

Midterm (Spring, 2009)

- 20 points are allotted to each problem.
 - There are 5 problems. See the back side also.
 - Your answer should not exceed 2 pages.
 - Provide as much details as possible.
- (1) You are given with a scalar field of $T(x, y) = (x-1)^2 + (y-3)^2$.
- (a) Plot the contour plot for $0 \leq T \leq 9$.
 - (b) At the point $(0, 2)$, compute T and ∇T . Draw ∇T on the contour plot in (a).
 - (c) At the point $(0, 2)$, compute and plot $(\nabla T \cdot \mathbf{a}_x)\mathbf{a}_x$ and $(\nabla T \cdot \mathbf{a}_y)\mathbf{a}_y$.
 - (d) Compute $\nabla^2 T$.
- (2) Consider an artificial excitable cell with a membrane that is permeable only to K^+ ions. Both intracellular and extracellular fluids are KCl solutions with different concentrations.
- (a) Derive an equation that determines the resting membrane potential (RMP) of the cell.
 - (b) Assume that $[\text{K}^+]_{out} = 40 \times [\text{K}^+]_{in}$. Compute the RMP and sketch the charge distributions inside and outside the membrane.
 - (c) Sketch an instrumentation system that measures the RMP.
- (3) We measure a bioimpedance $\mathbb{Z} = Z \angle \theta = R + jX$ between the left and right hand. Attaching a pair of electrodes on the hands, we inject current $i(t) = I \sin(2\pi \times 50000t)$ between them. We measure an induced voltage between the same electrodes and express it as $v(t)$.
- (a) Provide a mathematical expression of $v(t)$.
 - (b) Design a phase-sensitive demodulator that can extract R and X from $v(t)$. Explain how it works.
 - (c) Explain the technical problem of the method using two electrodes.
 - (d) Sketch the measurement system using the four-electrode method.

- (4) Design an ECG monitor following the procedure below.
- (a) Design an ECG amplifier that meets the following requirements.
- ECG signal: $\pm 5\text{mV}$, 0.5-30Hz
 - Electrode dc offset voltage: $\pm 300\text{mV}$
 - dc power supply: $\pm 6\text{V}$
 - Operational amplifiers saturate at $\pm 5\text{V}$
 - ADC input range: $\pm 5\text{V}$
- (b) Sketch the magnitude and phase response of the ideal ECG amplifier that produce no distortion of the input ECG signal.
- (c) The ADC converts the output of the amplifier to a 12-bit digital data. Assuming the same input range of the ADC as in (a), compute the quantization noise power.
- (d) Determine the allowable analog noise power at the output of the ECG amplifier to preserve the 12-bit accuracy.
- (5) Implement the following digital filters using the C language.
- (a) $y[n] = \frac{1}{4} \{x[n] + 2x[n-1] + x[n-2]\}.$
- (b) $y[n] = \sum_{k=0}^{99} x[n-k].$
- (c) $y[n] = x[n] - 0.5y[n-1].$