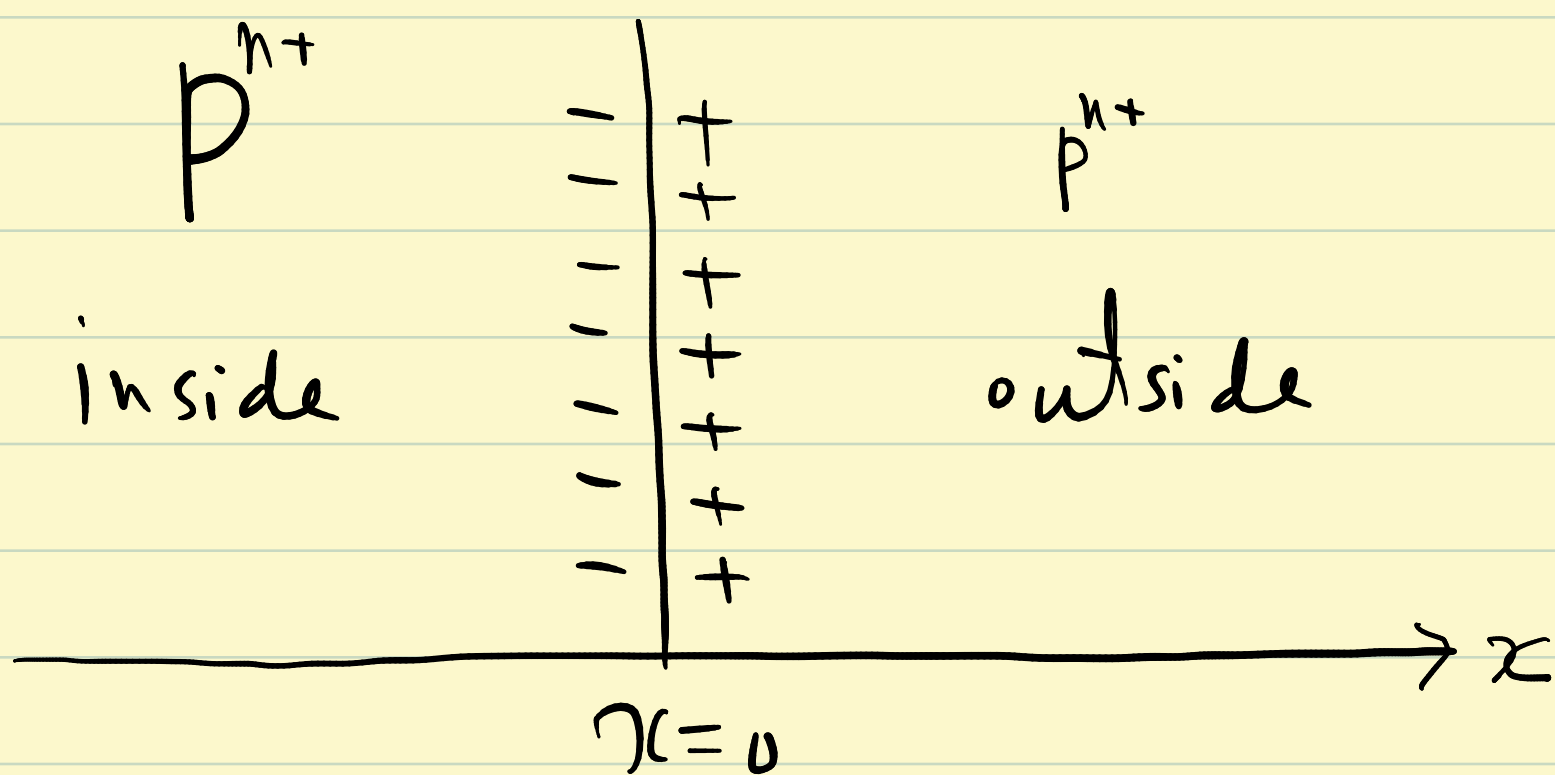


Resting Membrane Potential (RMP) ①



→ Diffusion

← Electric field

① For P^{n+} ions outside, Acceleration

Coulomb force $\vec{F} = nq\vec{E} = m\vec{a}$ Mobility

Drift Velocity $\vec{u} = \mu\vec{E} = -\mu\nabla V$ Voltage

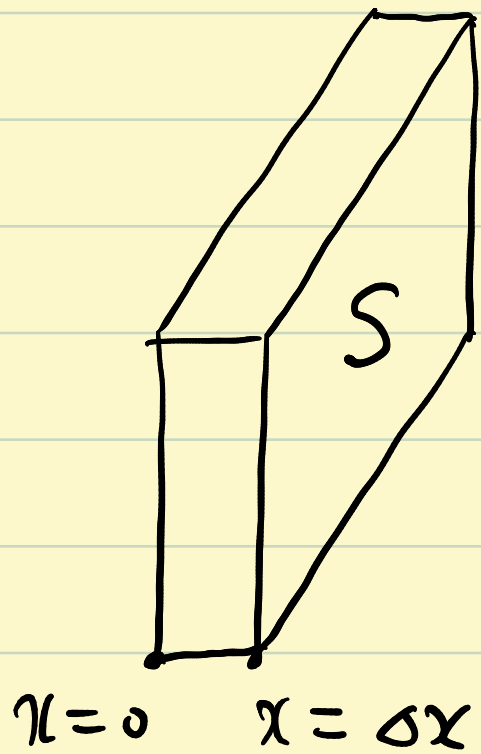
$$q = 1.6 \times 10^{-19} \text{ C}$$

For one-dimensional case,

$$u(x) = -\mu \frac{dV(x)}{dx}$$

(2)

Consider a volume at $x=0$;



$$\Delta x = u(x) \Delta t$$

$$\Delta V(x) = S u(x) \Delta t$$

↑
Volume

$$\Delta N(x) = S u(x) \Delta t A_v c(x)$$

Number of ions in $\Delta V(x)$ Molar concentration

$$A_v = 6.022 \times 10^{23}$$

: Avogadro number

$$\Delta Q(x) = n_f \Delta N(x) = n_f S u(x) \Delta t A_v c(x)$$

↓
Charge in ΔV

$$\dot{I}_E(x) = \frac{\Delta Q(x)}{\Delta t} = n_f S u(x) A_v c(x)$$

Current by Electric field

$$J_E(x) = \frac{\dot{I}_E(x)}{S} = -\mu n_f \frac{dV(x)}{dx} A_v c(x)$$

↓
Current Density by Electric field

(B) For p^{n+} ions inside,

(3)

$$J_D(x) = -DA_v \frac{dc(x)}{dx}$$

Current Density by Diffusion \rightarrow Diffusion constant

From (A) and (B),

$$J(x) = J_E(x) + J_D(x)$$
$$= -\mu nq \frac{dv(x)}{dx} A_v c(x) - DA_v \frac{dc(x)}{dx}$$

From Einstein's relation,

$$\mu = \frac{D}{T k_B}$$

$$\left\{ \begin{array}{l} k_B = 1.38 \times 10^{-23} \text{ J/K} \quad \text{Boltzmann constant} \\ F = A_v q = 96,485.3399 \text{ C/mol} \quad \text{Faraday constant} \\ R = k_B A_v = 8,314472 \text{ J/K}\cdot\text{mol} \quad \text{Universal gas constant} \end{array} \right.$$

$$\Rightarrow \mu nq = \frac{nF}{RT} D$$

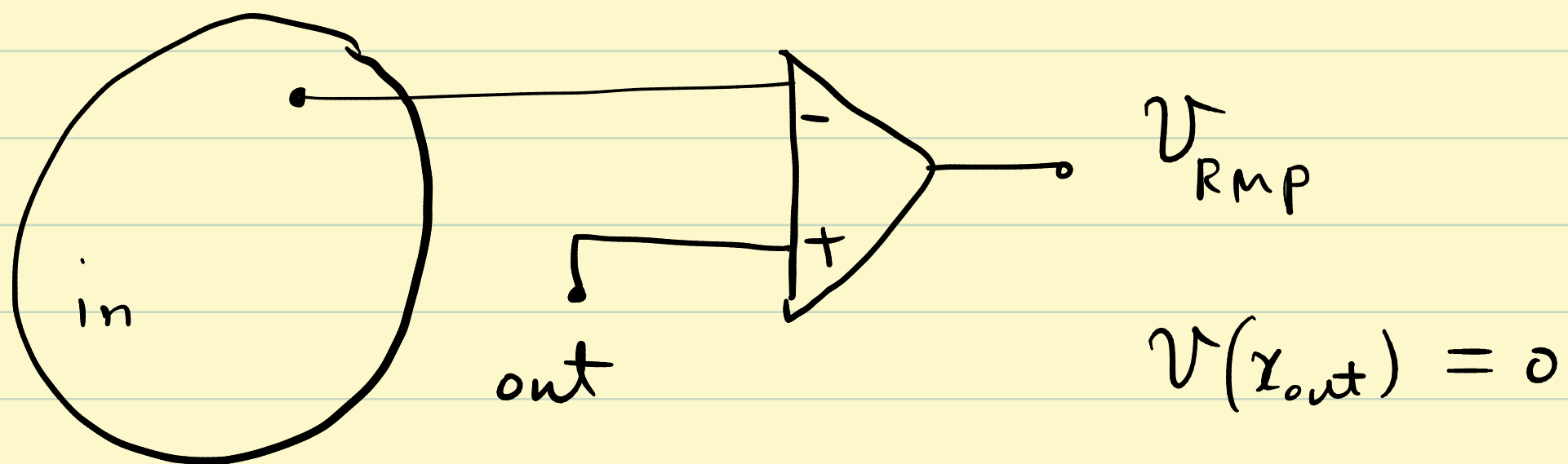
$$J(x) = -\frac{nF}{RT} A_r D \frac{dV(x)}{dx} c(x) - A_r D \frac{dC(x)}{dx} \quad (4)$$

At dynamic equilibrium, $J(x) = 0$.

$$\frac{nF}{RT} \frac{dV(x)}{dx} = -\frac{1}{C(x)} \frac{dC(x)}{dx}$$

$$\frac{nF}{RT} \int_{x_{in}}^{x_{out}} \frac{dV(x)}{dx} dx = - \int_{x_{in}}^{x_{out}} \frac{1}{C(x)} \frac{dC(x)}{dx} dx$$

$$\frac{nF}{RT} [V(x_{out}) - V(x_{in})] = - [\ln C(x_{out}) - \ln C(x_{in})]$$



$$V_{RMP} = V(x_{in}) = \frac{RT}{nF} \ln \frac{C_{out}}{C_{in}}$$

$$= 61.5 \log_{10} \frac{C_{out}}{C_{in}} \text{ [mV] at } 37^\circ\text{C}$$

5

Note

For negative ions,

$$V_{RMP} = \frac{RT}{nF} \ln \frac{C_{in}}{C_{out}}$$

Note

For multiple ions,

Use "Superposition".

Note

V_{RMP} : equilibrium voltage
or
Nernst potential

Note

$V_{RMP} = -90 \text{ mV}$ for muscle

$V_{RMP} = -60 \text{ mV}$ for nerve