Common Mode Rejection Ratio for Cascaded Differential Amplifier Stages

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***Abstract*—We provide a simple equation for calculating the total common mode rejection ratio, $CMRR_T$, for cascaded differential amplifier stages and apply it to several typical cases in instrumentation circuits. We define two factors for each stage, C and D, and show that, in general, $CMRR_T$ can be calculated by adding the reciprocals of the equivalent $CMRR$ for each stage, which we define as the product of its C factor and the product of D factors for the preceding stages.**

I. Introduction

The NEED for *CMRR* calculations in cascaded differential stages arises in many circuits dealing with differential signals. A simple case is the three-op-amp instrumentation amplifier (IA), for which a complete analysis has already been published [1]. No general rule is known, however, for cases with three or more stages. These cases arise, for example, in time-division multiplexed systems when there is a differential multiplexer between input buffers and a common differential amplifier (DA) based on a single op amp or an IA, or, more commonly, when the effect of input stray capacitance imbalance is considered in IAs. Another case is the use of voltage divider probes in oscilloscopes with differential inputs.

According to [2], a circuit stage with a differential input and a differential output can be described by means of four transfer functions. These are G_{DD}, G_{CC}, G_{CD}, and G_{DC}. G_{DD} is the differential mode gain: output differential mode signal/input differential mode signal, when there is no input common mode signal. G_{CC} is the common mode gain: output common mode signal/input common mode signal, when there is no input differential mode signal. G_{CD} is the differential to common mode gain, that is, the output common mode signal due to the input differential signal. G_{DC} is the common mode to differential gain, that is, the output differential signal due to the input common mode signal. When cascading more than two of these stages, however, the use of these transfer functions is very cumbersome.

In this paper we give a simple rule to calculate the total common mode rejection ratio ($CMRR_T$) for several cascaded differential input/differential output stages connected to a last stage with a single-ended output. We provide the equations that allow us to verify whether or not the approximate formulas are acceptable, and give application examples based on typical situations in instrumentation circuits.

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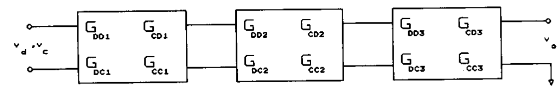

Fig. 1. Differential system consisting of three cascaded stages, the last one with a single-ended output.

II. *CMRR* Calculation

Fig. 1 shows a system consisting of three differential stages, the last stage having a single-ended output. As usual when dealing with transfer functions, we assume that no stage loads the preceding one. By applying the foregoing definitions for the transfer functions for each stage, we obtain

$$\begin{aligned} v_o = {} & v_d[G_{DD1}G_{DD2}G_{DD3} + G_{DD1}G_{CD2}G_{DC3} \\ & + G_{CD1}G_{DC2}G_{DD3} + G_{CD1}G_{CC2}G_{DC3}] \\ & + v_c[G_{DC1}G_{DD2}G_{DD3} + G_{DC1}G_{CD2}G_{DC3} \\ & + G_{CC1}G_{DC2}G_{DD3} + G_{CC1}G_{CC2}G_{DC3}]. \end{aligned} \quad (1)$$

The common mode rejection ratio for the system is defined as the quotient between the output due to a differential input voltage and the output due to a common mode voltage of the same magnitude. We have therefore,

$$CMRR_T = \frac{G_{DD1}(G_{DD2}G_{DD3} + G_{CD2}G_{DC3}) + G_{CD1}(G_{DC2}G_{DD3} + G_{CC2}G_{DC3})}{G_{DC1}(G_{DD2}G_{DD3} + G_{CD2}G_{DC3}) + G_{CC1}(G_{DC2}G_{DD3} + G_{CC2}G_{DC3})}. \quad (2)$$

Let us define three ratios for each stage:

$$C_i = G_{DDi}/G_{DCi} \quad (3)$$

$$D_i = G_{DDi}/G_{CCi} \quad (4)$$

$$E_i = G_{CDi}/G_{DDi}. \quad (5)$$

Now, if the second and third stages were both perfect from the point of view of differential amplification, $G_{CD2} = G_{DC2} = G_{DC3} = 0$, and $CMRR_T$ would be determined by the first stage, that is, $CMRR_T = CMRR_1$ in this case. By applying this in (2) we obtain

$$CMRR_1 = \frac{G_{DD1}}{G_{DC1}} = C_1. \tag{6}$$

If the first and third stages were the perfect stages, we would have

$$CMRR_2 = \frac{G_{DD1}}{G_{CC1}} \frac{G_{DD2}}{G_{DC2}} = D_1 C_2. \tag{7}$$

If the first and second stages were the perfect stages, we would have

$$CMRR_3 = \frac{G_{DD1}}{G_{CC1}} \frac{G_{DD2}}{G_{CC2}} \frac{G_{DD3}}{G_{DC3}} = D_1 D_2 C_3. \tag{8}$$

By rearranging (2) and using the definitions in (3) and (4), we obtain

$$\frac{1}{CMRR_T} = \frac{\frac{1}{C_1}\left(1 + \frac{G_{CD2}}{G_{DD2}}\frac{1}{C_3}\right) + \frac{1}{D_1}\left(\frac{1}{C_2} + \frac{1}{D_2}\frac{1}{C_3}\right)}{1 + \frac{G_{CD2}}{G_{DD2}}\frac{1}{C_3} + \frac{G_{CD1}}{G_{DD1}}\left(\frac{1}{C_2} + \frac{1}{D_2}\frac{1}{C_3}\right)}. \tag{9}$$

Whenever $E_i << C_{i+1}$ and $E_i << D_{i+1}C_{i+2}$, ($i = 1, 2, 3$), (9) simplifies to

$$\frac{1}{CMRR_T} \approx \frac{1}{CMRR_1} + \frac{1}{CMRR_2} + \frac{1}{CMRR_3}. \tag{10}$$

Therefore, the total *CMRR* can be calculated by adding the reciprocals of the equivalent *CMRRs* for each stage. The equivalent *CMRR* for each stage is the product of its C factor, times the product of the D factors for the preceding stages. Note that for stages with a single-ended output, i.e., without a common mode output voltage, the C factor coincides with the usual definition for the *CMRR*, D is infinite because $G_{CC} = 0$, and $E = 0$ because $G_{CD} = 0$. The extension of the rule formulated by (10) to more than three stages is straightforward.

III. Application

The preceding rule for $CMRR_T$ calculation is so simple that it is instructive to analyze whether the simplifications leading from (9) to (10) are acceptable in typical cases. One situation of interest consists of what we call "noncoupled paths stage," shown in Fig. 2. The differential signal goes through two independent paths, each described by its own transfer function. By applying the corresponding definitions for differential and common mode transfer functions, we obtain

$$G_{DD} = [H_1(s) + H_2(s)]/2 \tag{11a}$$

$$G_{CD} = [H_1(s) - H_2(s)]/4 \tag{11b}$$

$$G_{DC} = H_1(s) - H_2(s) \tag{11c}$$

$$G_{CC} = [H_1(s) + H_2(s)]/2. \tag{11d}$$

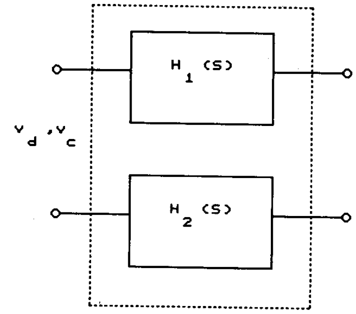

Fig. 2. A stage with noncoupled paths: The differential signal goes through two independent paths, each one described by its own transfer function.

From here we see that for noncoupled paths we have $D = 1$, $E = 1/4C$, regardless of the particular implementation. Factors C and E depend on transfer functions $H_1(s)$ and $H_2(s)$.

For a differential analog multiplexer integrated circuit, for example, the situation can be described by the model in Fig. 3(a). R_1 and R_2 are the respective ON resistances for each channel, and C_1' and C_2' are their output capacitances. Therefore, we have

$$H_1(s) = \frac{\omega_1}{s + \omega_1} \tag{12a}$$

$$H_2(s) = \frac{\omega_2}{s + \omega_2} \tag{12b}$$

where $\omega_1 = 1/R_1C_1'$ and $\omega_2 = 1/R_2C_2'$. We can take into account the mismatch between channels by defining average and incremental resistance capacitance as follows,

$$R_a = (R_1 + R_2)/2 \tag{13a}$$

$$R_i = R_1 - R_2 \tag{13b}$$

$$C_a = (C_1' + C_2')/2 \tag{14a}$$

$$C_i = C_1' - C_2'. \tag{14b}$$

Finally we obtain,

$$C \approx \frac{j\omega R_a C_a + 1}{-j\omega(R_a C_i + R_i C_a)}. \tag{15}$$

Specifications for the DG507A model (Siliconix), for example, are $R_a = 270\ \Omega$, $R_i/R_a = 6\%$, $C_a = 23$ pF. C_i is assumed to be zero. By applying (15), at 10 kHz we obtain $C = j42735 = 92.6$ dB$\underline{/90°}$. Then $E = -104.6$ dB$\underline{/-90°}$, and the assumptions leading to (10) will be correct even if the succeeding stage is far from ideal, i.e., its factor C is not very high. Note that for this stage $CMRR = C$, which has a 90° phase shift.

Another example of a noncoupled differential stage is provided by two voltage divider probes connected to an oscilloscope with a differential input (Fig. 3(b)). We assume that the probes are compensated before measuring, i.e., $R_{o1}C_{o1} = R_{p1}C_{p1}$, $R_{o2}C_{o2} = R_{p2}C_{p2}$. Then we have

$$H_1(s) = \frac{R_{o1}}{R_{o1} + R_{p1}} \tag{16a}$$

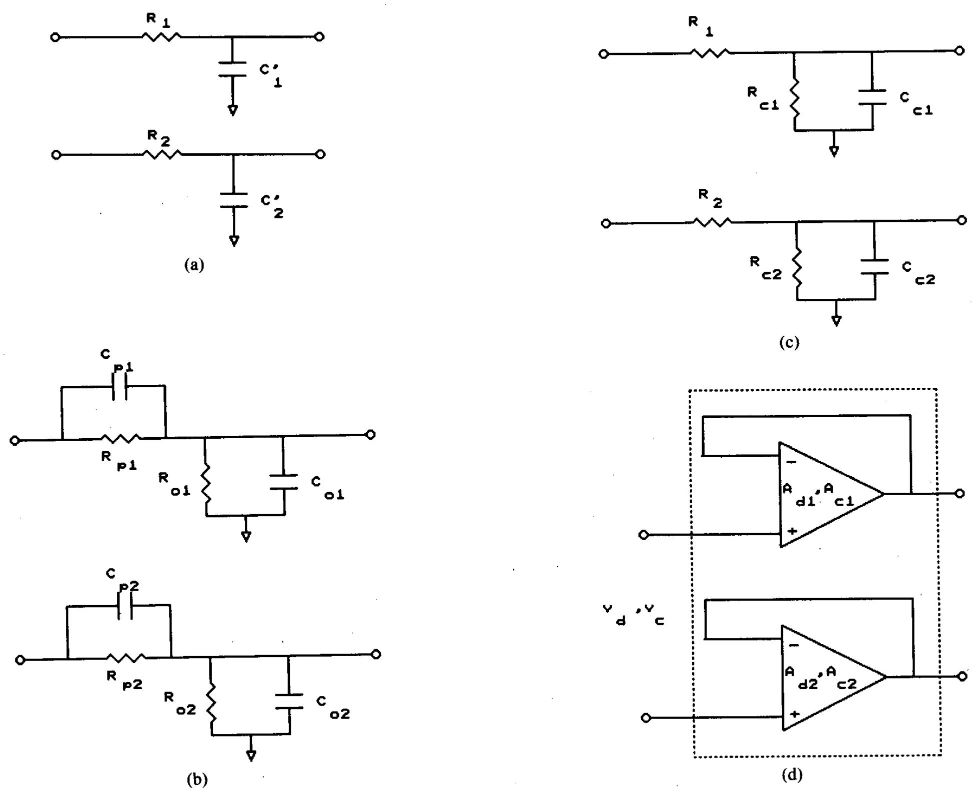

Fig. 3. Different examples of noncoupled differential stages. (a) differential multiplexer, (b) voltage divider probes connected to a differential oscilloscope, (c) source impedance and common mode input impedance in differential amplifier, (d) unity gain buffers pair.

$$H_2(s) = \frac{R_{o2}}{R_{o2} + R_{p2}} \tag{16b}$$

and

$$C = \frac{1}{2}\frac{2R_{o1}R_{o2} + R_{o1}R_{p2} + R_{o2}R_{p1}}{R_{o1}R_{p2} - R_{o2}R_{p1}}. \tag{17}$$

Resistor mismatching can be taken into account by considering their tolerance t. If we assume that t is the same for all the resistors involved and call k the ratio between nominal values, $k = R_{p0}/R_{o0}$, in a worst-case condition we have

$$C \approx \frac{1+k}{4kt}. \tag{18}$$

$$C = \frac{1}{2}\frac{2R_{c1}R_{c2} + R_2R_{c1} + R_1R_{c2} + sR_{c1}R_{c2}(R_1C_{c1} + R_2C_{c2})}{R_2R_{c1} - R_1R_{c2} + sR_{c1}R_{c2}(R_2C_{c2} - R_1C_{c1})}. \tag{20}$$

For a 10:1 probe, $k = 9$ and therefore the value for E will usually be low enough to accept (10), but at the same time we see that extremely low tolerances would be necessary to obtain a $CMRR_T$, say, larger than 60 dB. This suggests the provision for resistor adjustment in probes intended for differential measurements, in addition to the common probe compensation trimmer capacitor.

A third example of noncoupled paths is provided by input imbalance in differential amplifiers (Fig. 3(c)). We assume a resistive source impedance but otherwise a common mode input impedance with resistive and capacitive components. The corresponding transfer functions are

$$H_1(s) = \frac{R_{c1}}{R_1 + R_{c1} + R_1R_{c1}C_{c1}s} \tag{19a}$$

$$H_2(s) = \frac{R_{c2}}{R_2 + R_{c2} + R_2R_{c2}C_{c2}s} \tag{19b}$$

so that now we have (20).

Usually, we have $R_1 << R_{c1}$, $R_2 << R_{c2}$, and (20) simplifies to

$$C \approx \frac{1 + s(R_1C_{c1} + R_2C_{c2})/2}{s(R_2C_{c2} - R_1C_{c1})}. \tag{21}$$

If the main problem is stray capacitances, we can assume $R_1 = R_2 = R$, $R_{c1} = R_{c2} = R_c$, and define $C_{ci} = C_{c1} - C_{c2}$, $C_{c1} + C_{c2} = 2C_{ca}$. In this case,

$$C \approx -\frac{C_{ca}}{C_{ci}} + j/\omega RC_{ci} \tag{22}$$

and a 10% imbalance ($C_{ci}/C_{ca} = 0.1$), for example, would imply $C \approx -10 + j/\omega RC_{ci}$.

In a similar way, when the problem is mainly one of source imbalance, we can assume $R_{c1} = R_{c2} = R_c$, $C_{c1} = C_{c2} = C_c$, and define $R_i = R_1 - R_2$, $2R_a = R_1 + R_2$. In this case,

$$C \approx -R_a/R_i + j/\omega R_i C_c. \tag{23}$$

A pair of unity gain buffers, Fig. 3(d), offers a common example of active noncoupled paths. Here we assume that open loop differential and common mode gains (hence *CMRR*) are not identical for both op amps. Therefore,

$$H_1(s) = \frac{A_{d1}(2CMRR_{OA1} + 1)}{A_{d1}(2CMRR_{OA1} - 1) + 2CMRR_{OA1}} \tag{24a}$$

$$H_2(s) = \frac{A_{d2}(2CMRR_{OA2} + 1)}{A_{d2}(2CMRR_{OA2} - 1) + 2CMRR_{OA2}} \tag{24b}$$

where A_d is the open loop differential gain and $CMRR_{OA}$ is the common mode rejection ratio for each op amp.

Factor C is now

$$C = \frac{1}{2}\frac{2 + 1/A_{d1} + 1/A_{d2} + 1/2A_{d2}CMRR_{OA1} + 1/2A_{d1}CMRR_{OA2} - 1/2CMRR_{OA1}CMRR_{OA2}}{1/A_{d2} - 1/A_{d1} + 1/CMRR_{OA1} - 1/CMRR_{OA2} + 1/2A_{d2}CMRR_{OA1} - 1/2A_{d1}CMRR_{OA2}}. \tag{25}$$

Given the typical values for open loop differential and common mode gains for op amps at the working frequencies, (25) can be greatly simplified and leads to

$$\frac{1}{C} \approx \frac{1}{A_{d2}} - \frac{1}{A_{d1}} + \frac{1}{CMRR_{OA1}} - \frac{1}{CMRR_{OA2}}. \tag{26}$$

It is very important to notice the result if instead of unity gain buffers we used two amplifiers with gain. Then the transfer functions would have been determined by feedback (passive) components, and we would have obtained a value for C dependent on component tolerances. This would have led to very low values for C, as deduced from (3) and (11c).

If we assume that the op amp' open loop differential gain and *CMRR* both have a first-order low-pass frequency dependence, with respective corner frequencies ω_d and ω_c, we obtain

$$\frac{1}{C} \approx \frac{A_{10} - A_{20}}{A_{10}A_{20}}\frac{s + \omega_{de}}{\omega_{de}} + \frac{CMRR_{20} - CMRR_{10}}{CMRR_{20}CMRR_{10}}\frac{s + \omega_{ce}}{\omega_{ce}} \tag{27}$$

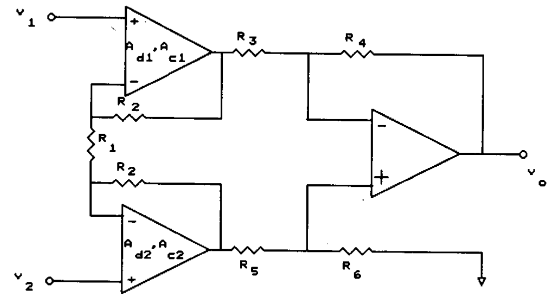

Fig. 4. Input stage for the three-op-amp instrumentation amplifier, as an example of a "coupled" differential stage.

where the "0" in subindices stands for dc values, and

$$\omega_{de} = \frac{\omega_{d1}\omega_{d2}(A_{10} - A_{20})}{A_{10}\omega_{d1} - A_{20}\omega_{d2}} \tag{28a}$$

$$\omega_{ce} = \frac{\omega_{c1}\omega_{c2}(CMRR_{20} - CMRR_{10})}{CMRR_{20}\omega_{c2} - CMRR_{10}\omega_{c1}}. \tag{28b}$$

If, for example, we consider two op amps whose dc gains and corner frequencies are 10% different, we have

$$\frac{1}{C} \approx \frac{1}{10A_0}\frac{s + \omega_d/2}{\omega_d/2} + \frac{1}{10C_0}\frac{s + \omega_c/2}{\omega_c/2}. \tag{29}$$

In most op amps $\omega_d << \omega_c$, and (29) can be approximated by the relation that at $\omega_d/2$, $C \approx 7A_0$ (−3 dB), and decreases at a rate of 20 dB/decade.

When the differential signal goes through "coupled paths," then $D \neq 1$ and the relation between C and E depends on the circuit. This is the case for the input stage of the classical three-op-amp IA (Fig. 4). From [1] we obtain for this stage that (26) applies, regardless of the values for resistors R_2 and R_1, for reasonable differential gains, and without requiring resistor matching. Also,

$$D = 1 + 2R_2/R_1. \tag{30}$$

Factor E can be obtained from [1]. This gives,

$$E = \frac{1}{2}\frac{A_{d1} - A_{d2} + A_{c1}/2 - A_{c2}/2 + (A_{d2}A_{c1} - A_{d1}A_{c2})/(1 + 2R_2/R_1)}{A_{d1} + A_{c1}/2 + A_{d2} + A_{c2}/2 + 2A_{d1}A_{d2} - A_{c1}A_{c2}/2}. \tag{31}$$

For op amps, A_d is very high as compared to the common mode gain, so that we can approximate

$$E \approx \frac{1}{2}\left[\frac{1}{A_{d2}} - \frac{1}{A_{d1}} + \frac{1}{1 + 2R_2/R_1} \cdot \left(\frac{1}{CMRR_{OA1}} - \frac{1}{CMRR_{OA2}}\right)\right]. \tag{32}$$

This means that E will normally be very small, so that the approximations leading to (10) are acceptable.

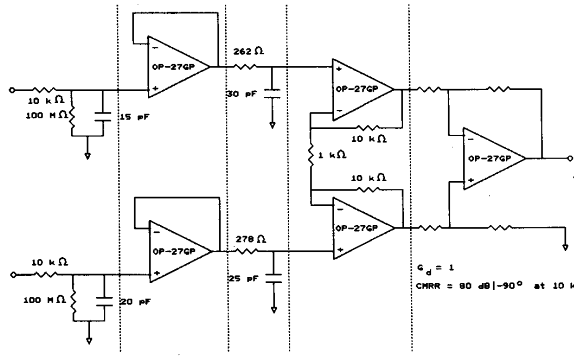

Fig. 5. A complete differential circuit consisting of several cascaded stages. A complete *CMRR* analysis is given in the text.

Example:

Fig. 5 shows a differential circuit consisting on five stages that can be analyzed by applying (10). It includes mismatched output source impedance and input (stray) capacitance; two unity-gain noncoupled buffers; a differential multiplexer; and a discrete-component three-op-amp IA. All op amps are assumed to be OP-27GP; for each pair we assume: $A_{10} = 8 \times 10^5$, $A_{20} = 3 \times 10^5$ (typical and minimal values in data sheet), $f_{d1} = 10$ Hz, $f_{d2} = 16$ Hz (thus resulting in the typical and minimal gain–bandwidth product), $CMRR_{OA1} = 116$ dB, $CMRR_{OA2} = 94$ dB, $f_{c1} = f_{c2} = 3$ kHz. The problem is to calculate $CMRR_T$ at 10 kHz.

For the first three stages we have $D_1 = D_2 = D_3 = 1$. From (21),

$$C_1 = -5.2 + j455.$$

From (27) we obtain

$$C_2 = -30 - j1352.$$

We assume that the fourth stage has the same op-amp mismatching, and, therefore, $C_4 = C_2$. From (15) we obtain

$$C_3 = -8.3 + j8891.$$

From (30), $D_4 = 21$. We have assumed for the fifth stage a *CMRR* of 80 dB, $-90°$ at 10 kHz. Therefore, $C_5 = -j10^4$.

The extension of (10) to the present case, where there are five stages, gives

$$\frac{1}{CMRR_T} \approx \frac{1}{C_1} + \frac{1}{D_1C_2} + \frac{1}{D_1D_2C_3} + \frac{1}{D_1D_2D_3C_4} + \frac{1}{D_1D_2D_3D_4C_5}$$
$$\approx \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{D_4C_5}.$$

By using the previous results, we obtain

$$\frac{1}{CMRR_T} = -58 \times 10^{-6} - j826 \times 10^{-6}.$$

In practical terms this means that an input common mode voltage results in an equivalent input error voltage with the in-phase component attenuated by 85 dB and the out-of-phase component attenuated by 62 dB.

If stages 2 and 4 were perfect, then the result would be

$$\frac{1}{CMRR_T} = -25 \times 10^{-6} - j2.3 \times 10^{-3}.$$

It is a bit surprising that now the result is worse than in the previous case, but this is a consequence of partial cancellations taking place when adding different terms. If we interchange the op-amp positions in stage 2 and also in stage 4, then the result is

$$\frac{1}{CMRR_T} = -7.6 \times 10^{-6} - j3.8 \times 10^{-3}.$$

In any case, we see that the result is far from the 80 dB, $-90°$ specified for the last stage. The degradation is mostly due to input stage imbalance, but in other situations a different stage can be responsible for it.

IV. Conclusion

Equation (10) provides a simple method to calculate the resulting common mode rejection ratio for three (or more by simple extension) cascaded differential amplifier stages. We have shown that for several typical circuits the assumptions made to obtain such a simplified equation are acceptable. Also the value for the parameter C for each of these circuits is given. Some of the resulting formulas are applied to calculate the total *CMRR* for the circuit in Fig. 5, which requires only a few calculations when using the rule formulated by (10), thus illustrating its usefulness.

References

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