Midterm (Spring, 2009)

- 20 points are allotted to each problem.
- There are 5 problems. See the back side also.
- Your answer should not exceed 2 pages.
- Provide as much details as possible.

(1) You are given with a scalar field of $T(x, y) = (x-1)^2 + (y-3)^2$.

(a) Plot the contour plot for $0 \le T \le 9$.

(b) At the point (0, 2), compute T and ∇T. Draw ∇T on the contour plot in (a).

(c) At the point (0, 2), compute and plot $(\nabla T \cdot \mathbf{a}_x)\mathbf{a}_x$ and $(\nabla T \cdot \mathbf{a}_y)\mathbf{a}_y$.

(d) Compute $\nabla^2 T$.

(2) Consider an artificial excitable cell with a membrane that is permeable only to K^+ ions. Both intracellular and extracellular fluids are KCl solutions with different concentrations.

(a) Derive an equation that determines the resting membrane potential (RMP) of the cell.

(b) Assume that $[K^+]_{out} = 40 \times [K^+]_{in}$. Compute the RMP and sketch the charge distributions inside and outside the membrane.

(c) Sketch an instrumentation system that measures the RMP.

(3) We measure a bioimpedance $\mathbb{Z} = Z\angle\theta = R + jX$ between the left and right hand. Attaching a pair of electrodes on the hands, we inject current $i(t) = I\sin(2\pi \times 50000t)$ between them. We measure an induced voltage between the same electrodes and express it as $v(t)$.

(a) Provide a mathematical expression of $v(t)$.

(b) Design a phase-sensitive demodulator that can extract R and X from $v(t)$. Explain how it works.

(c) Explain the technical problem of the method using two electrodes.

(d) Sketch the measurement system using the four-electrode method.

(4) Design an ECG monitor following the procedure below.

(a) Design an ECG amplifier that meets the following requirements.

- ECG signal: ±5mV, 0.5-30Hz
- Electrode dc offset voltage: ±300mV
- dc power supply: ±6V
- Operational amplifiers saturate at ±5V
- ADC input range: ±5V

(b) Sketch the magnitude and phase response of the ideal ECG amplifier that produce no distortion of the input ECG signal.

(c) The ADC converts the output of the amplifier to a 12-bit digital data. Assuming the same input range of the ADC as in (a), compute the quantization noise power.

(d) Determine the allowable analog noise power at the output of the ECG amplifier to preserve the 12-bit accuracy.

(5) Implement the following digital filters using the C language.

(a) $y[n] = \frac{1}{4}\{x[n] + 2x[n-1] + x[n-2]\}$.

(b) $y[n] = \sum_{k=0}^{99} x[n-k]$.

(c) $y[n] = x[n] - 0.5y[n-1]$.