Basics of Analog Differential Filters

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***Abstract*—We describe and analyze passive and active analog filters with differential input and differential output. They are implemented by coupling single-ended filters and provide very high common-mode rejection ratios. This makes it possible to place these filters before differential amplifiers, thus improving interference rejection and noise reduction.**

I. Introduction

FILTERING of analog signals in data acquisition systems is usually done at intermediate stages. Very commonly, to prevent signal aliasing, filtering is done just before sampling. Nevertheless, in order to reduce interference and bandwidth, and hence noise, in several cases, and particularly for ac signals, filtering should be done as soon as possible, even before amplification. However, many instrumentation signals are differential, and placing filters at the input of differential amplifiers can lead to a severe reduction in the common-mode rejection ratio (CMRR).

In this paper we analyze the CMRR for filters having differential input and differential output. We call them differential filters. We propose several passive and active differential filters. All of them are easily derived from conventional single-ended filter circuits and display a very high CMRR. In the integrated circuit design area, differential filters are sometimes implemented using fully differential op amps [1]. Here, we rely on common op amps. Nevertheless, our approach to CMRR analysis is valid for different implementation approaches.

II. The Concept of Differential Filter

A filter is any circuit that separates signals according to their frequency or another criterion. Here, we consider only filters in the frequency domain. Further, we assume the desired information is encoded in a voltage signal. Voltages are measured with respect to a reference point called the signal common or ground. If one of the signal terminals is ground, we have a single-ended signal. If neither of the signal terminals is ground, we have a differential signal.

A differential filter is any filter with a differential input and a differential output. A differential input allows us to apply the filter to differential signals. A differential output allows us to place the filter before another differential stage, such as a differential amplifier. Therefore, an important feature of a differential filter is its CMRR, whose relevance can be better understood by first analyzing the filtering of differential signals by means of noncoupled single-ended filters (Fig. 1).

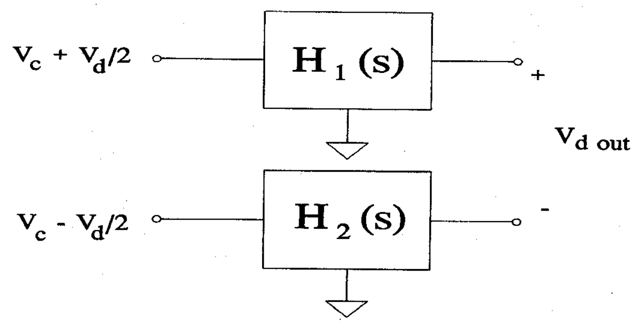

Fig. 1. Analog filter for differential signals implemented by single-ended filters.

The CMRR is defined as the quotient between the differential output due to a differential input voltage and the differential output due to a common-mode input voltage of the same magnitude. Because we have noncoupled stages [2], the CMRR for the filter in Fig. 1 is

$$\mathrm{CMRR}(s) = \frac{1}{2}\frac{H_1(s) + H_2(s)}{H_1(s) - H_2(s)}. \tag{1}$$

If we had $H_1(s) = H_2(s)$, then $\mathrm{CMRR} = \infty$, and the output voltage due to the input common-mode voltages would be zero. But, in practice, we will have $H_1(s) \neq H_2(s)$ and a finite CMRR. If, for example, we consider a passive RC first-order low-pass filter, the transfer function for each signal path will be

$$H_i(s) = \frac{1}{R_i C_i s + 1}. \tag{2}$$

By considering component tolerances in a worst case situation,

$$R_1 = R(1+\alpha) \qquad C_1 = C(1+\beta) \tag{3a}$$

$$R_1' = R(1-\alpha) \qquad C_1' = C(1-\beta) \tag{3b}$$

we find that the minimum CMRR is

$$\mathrm{CMRR}(s') = \frac{1}{2}\frac{(1+\alpha\beta)s' + 1}{(\alpha+\beta)s'} \tag{4}$$

where $s' = sRC$. If, for example, $\alpha = 1\%$ and $\beta = 5\%$, in order to ensure $\mathrm{CMRR} \approx 60$ dB, we need $s' = 0.01$. That is, the corner frequency for each filter should be 100 times larger than the signal frequency we accept to be attenuated by 3 dB. Such a filter would not be very effective for interference and noise reduction. We are interested in filters whose bandpass be dictated only by the differential signal, not by the CMRR we wish to obtain.

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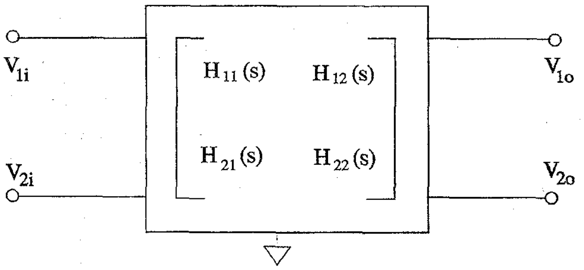

Fig. 2. Analog differential filter described by means of four transfer functions.

Fig. 2 shows how to describe a differential filter by means of four transfer functions. The output voltages will be

$$\begin{pmatrix} V_{1o} \\ V_{2o} \end{pmatrix} = \begin{pmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{pmatrix} \begin{pmatrix} V_{1i} \\ V_{2i} \end{pmatrix}. \tag{5}$$

Writing the input and output voltages in terms of their differential and common-mode components,

$$V_{\mathrm{di}} = V_{1i} - V_{2i} \tag{6a}$$

$$V_{\mathrm{ci}} = \frac{(V_{1i} + V_{2i})}{2} \tag{6b}$$

we can determine the gain matrix $[G(s)]$, shown in (7) and (8) at the bottom of the page.

From (8) we can calculate the three ratios that characterize any differential stage [2],

$$C = \frac{G_{\mathrm{dd}}}{G_{\mathrm{dc}}} = \frac{1}{2}\frac{[H_{11}(s) - H_{21}(s)] + [H_{22}(s) - H_{12}(s)]}{[H_{11}(s) - H_{21}(s)] - [H_{22}(s) - H_{12}(s)]} \tag{9}$$

$$D = \frac{G_{\mathrm{dd}}}{G_{\mathrm{cc}}} = \frac{[H_{11}(s) + H_{22}(s)] - [H_{12}(s) + H_{21}(s)]}{[H_{11}(s) + H_{22}(s)] + [H_{12}(s) + H_{21}(s)]} \tag{10}$$

$$E = \frac{G_{\mathrm{cd}}}{G_{\mathrm{dd}}} = \frac{1}{2}\frac{[H_{11}(s) - H_{12}(s)] - [H_{22}(s) - H_{21}(s)]}{[H_{11}(s) - H_{12}(s)] + [H_{22}(s) - H_{21}(s)]} \tag{11}$$

C is the CMRR for the stage when considered alone, and D is the discrimination factor [3]. For an ideal differential filter, the aim is to obtain $C = \infty$, $D = H(s)$, the desired transfer function for differential signals, and $E = 0$. Aiming to have $D = H(s)$ is equivalent to obtaining $G_{\mathrm{cc}} = 1$. For real filters, C, D and E will depend on their structure. In general, however, by comparing (9) to (1) we see that, in order to have a large CMRR, if noncoupled stages are used, in (1) we need to match them, but in (9) we need only the difference between two differences to be small.

III. Passive Differential Filters

Fig. 3 shows a simple first-order low-pass differential filter. It is based on two single-ended filters that have been "coupled," that is, connected to a common point P that is not connected to ground. Obviously, a single capacitor $C_1/2$ can be used, but we represent two separate capacitors for analysis purposes. The corresponding input-output relation is

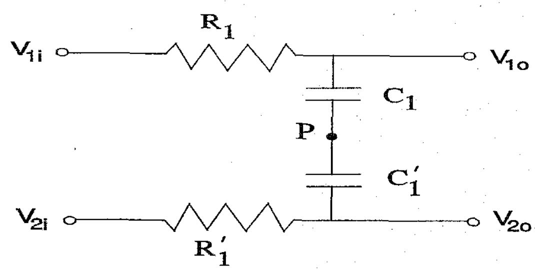

Fig. 3. Low-pass, first-order passive differential filter.

$$\begin{pmatrix} V_{1o} \\ V_{2o} \end{pmatrix} = \frac{1}{\det(M)} \begin{pmatrix} 1 + R_1' C_{11} s & R_1 C_{11} s \\ R_1' C_{11} s & 1 + R_1 C_{11} s \end{pmatrix} \begin{pmatrix} V_{1i} \\ V_{2i} \end{pmatrix} \tag{12}$$

where

$$\det(M) = 1 + (R_1 + R_1')C_{11}s \tag{13a}$$

$$C_{11} = \frac{C_1 C_1'}{C_1 + C_1'}. \tag{13b}$$

From (9) we obtain $C = \infty$, irrespective of component tolerances. We can obtain the same result by circuit inspection, because in Fig. 3 there is no way a common-mode input signal can produce a differential output signal. For D, from (10) we have

$$D = \frac{2}{2 + 2R_1 C_{11} s + 2R_1' C_{11} s} \tag{14}$$

and by assuming $C_1 \approx C_1'$ and $R_1 \approx R_1'$, we finally have

$$D \approx \frac{1}{1 + R_1 C_1 s} \tag{15}$$

which is the desired low-pass transfer function. For E, from (11), (3a), and (3b), we have

$$|E| = \frac{1}{2}\alpha(1 - \beta^2)RCs = \frac{1}{2}\alpha(1 - \beta^2)s'. \tag{16}$$

If, for example, component tolerances are $\alpha = 5\%$ and $\beta = 20\%$, $|E|$ ranges from 2.4×10^{-5} when $s' = 0.001$ to 2.4×10^{-2} when $s' = 1$. In the band pass, therefore, $|E|$ is very small, as desired.

This simple approach based on coupling single-ended stages, however, can not always be applied to other filter circuits. If, for example, in Fig. 3 we interchange R's and C's, we obtain a high-pass passive filter. If we placed this filter at the input of an amplifier, there would not be any path for bias currents into or from the amplifier. A possible solution is to place bias resistors from each amplifier input to ground, but

$$\begin{pmatrix} V_{\mathrm{do}}(s) \\ V_{\mathrm{co}}(s) \end{pmatrix} = [G(s)] \begin{pmatrix} V_{\mathrm{di}}(s) \\ V_{\mathrm{ci}}(s) \end{pmatrix} = \begin{pmatrix} G_{\mathrm{dd}}(s) & G_{\mathrm{dc}}(s) \\ G_{\mathrm{cd}}(s) & G_{\mathrm{cc}}(s) \end{pmatrix} \begin{pmatrix} V_{\mathrm{di}}(s) \\ V_{\mathrm{ci}}(s) \end{pmatrix} \tag{7}$$

$$[G(s)] = \begin{pmatrix} \frac{1}{2}(H_{11}(s) - H_{12}(s) - H_{21}(s) + H_{22}(s)) & (H_{11}(s) + H_{12}(s) - H_{21}(s) - H_{22}(s)) \\ \frac{1}{4}(H_{11}(s) - H_{12}(s) + H_{21}(s) - H_{22}(s)) & \frac{1}{2}(H_{11}(s) + H_{12}(s) + H_{21}(s) + H_{22}(s)) \end{pmatrix}. \tag{8}$$

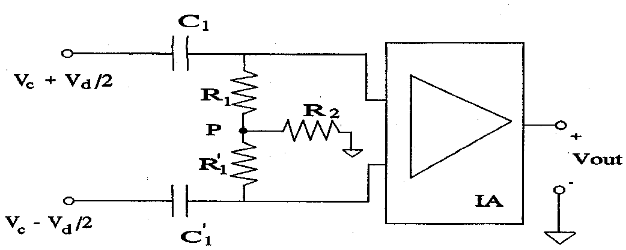

Fig. 4. High-pass, first-order passive differential filter with bias resistor R_2 for the ensuing amplifier.

their values should be very large so that the transfer function is not modified. If very high CMRR values are desired at low frequencies, this method can become impractical.

Fig. 4 shows how to add a single biasing resistor to a high-pass differential filter. For the filter alone, we have

$$C(s) = \frac{1}{2}\frac{2 + \left(\frac{1}{R_1'C_1's} + \frac{1}{R_1C_1s}\right)(1 - A - B)}{\left(\frac{1}{R_1'C_1's} - \frac{1}{R_1C_1s}\right)(1 - A - B)} \tag{17}$$

where

$$A = \frac{R_2R_1'}{R_1'R_1 + R_2R_1' + R_2R_1} \tag{18a}$$

$$B = \frac{R_2R_1}{R_1'R_1 + R_2R_1' + R_2R_1}. \tag{18b}$$

We will normally have

$$\left(\frac{1}{R_1'C_1's} + \frac{1}{R_1C_1s}\right)(1 - A - B) \ll 2 \tag{19}$$

and then (17) simplifies to

$$C(s) \approx \frac{1}{\left(\frac{1}{R_1'C_1's} - \frac{1}{R_1C_1s}\right)(1 - A - B)}. \tag{20}$$

This means that in order to obtain a large CMRR(s), it is not necessary to have a small value for $(1/R_1'C_1's - 1/R_1C_1s)$, as in noncoupled filters. We can instead design a small value for $(1 - A - B)$,

$$(1 - A - B) = \frac{R_1R_1'}{R_1R_1' + R_2(R_1 + R_1')}. \tag{21}$$

Therefore, if we design $R_2 \gg R_1, R_1'$, the CMRR will be very large.

If the value for R_2 is yet too large, we can use the circuit in Fig. 5 instead. Fig. 5 provides a biasing path that relies on low-value resistors and an op amp. The input impedance is

$$Z = \frac{1 + \frac{R_1}{R_3} + \frac{R_1}{R_2}}{\frac{1}{R_3} + \frac{1}{(1+A_o)R_2}} \tag{22}$$

where A_o is the open-loop gain for the op amp. R_3 prevents the circuit from oscillating. If the op amp and resistors are considered such that

$$(1 + A_o)R_2 \gg R_3 \tag{23}$$

then (22) reduces to

$$Z \simeq R_3 + R_1 + \frac{R_3R_1}{R_2}. \tag{24}$$

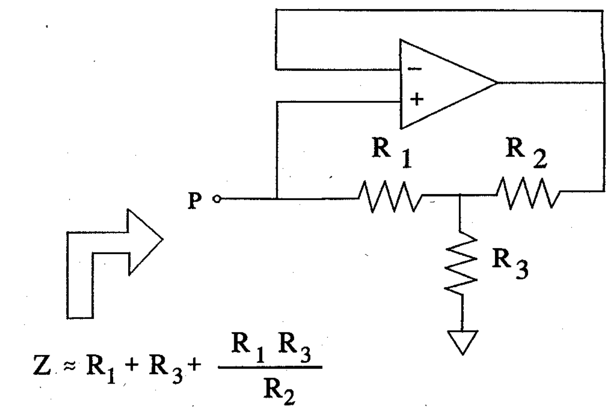

Fig. 5. Active circuit that behaves like a resistor.

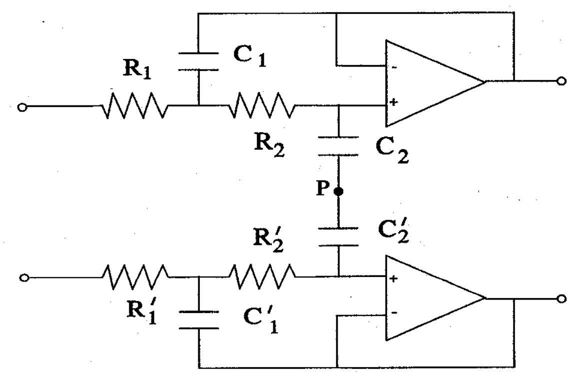

Fig. 6. Low-pass, second-order active differential filter.

By choosing $R_3, R_1 \gg R_2$, we can obtain the desired impedance. The op amp must have a large open-loop gain, small input currents and large input impedances.

IV. Active Differential Filters

The design of differential filters by coupling single-ended filters, can also be applied to active filters. Fig. 6 shows a low-pass, second-order, active differential filter, based upon a voltage-controlled voltage source (VCVS) proposed by Sallen and Key [4]. By the same reason given for the filter in Fig. 3, the CMRR for the filter in Fig. 6 is infinite, provided we use matched op amps. This is, in fact, the same method that provides the excellent common-mode performance for the input stage of the classic three-op-amp instrumentation amplifier. Higher order filters can be implemented by coupling them through the connection of all impedances that go to ground in a single-ended filter (and substituting them by the equivalent impedance). Other filter circuits can be coupled in the same way.

Here too, when applying this method, care must be taken in order to keep a bias path for any amplifier present in the circuit. Fig. 7 shows how to apply the method in Fig. 4 to a high-pass, second-order, active differential filter. Alternatively, the circuit in Fig. 5 could be used instead of R_3 in order to either improve the CMRR even more or to use only low- value resistors.

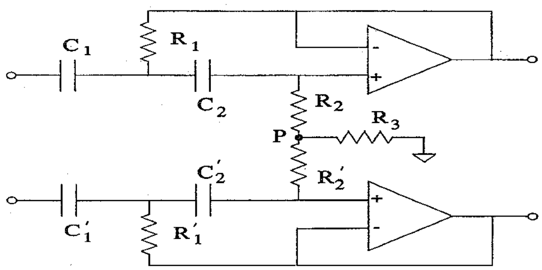

Fig. 7. High-pass, second-order active differential filter.

V. Experimental Results

Measuring the CMRR for differential filters using standard equipment, requires an instrumentation amplifier (IA) providing a single-ended output. From [2] we know that for cascaded differential amplifier stages the reciprocal of the CMRR for the system equals the sum of reciprocals of the CMRR for each stage. For a filter based on noncoupled stages (ncf), we will have

$$\frac{1}{\mathrm{CMRR}_T} = \frac{1}{\mathrm{CMRR}_{\mathrm{ncf}}} + \frac{1}{\mathrm{CMRR}_{\mathrm{IA}}} \approx \frac{1}{\mathrm{CMRR}_{\mathrm{ncf}}}. \quad (25)$$

For a filter based on coupled stages (cf), $C = \infty$, and the total CMRR will be

$$\frac{1}{\mathrm{CMRR}_T} = \frac{1}{\mathrm{CMRR}_{\mathrm{cf}}} + \frac{1}{\mathrm{CMRR}_{\mathrm{IA}}} = \frac{1}{D_1 C_2} \quad (26)$$

where D_1 is the differential gain of the filter, that is, its normal transfer function, and C_2 is the CMRR for the IA alone. $\mathrm{CMRR}_{\mathrm{IA}}$ is the CMRR for the IA once in the circuit, that is, when preceded by stages with a given differential gain. In practical terms, (26) means that the measured CMRR will approximately be equal either to the CMRR for the filter or the CMRR for the IA, whichever is lower.

We use the INA 110AG (Burr-Brown) IA, with a gain of 500. Its CMRR is 100 dB, minimum, at dc and decreases by 20 dB/decade from 100 Hz up. When connecting this IA to the filter in Fig. 3, with $R_1 = R_1' = 1$ kΩ, $\alpha = 5\%$, $C_1 = C_1' = 150$ nF, $\beta = 20\%$, (-3 dB frequency, 1 kHz), we obtain the experimental results in Fig. 8. The upper curve is for the coupled stages (cf); the lower curve is for noncoupled stages (ncf) (obtained by grounding P in Fig. 3). We see that for coupled stages the CMRR is determined by the IA (and the differential gain for the filter) and is up to 55 dB larger than the CMRR for noncoupled stages. This is in agreement with the theoretically infinite CMRR for passive filters based on coupled stages.

If resistors and capacitors in Fig. 3 interchange their positions, we have a high-pass filter. Then, in order to bias the IA, we place $R_2 = 10$ MΩ from P to ground. We still have a 50 dB improvement with respect to a filter based on noncoupled stages, but there is a 10 dB loss because of R_2. In order to test the advantages of the circuit in Fig. 5, we changed $R_1 = R_1' = 10$ kΩ. We obtained up to 9 dB improvement with respect to using $R_2 = 10$ MΩ as in Fig. 4.

We have implemented the filter in Fig. 6 by using TL072 op amps and designing $R_1 = R_1' = 1.5$ kΩ, $R_2 = R_2' = 5.6$ kΩ, $C_1 = C_1' = 100$ nF, $C_2 = C_2' = 33$ nF, thus resulting in a -3 dB corner frequency of 1 kHz. By grounding P we obtained a noncoupled filter. The respective experimental results are shown in Fig. 9. We can see the large benefit of using coupled stages. It is also worth noticing that from 100 Hz up, and for both the coupled and noncoupled filters, the results for the CMRR are influenced by the matching of the op amps used in the active filters, thus resulting in a somewhat smaller CMRR as compared with that of passive filters.

We have also implemented the filter in Fig. 7 by designing $R_1 = R_1' = 110$ Ω, $R_2 = R_2' = 220$ Ω, $C_1 = C_1' = C_2 = C_2' = 1$ μF and $R_3 = 10$ MΩ. We again obtain a 50 dB improvement in the CMRR when using coupled stages.

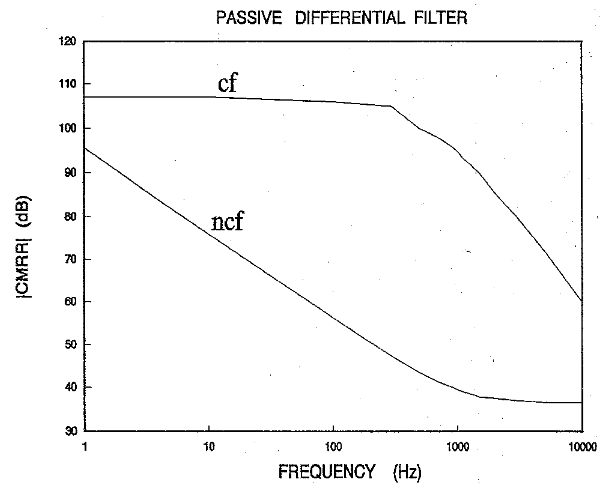

Fig. 8. CMRR for a passive differential filter (Fig. 3) followed by an INA 110AG instrumentation amplifier, when using coupled stages (upper curve, coupled filter) or noncoupled stages (lower curve, noncoupled filter).

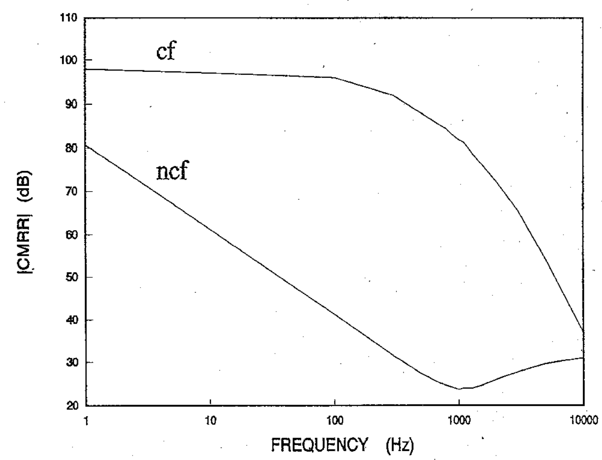

Fig. 9. CMRR for an active differential filter (Fig. 6) followed by an INA 110AG instrumentation amplifier, when using coupled stages (upper curve, coupled filter) or noncoupled stages (lower curve, noncoupled filter).

VI. Conclusion

We have presented a method for designing analog differential filters that provides a very large CMRR. The method is based on coupling single-ended circuits that are common in analog filters. In some cases, a bias path must be provided, and we propose two simple solutions. Others have proposed active filter circuits, intended for IC realization, that look similar to ours but using fully differential op amps [5]. We use common op amps. Our approach of coupling single-ended circuits, however, can also be applied in designing integrated-circuit filters.

The ability of placing differential filters at the input of differential amplifiers, or at any other place in a signal acquisition system, opens new horizons for analog signal processing, because we can more efficiently reduce interference and bandwidth. A differential input signal can thus be processed by a fully differential system and converted to a single-ended signal at the most beneficial place, not necessarily at the input amplifier.

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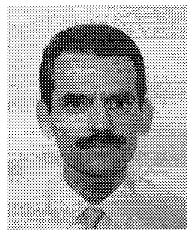

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