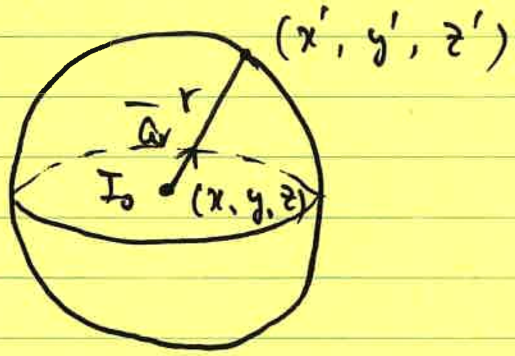
* Monopole Field



$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\bar{a}_r = \frac{(x'-x)\bar{a}_x + (y'-y)\bar{a}_y + (z'-z)\bar{a}_z}{r}$$

$$\bar{J} = \frac{I_0}{4\pi r^2}\bar{a}_r = \sigma \bar{E} = -\sigma \nabla' \Phi_m$$

$$\nabla' \Phi_m = -\frac{I_0}{4\pi\sigma r^2}\bar{a}_r = \frac{\partial \Phi_m}{\partial r}\bar{a}_r$$

$$\Phi_m(r) = \frac{I_0}{4\pi\sigma r}$$

또는

$$\nabla'\left(\frac{1}{r}\right) = -\frac{\bar{a}_r}{r^2}$$

$$\nabla' \Phi_m = \frac{I_0}{4\pi\sigma}\nabla'\left(\frac{1}{r}\right) = \nabla'\left(\frac{I_0}{4\pi\sigma r}\right)$$

$$\Phi_m(r) = \frac{I_0}{4\pi\sigma r}$$

* Source Strength (SS)

$$\bar{J} = -\sigma \nabla \Phi \quad (\Phi \text{ at field point})$$

$$\nabla \cdot \bar{J} = -\sigma \nabla^2 \Phi$$

$$SS = \int_v \nabla \cdot \bar{J} \, dv = -\sigma \int_v \nabla^2 \Phi \, dv$$

$$\Phi = \Phi_m(r) = \frac{I_o}{4\pi\sigma r} \quad \text{for monopole}$$

$$SS = -\frac{I_o}{4\pi} \int_v \nabla^2 \left(\frac{1}{r}\right) dv$$

$$= -\frac{I_o}{4\pi} \int_v \nabla \cdot \nabla \left(\frac{1}{r}\right) dv$$

$$= -\frac{I_o}{4\pi} \oint_s \nabla \left(\frac{1}{r}\right) \cdot \bar{ds}$$

$$= -\frac{I_o}{4\pi} \oint_s \left(-\frac{\bar{a}_r}{r^2}\right) \cdot \bar{ds}$$

(Choose a sphere with $r = a$)

$$= \frac{I_o}{4\pi} \cdot \frac{1}{a^2} \cdot 4\pi a^2 = I_o$$

Note $\nabla^2 \left(\frac{1}{r}\right) = -4\pi \delta(r)$, $\int \delta(r) \, dv = 1$

$$SS = I_o \int \delta(r) \, dv = I_o$$

③

$$* \quad \nabla \cdot \nabla\left(\frac{1}{r}\right) = \nabla^2\left(\frac{1}{r}\right)$$

$$= \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\right)$$

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = \frac{\partial}{\partial x}\left\{(x-x')^2 + (y-y')^2 + (z-z')^2\right\}^{-\frac{1}{2}}$$

$$= -\frac{1}{2}\left\{(x-x')^2 + (y-y')^2 + (z-z')^2\right\}^{-\frac{3}{2}} \cdot 2(x-x')$$

$$= -\frac{x-x'}{r^3}$$

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) = -\frac{r^3 - (x-x')\, 3r^2 \frac{\partial r}{\partial x}}{r^6}$$

$$= -\frac{1}{r^3} + \frac{3(x-x')\, \frac{1}{2}\left\{(x-x')^2 + (y-y')^2 + (z-z')^2\right\}^{-\frac{1}{2}} \cdot 2(x-x')}{r^4}$$

$$= -\frac{1}{r^3} + \frac{3(x-x')^2}{r^5}$$

$$\nabla^2\left(\frac{1}{r}\right) = -\frac{1}{r^3} + \frac{3(x-x')^2}{r^5} - \frac{1}{r^3} + \frac{3(y-y')^2}{r^5} - \frac{1}{r^3} + \frac{3(z-z')^2}{r^5}$$

$$= -\frac{3}{r^3} + \frac{3}{r^3} = 0$$

$$SS = \int_v \nabla \cdot \bar{J} \, dv = -\sigma \int_v \nabla^2 \Phi \, dv = I_o$$

for a distributed current,

$$I_o = \int_v I_v \, dv$$

Then,

$$\nabla \cdot \bar{J} = I_v = -\sigma \nabla^2 \Phi$$

$$\begin{cases} \text{Poisson equation:} & \nabla^2 \Phi = -\dfrac{I_v}{\sigma} \\ \text{Laplace equation:} & \nabla^2 \Phi = 0 \end{cases}$$

$$\Phi \underbrace{(x', y', z')}_{\text{field point}} = \frac{1}{4\pi\sigma} \int_v \frac{I_v}{r} \, dv$$

For multipole field,

$$\Phi (x', y', z') = \frac{1}{4\pi\sigma} \sum \frac{I_{oj}}{r_j}$$

* Duality between free space and volume conductor

$Q \cdot$ ε, $\sigma = 0$

$I_0 \cdot$ $\sigma \neq 0$, ε : negligible

$$\bar{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\bar{a}_r$$

$$\bar{D} = \varepsilon\bar{E} = \frac{1}{4\pi}\frac{Q}{r^2}\bar{a}_r \qquad \bar{J} = \frac{I_0}{4\pi r^2}\bar{a}_r$$

$$\oint_s \bar{D}\cdot\bar{d_s} = Q \qquad \oint_s \bar{J}\cdot d_s = I_0$$

$$\bar{E} = -\nabla\Phi = \frac{1}{\varepsilon}\bar{D} \qquad \bar{E} = -\nabla\Phi$$

$$\nabla\Phi = -\frac{1}{\varepsilon}\frac{1}{4\pi}\frac{Q}{r^2}\bar{a}_r \qquad \bar{J} = -\sigma\nabla\Phi$$

$$= \nabla\left(\frac{Q}{4\pi\varepsilon r}\right) \qquad \nabla\Phi = -\frac{1}{\sigma}\frac{1}{4\pi}\frac{I_0}{r^2}\bar{a}_r$$

$$= \nabla\left(\frac{I_0}{4\pi\sigma r}\right)$$

$$\Phi = \frac{Q}{4\pi\varepsilon r} \qquad \Phi = \frac{I_0}{4\pi\sigma r}$$