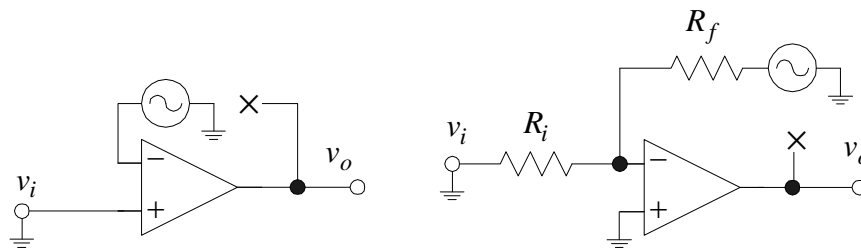
Closed-Loop Gain

- Closed-loop gain is usually much smaller than the maximal open-loop gain of op amp.
- Closed-loop gain is determined by external elements forming negative feedback.

- Closed-loop gain can never exceed open-loop gain.

Loop Gain

- Loop gain = (open-loop gain of op amp) – (closed-loop gain of op amp circuit)
 - At low frequency: high loop gain, external feedback circuit determines the op amp circuit
 - At high frequency: low loop gain, the op amp circuit follows the op amp open-loop gain
 - High loop gain \Rightarrow high accuracy and stability
- Measurement of loop gain
 - Break feedback loop at any point in the loop
 - Inject a signal
 - Measure the gain around the loop
 - Examples
 - Unity gain follower: loop gain = open-loop gain



- Inverting amplifier with gain of -1 : loop gain = (open-loop gain)/2

Gain-Bandwidth Product

- Gain-bandwidth product = (gain at f) \times (bandwidth at f)
- Unity-gain-bandwidth product is given in specification of op amp
- Compensated op amp has gain slope of $-1 \Rightarrow$

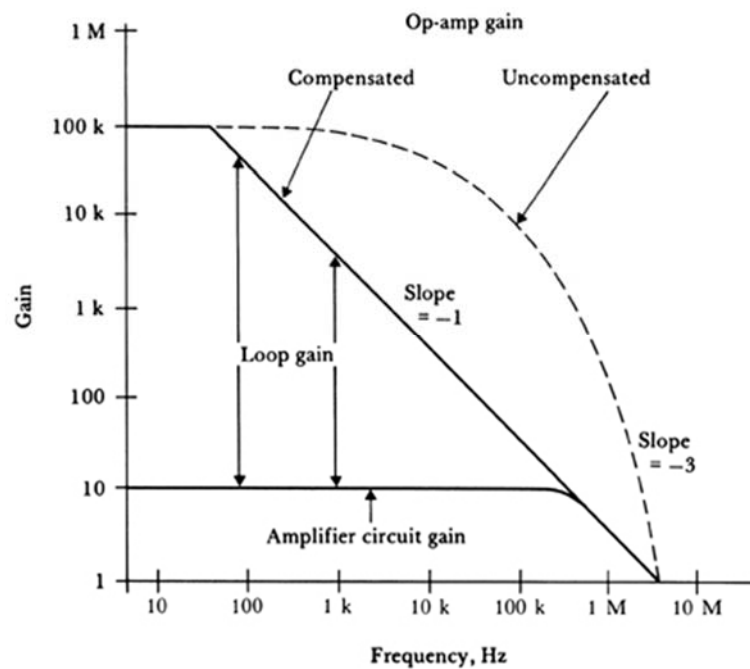
$$\text{Bandwidth of op amp circuit} = \frac{\text{Unity-gain-bandwidth product in Hz}}{\text{Op amp circuit gain}}$$

Slew Rate

- For an op amp, internal current source has its I_{\max} .
- Change in voltage across the compensation capacitor: $\frac{dv_c}{dt} = \frac{I_{\max}}{C} \Rightarrow \frac{dv_o}{dt}$ is limited

- Slew rate $S_r = \left. \frac{dv_o}{dt} \right|_{\max}$
- For sinusoidal input, full-power response or maximal frequency for rated output is $f_p = \frac{S_r}{2\pi V_{or}}$ where V_{or} is the rated output voltage.
- Uncompensated op amp is faster \Rightarrow useful for comparators

Figure 3.13 Op-amp frequency characteristics
 early op amps (such as the 709) were uncompensated, had a gain greater than 1 when the phase shift was equal to -180° , and therefore oscillated unless compensation was added externally. A popular op amp, the 411, is compensated internally, so for a gain greater than 1, the phase shift is limited to -90° . When feedback resistors are added to build an amplifier circuit, the loop gain on this log-log plot is the difference between the op-amp gain and the amplifier-circuit gain.



3.12 Offset Voltage

- For a real op amp, $v_2 - v_1 \neq 0$ to produce $v_o = 0$.
- Offset voltage = $v_2 - v_1 \neq 0$ must be considered for small input signals.

Nulling

- Add an external nulling pot.
- Adjust the pot \Rightarrow increase I_E at one input and decrease at the other $\Rightarrow v_2 - v_1 = 0$

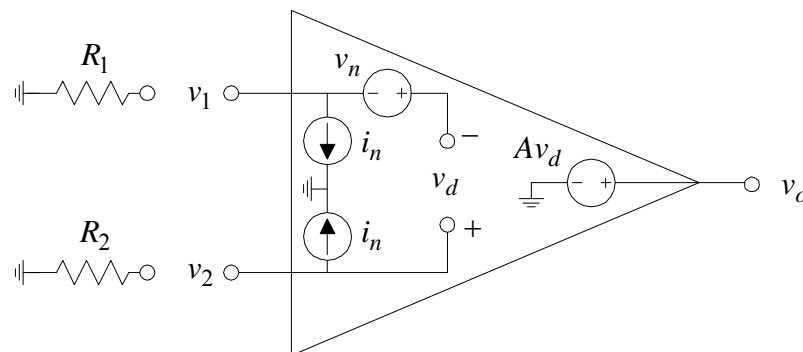
Drift

- Temperature change (environment or self-heating) \Rightarrow change in offset voltage, $(v_2 - v_1)$

- Specification
 - Maximal offset voltage change per $^{\circ}\text{C}$ such as $0.1 \mu\text{V}/^{\circ}\text{C}$
 - Maximal offset voltage over a given temperature range such as -25 to $+85 \text{ }^{\circ}\text{C}$

Noise

- Semiconductor junctions \Rightarrow noise voltage sources and noise current sources
- For low source impedance (R_1 and R_2 small), v_n dominates.
- Characteristics
 - Random
 - At low frequency \Rightarrow amplitude $\propto 1/f$ (flicker noise)
 - At midfrequency \Rightarrow smaller amplitude expressed in rms units of $\text{V} \cdot \text{Hz}^{-1/2}$
 - Some op amps exhibit bursts of noise (popcorn noise).

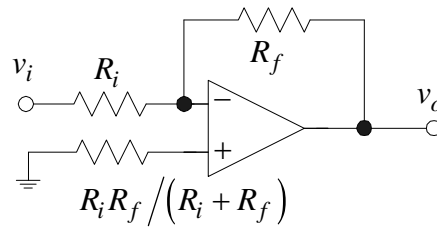


3.13 Bias Current

- Base or gate current to keep transistors turned on \Rightarrow bias current $\neq 0$
- Bias current flows through feedback resistors \Rightarrow smaller resistors are desirable (about $10 \text{ k}\Omega$)
- Caution: current flowing through feedback resistors plus current flowing through loads must be smaller than op amp output current rating \Rightarrow too small resistors cannot be used

Differential Bias Current

- Difference between two input bias currents \ll each bias current
- Compensation resistor minimizes the effect of bias currents



Drift

- Change of bias currents due to temperature
- Compensation resistor also minimizes the effect of bias current drift

Noise

- Noise currents flow through external equivalent resistors.

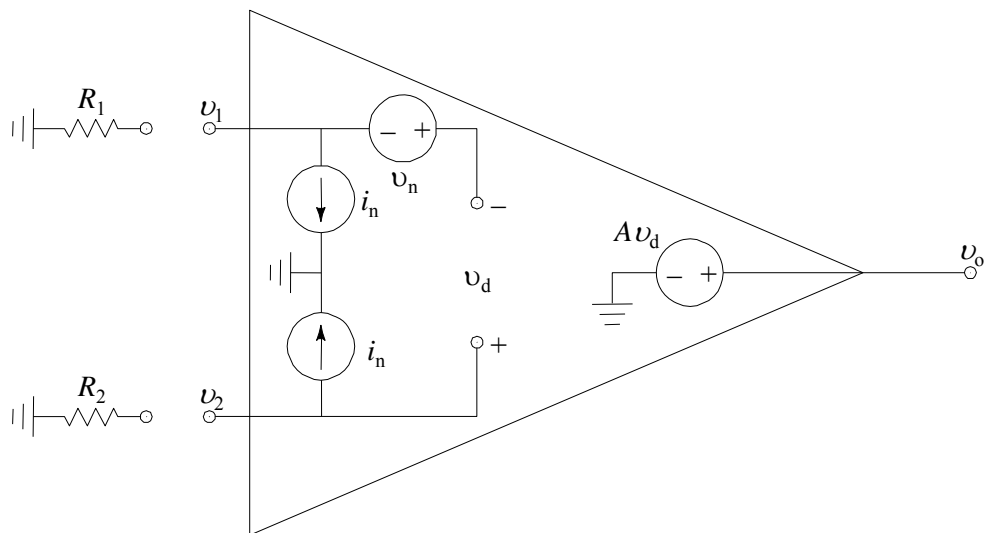
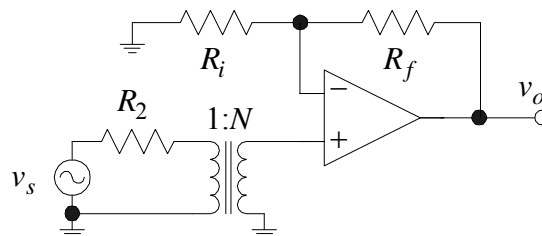


Figure 3.14 Noise sources in an op amp The noise-voltage source v_n is in series with the input and cannot be reduced. The noise added by the noise-current sources i_n can be minimized by using small external resistances.

- Total rms noise voltage is $v_t \cong \left[\left\{ v_n^2 + (i_n R_1)^2 + (i_n R_2)^2 + 4kTR_1 + 4kTR_2 \right\} \times BW \right]^{1/2}$
 - R_1 and R_2 : equivalent source resistances
 - v_n : mean value of rms noise voltage in $V \cdot \text{Hz}^{-1/2}$ over a frequency range
 - i_n : mean value of rms noise voltage in $A \cdot \text{Hz}^{-1/2}$ over a frequency range

- k : Boltzmann's constant
- T : temperature, K
- BW : noise bandwidth, Hz
- Types of op amp
 - Small (10 k Ω) source resistances \Rightarrow BJT input op amp produces smaller noise
 - Large source resistances \Rightarrow FET input op amp produces smaller noise due to smaller noise current
- Low noise ac amplifier design (noninverting amplifier) by impedance matching
 - Characteristic noise resistance is $R_n = v_n/i_n$
 - Set $R_n = R_2$ using a transformer with turns ratio 1: N where $N = (R_n/R_2)^{1/2}$



3.14 Input and Output Resistance

Input Resistance

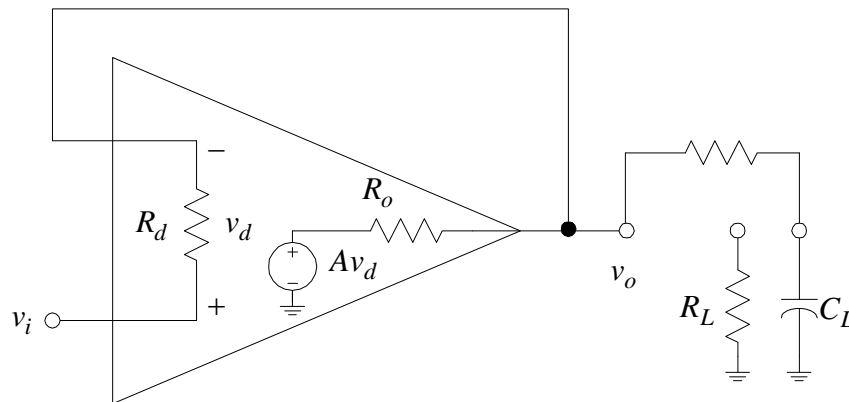
- Op amp differential input resistance, R_d : T Ω for FET, M Ω for BJT
- Amplifier-circuit input resistance, R_{ai}

$$\square \text{ Unity-gain follower: } \Delta v_o = A\Delta v_d = A(\Delta v_i - \Delta v_o) \Rightarrow \Delta v_o = \frac{A\Delta v_i}{A+1}$$

$$\Delta i_i = \frac{\Delta v_d}{R_d} = \frac{\Delta v_i - \Delta v_o}{R_d} = \frac{\Delta v_i}{(A+1)R_d}$$

$$R_{ai} = \frac{\Delta v_i}{\Delta i_i} = (A+1)R_d \approx AR_d, \quad R_{ai} \text{ could be } > \text{T}\Omega$$

- Noninverting amplifiers: $R_{ai} = R_d \times (\text{loop gain})$, very high, limited by surface leakage current
- Inverting amplifier: $R_{ai} = \frac{\Delta v_i}{\Delta i_i} = R_i$, usually small



Output Resistance

- Op amp output resistance, $R_o \approx 40 \Omega$
- Amplifier-circuit output resistance, R_{ao} for unity-gain follower with resistive load
 - Resistive load, $R_L \Rightarrow$ change in output current, Δi_o

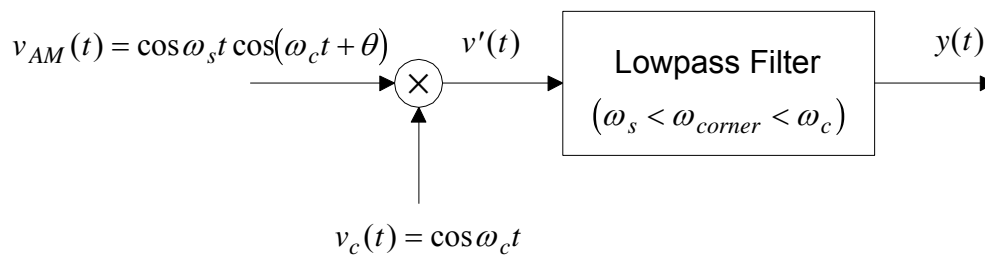
$$-\Delta v_d = \Delta v_o = A\Delta v_d + \Delta i_o R_o = -A\Delta v_o + \Delta i_o R_o \Rightarrow (A+1)\Delta v_o = \Delta i_o R_o$$

$$R_{ao} = \frac{\Delta v_o}{\Delta i_o} = \frac{R_o}{A+1} \approx \frac{R_o}{A}, R_{ao} \text{ could be } < 10\text{-}3 \Omega$$
 - All noninverting and inverting amplifiers: $R_{ao} = R_d / (\text{loop gain})$, very small, load resistance is limited by maximal output current of op amp (too small load resistance \Rightarrow op amp saturates internally)
- R_{ao} for unity-gain follower with capacitive load
 - Capacitive load, $C_L \Rightarrow i_o = C_L \frac{dv_o}{dt}$, limited by maximal output current of op amp and slew rate
 - $R_o - C_L \Rightarrow$ lowpass filter \Rightarrow additional phase shift around the loop \Rightarrow possible oscillation
 - To prevent oscillation, add a small resistor between v_o and C_L .
- Current booster for large output current: op amp + high-power transistors

3.15 Phase-Sensitive Demodulators

- Consider the amplitude-modulated (AM) signal, $v_{AM}(t) = x(t) \cos \omega_c t$
 - Signal: $x(t)$ with maximal frequency much less than $f_c = \omega_c / 2\pi$
 - Carrier: $\cos \omega_c t$ with $\omega_c = 2\pi f_c$

- Detection (or demodulation) of the sign
 - Envelope detection
 - Rectification and lowpass filtering
 - Noise at various frequencies cannot be rejected
 - Tuned amplifier or bandpass filter can remove some noise.
 - Phase-sensitive demodulation or synchronous detection
 - Multiplication or switching and lowpass filtering
 - Excellent noise rejection
- Phase-sensitive demodulation for $x(t) = \cos \omega_s t$
 - Assume the following,



$$\begin{aligned}
 v'(t) &= v_{AM}(t) \cos \omega_c t \\
 &= x(t) \cos(\omega_c t + \theta) \cos \omega_c t \\
 &= x(t) \frac{1}{2} \{ \cos(2\omega_c t + \theta) + \cos \theta \}
 \end{aligned}$$

- $y(t) = \frac{1}{2} \cos \theta x(t)$
 - If $\theta \neq \frac{\pi}{2}(2n+1)$ with $n = 0, 1, 2, \dots$ and θ is constant, then we can detect $x(t)$ from $v_{AM}(t) = x(t) \cos \omega_c t$.
 - Noise not synchronized with $v_c(t) = \cos \omega_c t$ is rejected.

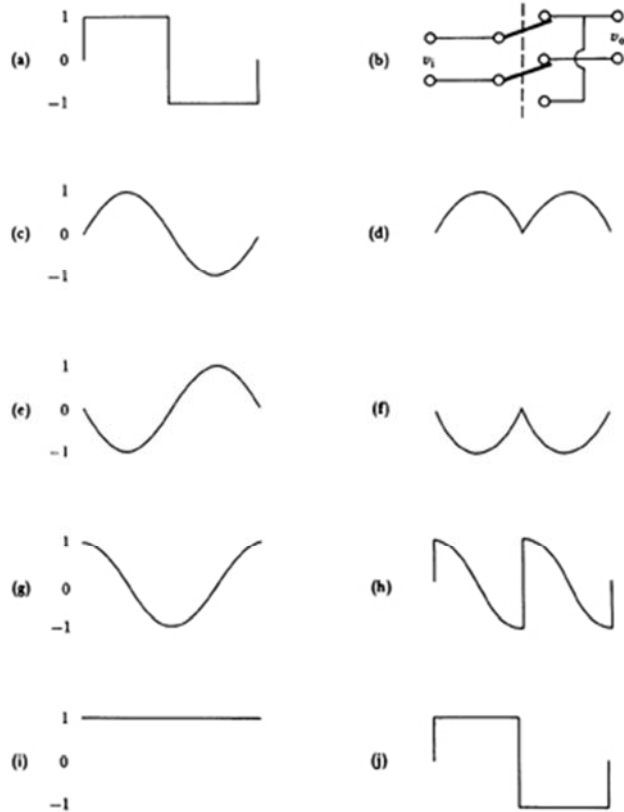


Figure 3.16 Functional operation of a phase-sensitive demodulator (a) Switching function. (b) Switch. (c), (e), (g), (i) Several input voltages. (d), (f), (h), (j) Corresponding output voltages.

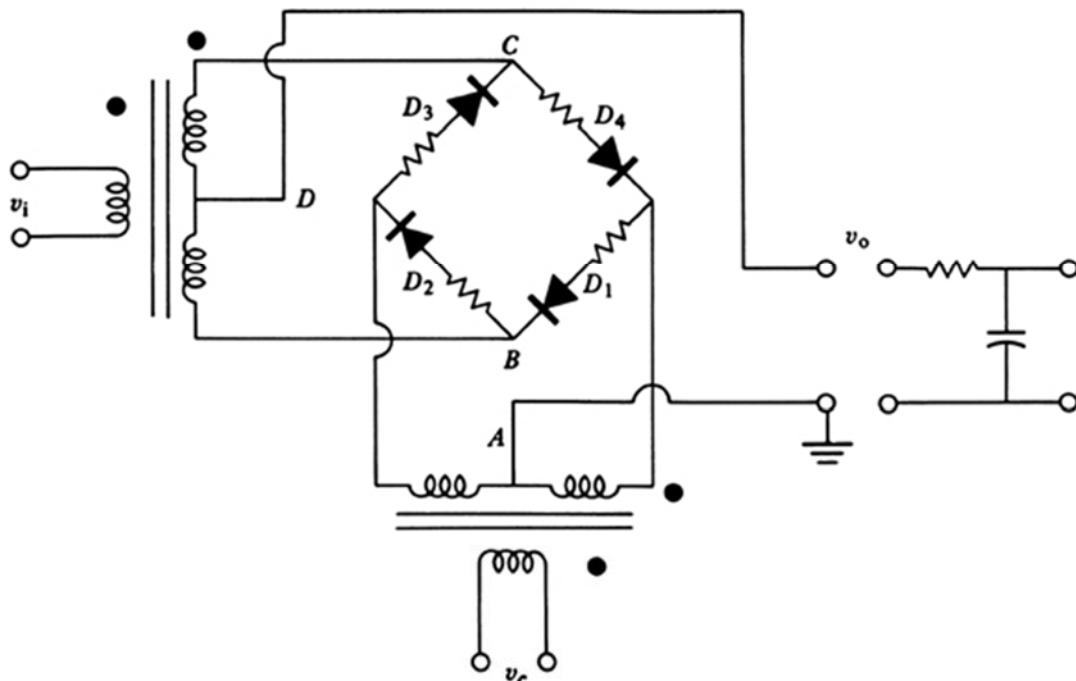
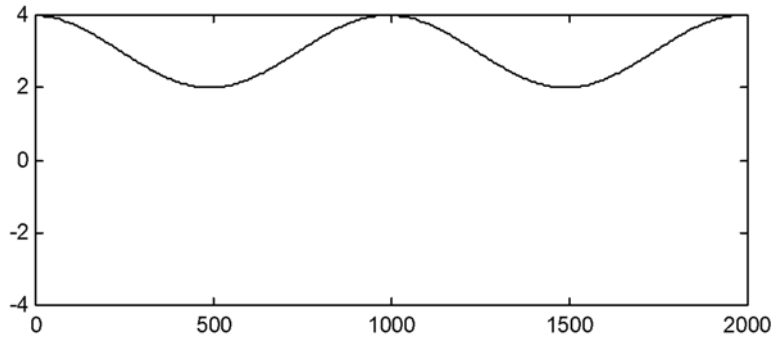
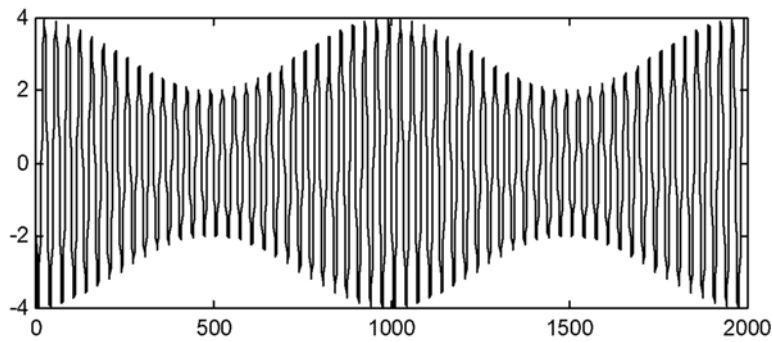


Figure 3.17 A ring demodulator This phase-sensitive detector produces a full-wave-rectified output v_o that is positive when the input voltage v_i is in phase with the carrier voltage v_c and negative when v_i is 180° out of phase with v_c .

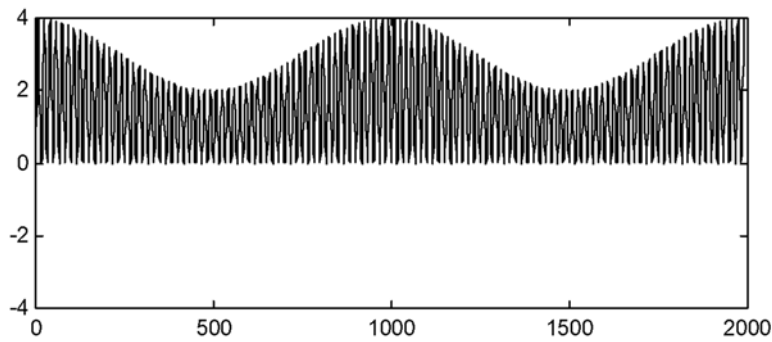
- Circuits: analog multiplier, balanced modulator/demodulator, ring demodulator



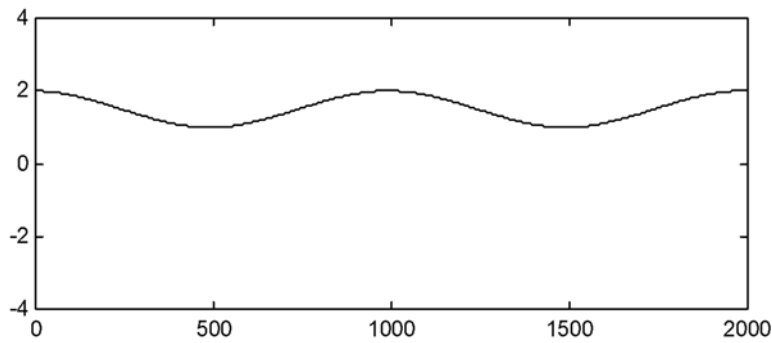
(a)



(b)



(c)



(d)

○ Example

- (a) $x(t) = \cos(2\pi t) + 3$
- (b) $v_{AM}(t) = \{\cos(2\pi t) + 3\} \cos(2\pi \times 30t)$
- (c) $v'(t) = \{\cos(2\pi t) + 3\} \cos^2(2\pi \times 30t)$
- (e) $y(t) = \frac{1}{2} \{\cos(2\pi t) + 3\}$

3.16 Microcomputers in Medical Instrumentation