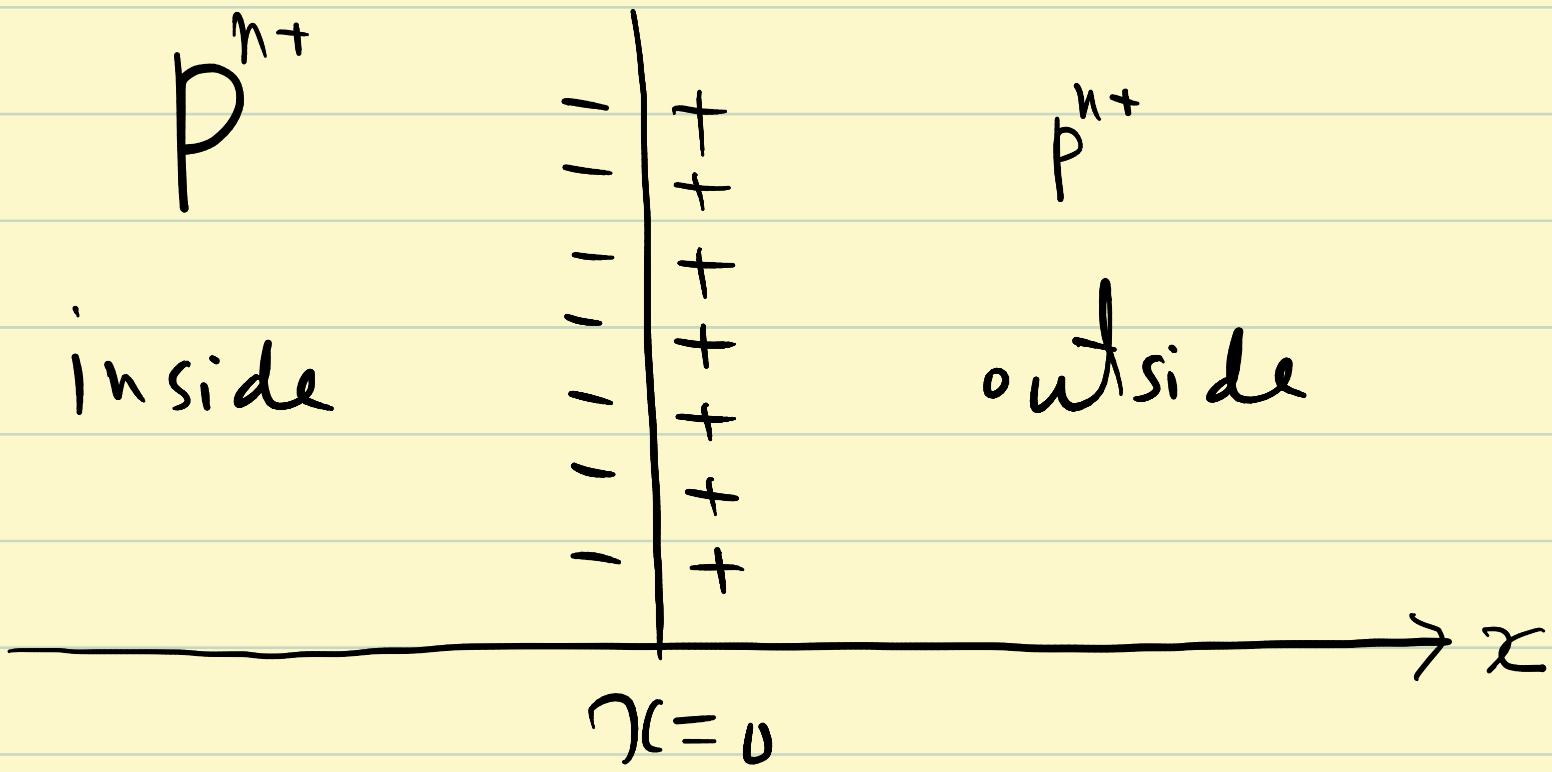
Resting Membrane Potential (RMP) ①



$\longrightarrow$ Diffusion

$\longleftarrow$ Electric field

Ⓐ For p^{n+} ions outside,

Coulomb force $\quad \bar{F} = nq\bar{E} = m\bar{a}$ $\quad$ Acceleration

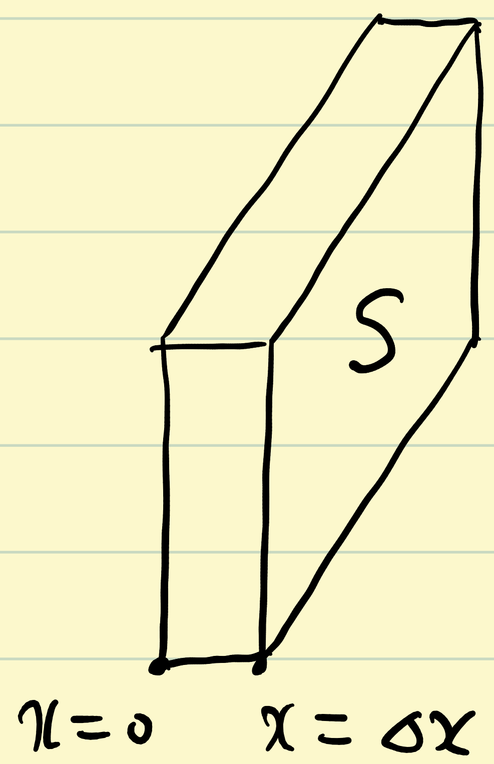
Drift Velocity $\quad \bar{u} = \mu\bar{E} = -\mu\nabla V$ $\quad$ Mobility $\quad$ Voltage

$$q = 1.6 \times 10^{-19}\ C$$

For one-dimensional case,

$$u(x) = -\mu\frac{dV(x)}{dx}$$

Consider a volume at $x=0$;



$$\Delta x = u(x)\,\Delta t$$

$$\Delta V(x) = S\,u(x)\,\Delta t$$

Volume

$$\Delta N(x) = S\,u(x)\,\Delta t\;A_v\,c(x)$$

Number of ions in $\Delta V(x)$

Molar concentration

$A_v = 6.022 \times 10^{23}$: Avogardro number

$$\Delta Q(x) = nq\,\Delta N(x) = nq\,S\,u(x)\,\Delta t\,A_v\,c(x)$$

Charge in ΔV

$$i_E(x) = \frac{\Delta Q(x)}{\Delta t} = nq\,S\,u(x)\,A_v\,c(x)$$

Current by Electric field

$$J_E(x) = \frac{i_E(x)}{S} = -\mu nq\,\frac{dv(x)}{dx}\,A_v\,c(x)$$

Current Density by Electric field

B for p^{n+} ions inside,

$$J_D(x) = -DA_v \frac{dc(x)}{dx}$$

Current Density by Diffusion

Diffusion constant

from A and B,

$$J(x) = J_E(x) + J_D(x)$$

$$= -\mu n q \frac{dv(x)}{dx} A_v c(x) - DA_v \frac{dc(x)}{dx}$$

From Einstein's relation,

$$\mu = \frac{D}{T k_B}$$

$$\begin{cases} k_B = 1.38 \times 10^{-23} \text{ J/K} & \text{Boltzmann constant} \\ F = A_v q = 96,485.3399 \text{ C/mol} & \text{Faraday constant} \\ R = k_B A_v = 8.314472 \text{ J/K·mol} & \end{cases}$$

Universal gas constant

$$\Rightarrow \mu n q = \frac{nF}{RT} D$$

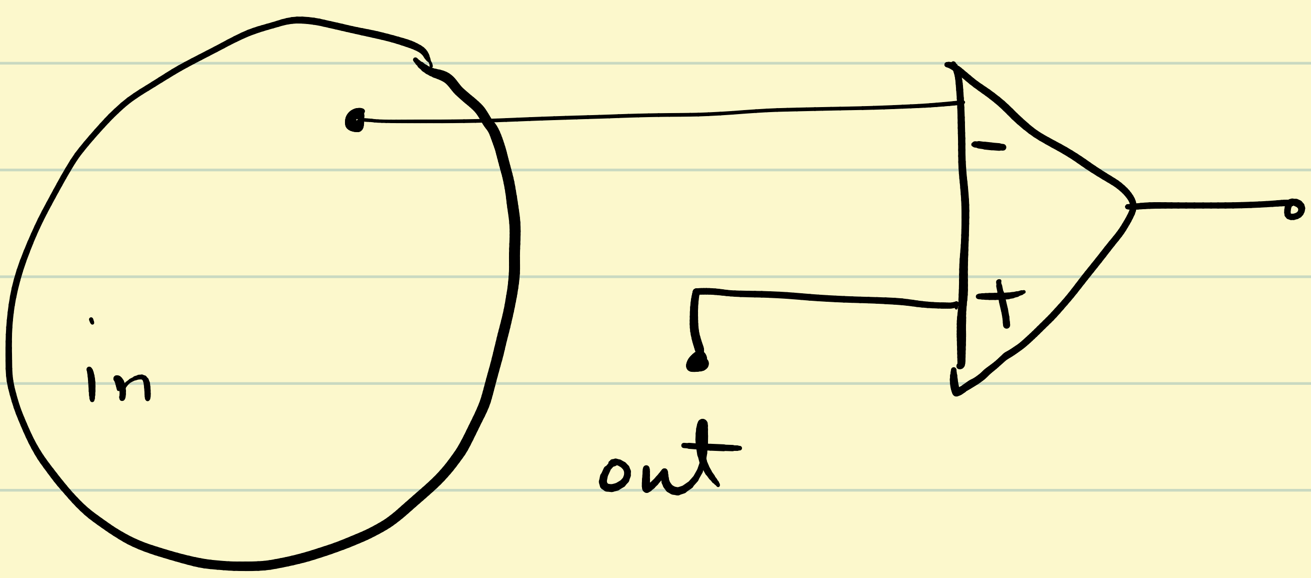
$$J(x) = -\frac{nF}{RT} A_v D \frac{dv(x)}{dx} c(x) - A_v D \frac{dc(x)}{dx}$$

At dynamic equilibrium, $J(x) = 0$.

$$\frac{nF}{RT}\frac{dv(x)}{dx} = -\frac{1}{c(x)}\frac{dc(x)}{dx}$$

$$\frac{nF}{RT}\int_{x_{in}}^{x_{out}} \frac{dv(x)}{dx}\,dx = -\int_{x_{in}}^{x_{out}} \frac{1}{c(x)}\frac{dc(x)}{dx}\,dx$$

$$\frac{nF}{RT}\left[v(x_{out}) - v(x_{in})\right] = -\left[\ln c(x_{out}) - \ln c(x_{in})\right]$$



v_{RMP}

$v(x_{out}) = 0$

$$v_{RMP} = v(x_{in}) = \frac{RT}{nF}\ln\frac{C_{out}}{C_{in}}$$

$$= 61.5 \log_{10}\frac{C_{out}}{C_{in}} \quad [mV] \quad \text{at } 37^\circ C$$

$\boxed{\text{Note}}$ For negative ions,

$$V_{RMP} = \frac{RT}{nF} \ln \frac{C_{in}}{C_{out}}$$

$\boxed{\text{Note}}$ For multiple ions,

use "Superposition".

$\boxed{\text{Note}}$ V_{RMP} : equilibrium voltage

or

Nernst potential

$\boxed{\text{Note}}$ $V_{RMP} = -90 \text{ mV}$ for muscle

$V_{RMP} = -60 \text{ mV}$ for nerve