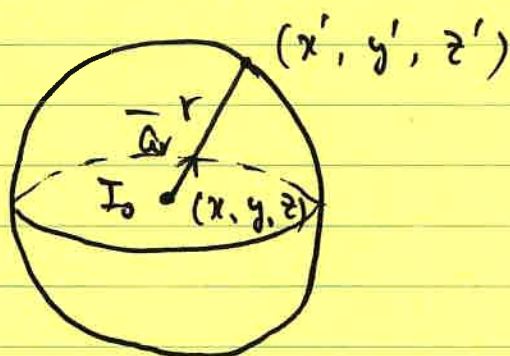


①

* Monopole Field



$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\vec{a}_r = \frac{(x'-x)\vec{a}_x + (y'-y)\vec{a}_y + (z'-z)\vec{a}_z}{r}$$

$$\vec{J} = \frac{I_0}{4\pi r^2} \vec{a}_r = \sigma \vec{E} = -\sigma \nabla' \Phi_m$$

$$\nabla' \Phi_m = -\frac{I_0}{4\pi\sigma r^2} \vec{a}_r = \frac{\partial \Phi_m}{\partial r} \vec{a}_r$$

$$\Phi_m(r) = \frac{I_0}{4\pi\sigma r}$$

22/2

$$\nabla' \left(\frac{1}{r} \right) = -\frac{\vec{a}_r}{r^2}$$

$$\nabla' \Phi_m = \frac{I_0}{4\pi\sigma} \nabla' \left(\frac{1}{r} \right) = \nabla' \left(\frac{I_0}{4\pi\sigma r} \right)$$

$$\Phi_m(r) = \frac{I_0}{4\pi\sigma r}$$

(2)

* Source strength (SS)

$$\vec{J} = -\sigma \nabla \Phi \quad (\Phi \text{ at field point})$$

$$\nabla \cdot \vec{J} = -\sigma \nabla^2 \Phi$$

$$SS = \int_V \nabla \cdot \vec{J} \, dV = -\sigma \int_V \nabla^2 \Phi \, dV$$

$$\Phi = \Phi_m(r) = \frac{I_0}{4\pi\sigma r} \quad \text{for monopole}$$

$$SS = -\frac{I_0}{4\pi} \int_V \nabla^2 \left(\frac{1}{r}\right) \, dV$$

$$= -\frac{I_0}{4\pi} \int_V \nabla \cdot \nabla \left(\frac{1}{r}\right) \, dV$$

$$= -\frac{I_0}{4\pi} \oint_S \nabla \left(\frac{1}{r}\right) \cdot \vec{ds}$$

$$= -\frac{I_0}{4\pi} \oint_S \left(-\frac{\vec{a}_r}{r^2}\right) \cdot \vec{ds}$$

(choose a sphere with $r=a$)

$$= \frac{I_0}{4\pi} \cdot \frac{1}{a^2} \cdot 4\pi a^2 = I_0$$

Note $\nabla^2 \left(\frac{1}{r}\right) = -4\pi \delta(r)$, $\int \delta(r) \, dV = 1$

$$SS = I_0 \int \delta(r) \, dV = I_0$$

(3)

$$* \quad \nabla \cdot \nabla \left(\frac{1}{r} \right) = \nabla^2 \left(\frac{1}{r} \right)$$

$$= \frac{\partial^2}{\partial x^2} \left(\frac{1}{r} \right) + \frac{\partial^2}{\partial y^2} \left(\frac{1}{r} \right) + \frac{\partial^2}{\partial z^2} \left(\frac{1}{r} \right)$$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\frac{\partial}{\partial x} \left(\frac{1}{r} \right) = \frac{\partial}{\partial x} \left\{ (x-x')^2 + (y-y')^2 + (z-z')^2 \right\}^{-\frac{1}{2}}$$

$$= -\frac{1}{2} \left\{ (x-x')^2 + (y-y')^2 + (z-z')^2 \right\}^{-\frac{3}{2}} \cdot 2(x-x')$$

$$= -\frac{x-x'}{r^3}$$

$$\frac{\partial^2}{\partial x^2} \left(\frac{1}{r} \right) = -\frac{r^3 - (x-x')^3 r^2 \frac{\partial r}{\partial x}}{r^6}$$

$$= -\frac{1}{r^3} + \frac{3(x-x') \frac{1}{2} \left\{ (x-x')^2 + (y-y')^2 + (z-z')^2 \right\}^{-\frac{1}{2}} \cdot 2(x-x')}{r^4}$$

$$= -\frac{1}{r^3} + \frac{3(x-x')^2}{r^5}$$

$$\nabla^2 \left(\frac{1}{r} \right) = -\frac{1}{r^3} + \frac{3(x-x')^2}{r^5} - \frac{1}{r^3} + \frac{3(y-y')^2}{r^5} - \frac{1}{r^3} + \frac{3(z-z')^2}{r^5}$$

$$= -\frac{3}{r^3} + \frac{3}{r^3} = 0$$

$$\oint_S \nabla \cdot \vec{J} \, d\tau = -\sigma \int_V \nabla^2 \Phi \, d\tau = I_0 \quad (4)$$

for a distributed current,

$$I_0 = \int_V I_r \, d\tau$$

Then,

$$\nabla \cdot \vec{J} = I_r = -\sigma \nabla^2 \Phi$$

$$\left\{ \begin{array}{l} \text{Poisson equation: } \nabla^2 \Phi = -\frac{I_r}{\sigma} \\ \text{Laplace equation: } \nabla^2 \Phi = 0 \end{array} \right.$$

$$\Phi(\underbrace{x', y', z'}_{\text{field point}}) = \frac{1}{4\pi\sigma} \int_V \frac{I_r}{r} \, d\tau$$

for multipole field,

$$\Phi(x', y', z') = \frac{1}{4\pi\sigma} \sum \frac{I_{0j}}{r_j}$$

(5)

* Duality between free space and volume conductor

$$\rho \cdot \begin{matrix} \epsilon \\ \sigma = 0 \end{matrix}$$

$$I_0 \cdot \begin{matrix} \sigma \neq 0 \\ \epsilon : \text{negligible} \end{matrix}$$

$$\vec{E} = \frac{1}{4\pi\epsilon} \frac{\rho}{r^2} \vec{a}_r$$

$$\vec{D} = \epsilon \vec{E} = \frac{1}{4\pi} \frac{\rho}{r^2} \vec{a}_r$$

$$\vec{J} = \frac{I_0}{4\pi r^2} \vec{a}_r$$

$$\oint_S \vec{D} \cdot d\vec{s} = \rho$$

$$\oint_S \vec{J} \cdot d\vec{s} = I_0$$

$$\vec{E} = -\nabla\Phi = \frac{1}{\epsilon} \vec{D}$$

$$\vec{E} = -\nabla\Phi$$

$$\nabla\Phi = -\frac{1}{\epsilon} \frac{1}{4\pi} \frac{\rho}{r^2} \vec{a}_r$$

$$\vec{J} = -\sigma \nabla\Phi$$

$$= \nabla \left(\frac{\rho}{4\pi\epsilon r} \right)$$

$$\nabla\Phi = -\frac{1}{\sigma} \frac{1}{4\pi} \frac{I_0}{r^2} \vec{a}_r$$

$$= \nabla \left(\frac{I_0}{4\pi\sigma r} \right)$$

$$\Phi = \frac{\rho}{4\pi\epsilon r}$$

$$\Phi = \frac{I_0}{4\pi\sigma r}$$