1. Vector Algebra

1.1 Introduction

전자기학 (Electromagnetics):

전장과 자장을 연구하는 물리학 분야의 기초 학문.

응용 분야는 전기와 자석을 사용하는 장치.

좁은 의미의 전자기학:

Maxwell Equation을 유도, 이해, 응용하는 학문.

1.2 A preview of the book

Maxwell Equation

$\nabla \cdot \vec{D} = \rho_v$ Gauss Eq

$\nabla \cdot \vec{B} = 0$ Gauss Eq

$\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ Faraday Eq

$\nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$ Ampere Eq

cgs 단위 (cm, g, sec) ↔ MKS 단위 (m, kg, sec)

$\vec{H} = \vec{B}/\mu$: Magnetic field [A/m]

$\vec{B}$: Magnetic flux [Tesla = Wb/m^2 = 10,000 Gauss]

$\vec{D} = \varepsilon\vec{E}$: Electric flux density [C/m^2]

$\vec{E}$: Electric field [volt/m]

$\mu_0 = 4\pi \times 10^{-7}$ [H/m]: Permeability (투자율)

$\varepsilon_0 = 10^{-9}/36\pi$ [F/m]: Permittivity (유전율)

$= 8.854 \times 10^{-12}$ [F/m]

$\vec{J}$: Current density [A/m^2]

ρ_v : Volume charge density [C/m^3]

1.3 Scalars and Vectors

- A Scalar is a quantity that has only magnitude

  ex) 1, -23.56, 30, $\sqrt{2}$ · · ·

- A Vector is a quantity that has both magnitude and direction

  ex) (1,2.1,3) (1,3) · · ·

- A Field : 들판, 공간 · · ·

  Scalar 장: ex: 건물 내의 온도.

  Vector 장: ex: 대기중의 빗방울 속도.

1.4 Unit Vector

$$\text{Unit Vector of } \vec{A} : \vec{a}_A \equiv \frac{\vec{A}}{|\vec{A}|} \equiv \frac{\vec{A}}{A} \qquad (1.5)$$

$$\vec{A} = A\vec{a}_A \qquad (1.6)$$

$$\equiv (A_x, A_y, A_z) \qquad (1.7)$$

$$= A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \qquad (1.7)$$

$$\text{절대값} : A \equiv \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad (1.8)$$

$$\vec{a}_A = \frac{A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \qquad (1.9)$$

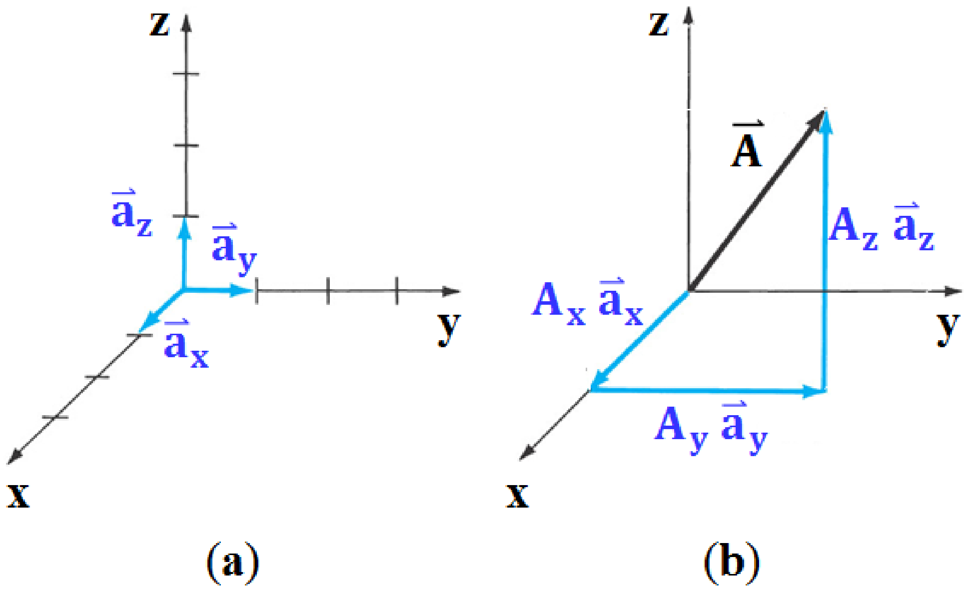


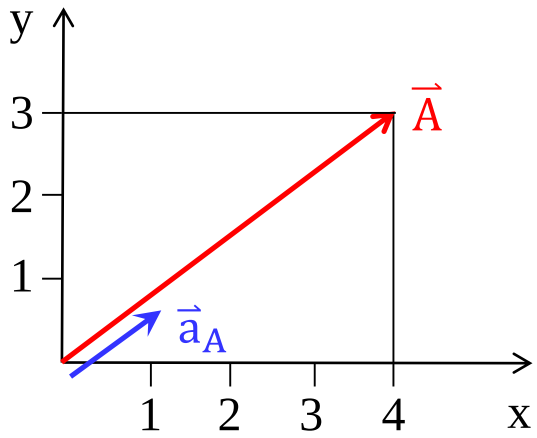
Fig. 1.1 (**a**) Unit vectors $\vec{a}_x, \vec{a}_y$, and $\vec{a}_z$,
(**b**) components of $\vec{A}$ along $\vec{a}_x, \vec{a}_y$, and $\vec{a}_z$.

$$\vec{A} = (4, 3)$$

절대값 : $A = \sqrt{A_x^2 + A_y^2}$

$$= \sqrt{4^2 + 3^2}$$

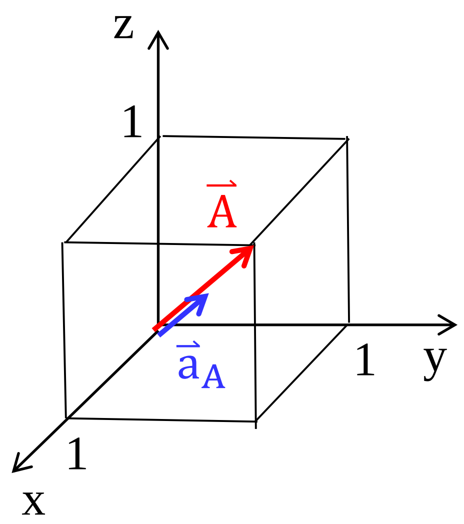
$$= 5$$



Unit Vector of $\vec{A}$:

$$\vec{a}_A \equiv \frac{\vec{A}}{|\vec{A}|} = \frac{(4, 3)}{5} = (\frac{4}{5}, \frac{3}{5})$$

$\vec{A} = (1, 1, 1)$



절대값 : $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{1^2 + 1^2 + 1^2}$

$= \sqrt{3}$

Unit Vector of $\vec{A}$:

$$\vec{a}_A \equiv \frac{\vec{A}}{|\vec{A}|} = \frac{(1, 1, 1)}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

1.5 Vector Addition and Subtraction

Vector Addition

$$\vec{C} = \vec{A} + \vec{B} \qquad (1.10)$$

$$\begin{pmatrix} \vec{A} = (A_x, A_y, A_z) \\ \vec{B} = (B_x, B_y, B_z) \end{pmatrix} \rightarrow$$

$$\vec{C} = (A_x + B_x)\vec{a}_x + (A_y + B_y)\vec{a}_y + (A_z + B_z)\vec{a}_z \quad (1.11)$$

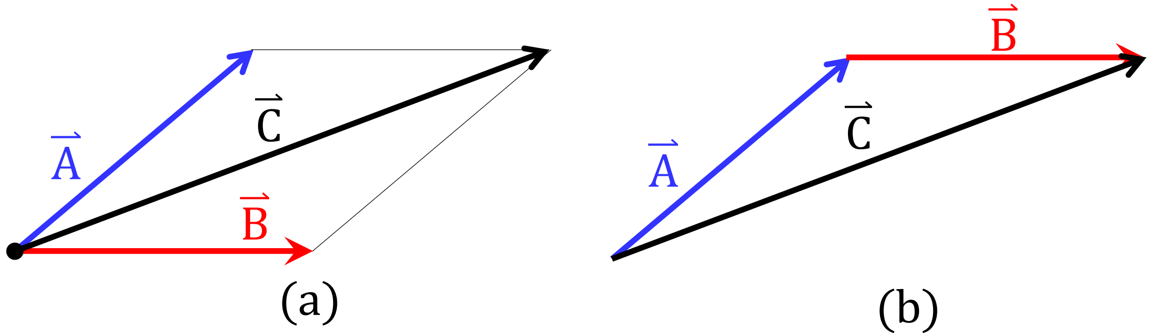


Fig. 1.2 Vector Addition

Vector Subtraction

$$\vec{D} = \vec{A} - \vec{B}$$
$$= \vec{A} + (-\vec{B})$$

$$= (A_x - B_x)\vec{a}_x$$
$$+ (A_y - B_y)\vec{a}_y$$
$$+ (A_z - B_z)\vec{a}_z \quad (1.12)$$

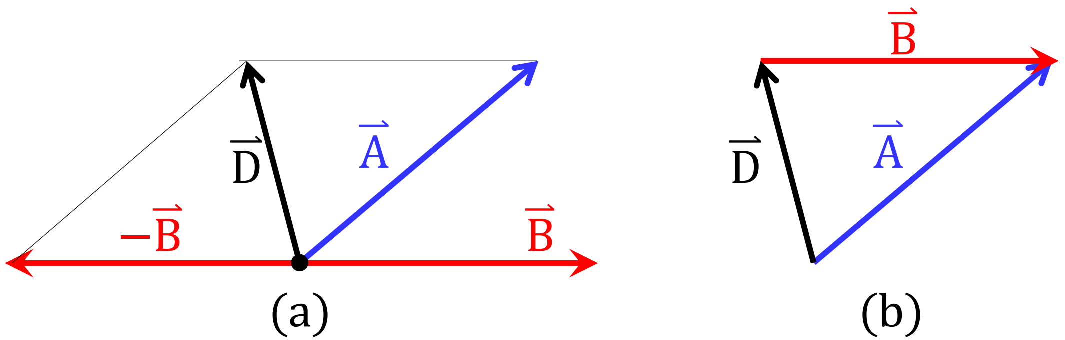
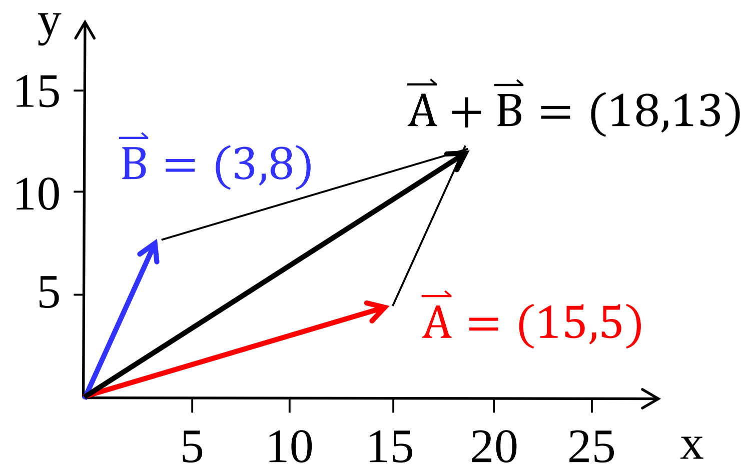


Fig. 1.3 Vector Subtraction

Vector Addition

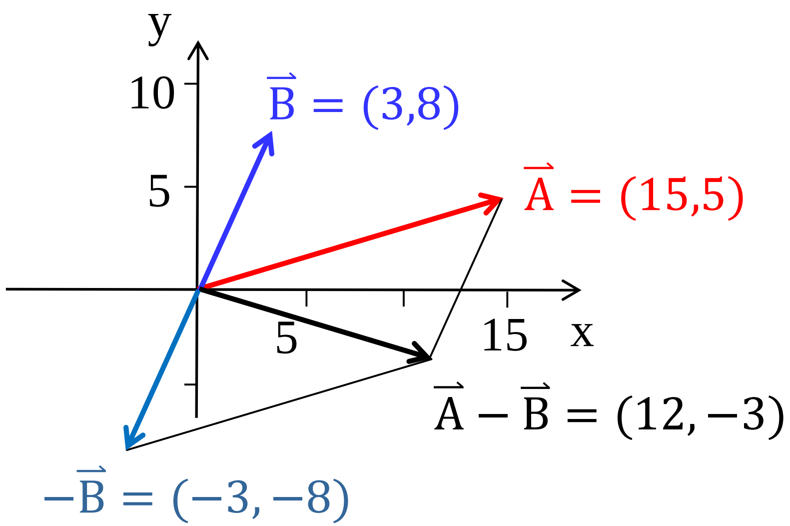
$$\begin{cases} \vec{A} = (15, 5) \\ \vec{B} = (3,8) \end{cases}$$

$$\vec{A} + \vec{B} = (15,5) + (3,8)$$
$$= (18,13)$$



Vector Subtraction

$$\vec{A} - \vec{B} = (15,5) - (3,8)$$
$$= (12,-3)$$



Law	Addition	Multiplication
Commutative	$\vec{A} + \vec{B} = \vec{B} + \vec{A}$	$k\vec{A} = \vec{A}k$
Associative	$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$	$k(j\vec{A}) = (kj)\vec{A}$
Distributive	$k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$	

1.6 Position and Distance Vectors

The **position vector $\vec{r}_p$** (or radius vector) of point P is defined as the directed distance from the origin O to P.

$$\vec{r}_p = \overrightarrow{OP} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z \qquad (1.14)$$

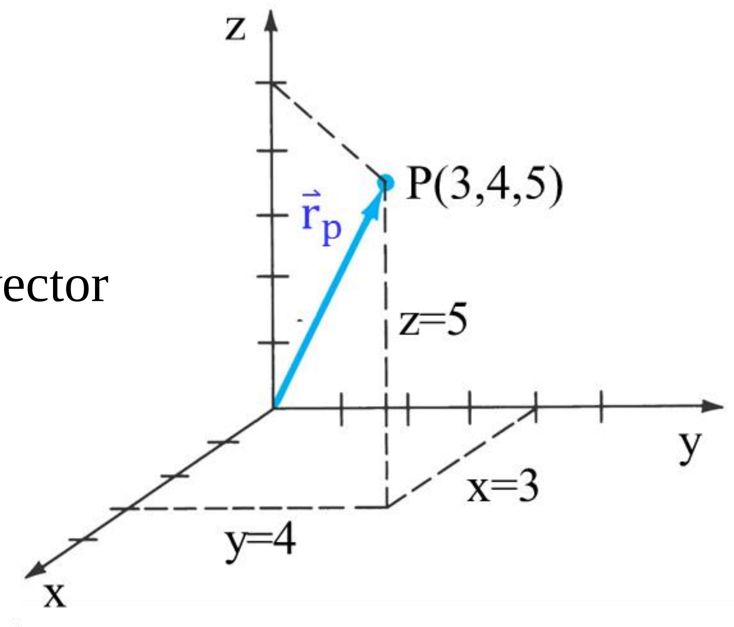


Fig. 1.4 Illustration of position vector $\vec{r}_p = 3\vec{a}_x + 4\vec{a}_y + 5\vec{a}_z$

The **distance vector** is the displacement from one point to another.

$$\begin{aligned}\vec{r}_{PQ} &= \vec{r}_Q - \vec{r}_P \\ &= (x_Q - x_P)\vec{a}_x \\ &\quad + (y_Q - y_P)\vec{a}_y \\ &\quad + (z_Q - z_P)\vec{a}_z \qquad (1.15)\end{aligned}$$

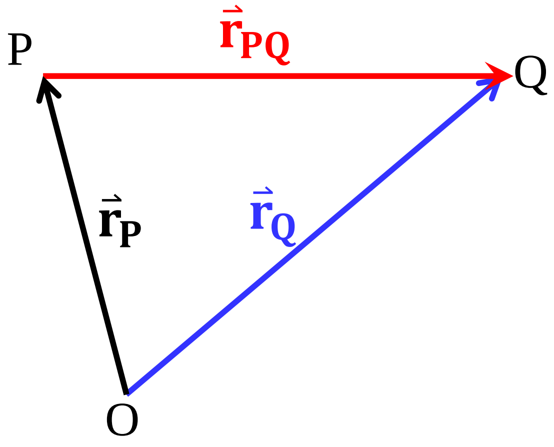
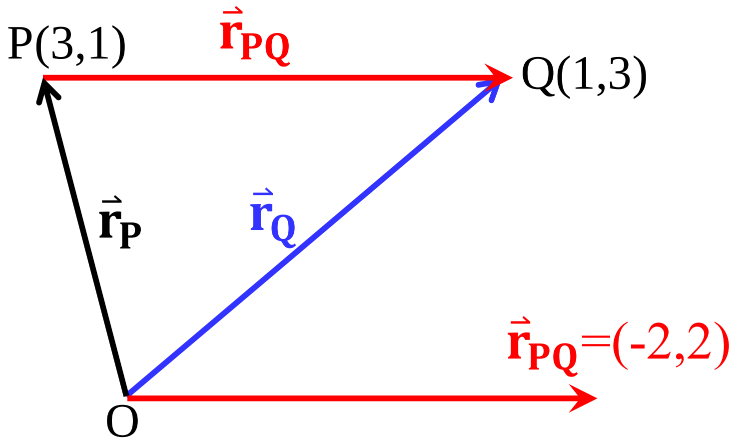


Fig. 1.5 Distance vector $\vec{\mathbf{r}}_{PQ}$.

$P(3,1),\ \ Q(1,3)$

$$\vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P$$
$$= (1,3) - (3,1)$$
$$= (-2,2)$$



예제 1.1 $\vec{A} = (10, -4, 6),\ \vec{B} = (2, 1, 0)$ 일 경우에

(a) $\vec{a}_y$ 방향의 $\vec{A}$의 성분

(b) $3\vec{A} - \vec{B}$

(c) $\vec{A} + 2\vec{B}$ 방향의 unit vector

(a) $\vec{a}_y \cdot \vec{A} = \vec{a}_y \cdot (10\vec{a}_x - 4\vec{a}_y + 6\vec{a}_z) = -4$

(b) $3\vec{A} - \vec{B}$=3(10, -4, 6)-(2, 1, 0)

=(30, -12, 18)-(2, 1, 0)

=(28, -13, 18)

(c) $\vec{C} = \vec{A} + 2\vec{B}$ =(10, -4, 6)+2(2, 1, 0)=(14, -2, 6)

$$\vec{a}_C = \frac{\vec{C}}{C} = \frac{(14, -2, 6)}{\sqrt{14^2 + (-2)^2 + 6^2}} = (0.9113, -0.130, 0.3906)$$

예제 1.2 Point P(0,2,4), Point Q(-3,1,5) 일 때 다음을 구하라

(a) $\overrightarrow{\mathrm{OP}}$

(b) $\overrightarrow{\mathrm{PQ}}$

(c) $\left|\overrightarrow{\mathrm{PQ}}\right|$

(d) $10\vec{\mathrm{a}}_{\mathrm{PQ}}$

(a) $\overrightarrow{\mathrm{OP}} = (0,2,4)$

(b) $\overrightarrow{\mathrm{PQ}}$ =(-3,1,5)-(0,2,4)=(-3,-1,1)

(c) $\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{(-3)^2+(-1)^2+1^2} = 3.317$

(d) $10\vec{\mathrm{a}}_{\mathrm{PQ}} = (-9.045,\ -3.015,\ 3.015)$

예제 1.3 Boat가 강물을 따라 남동쪽으로 10 km/hr로 움직이고 그 위에서 사람이 Boat의 가는 방향의 오른 쪽 수직으로 2 km/hr로 움직일 때 그 사람의 절대 속도와 가는 방향은?

Boat의 속도

$$\vec{u}_b = 10(\cos\frac{\pi}{4}, \ -\sin\frac{\pi}{4}) = (7.071, -7.071)$$

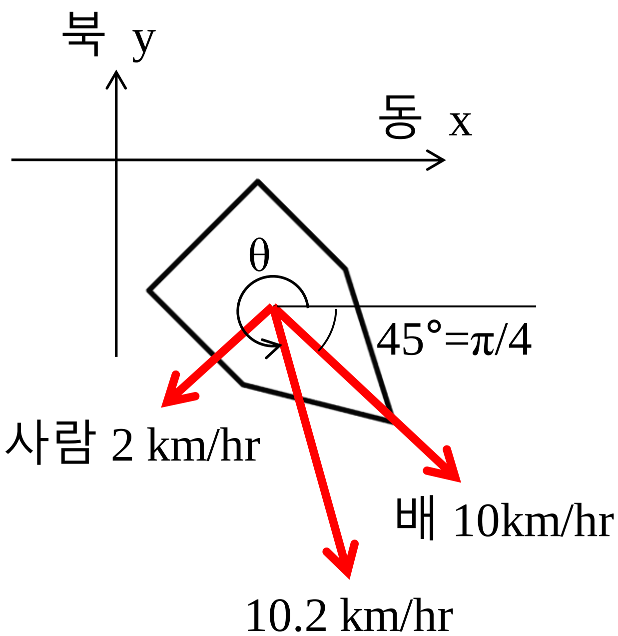
사람의 상대속도

$$\vec{u}_m = 2(-\cos\frac{\pi}{4}, \ -\sin\frac{\pi}{4}) = (-1.414, -1.414)$$

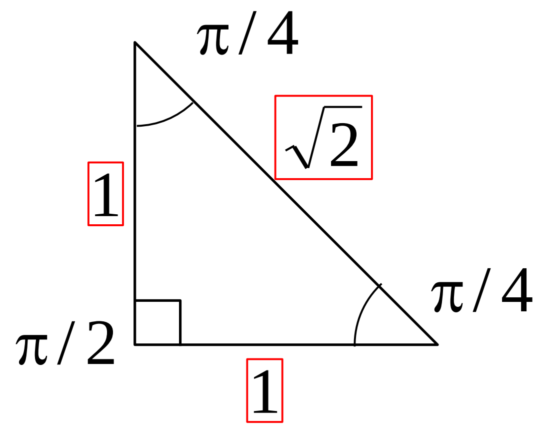
사람의 절대속도

$$\vec{u}_b + \vec{u}_m = (7.071, \ -7.071) + (-1.414, -1.414)$$
$$= (5.657, -8.485)$$

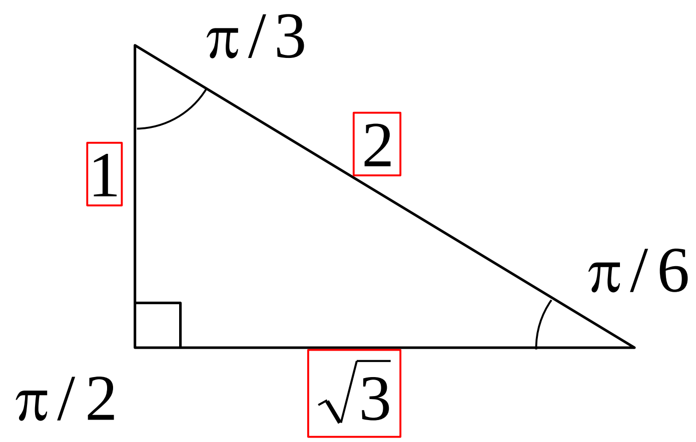
$$\left|\vec{u}_b + \vec{u}_m\right| = 10.2 \ \text{km/hr}$$



$$\begin{cases} \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} \\ \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} \end{cases}$$



$$\begin{cases} \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2} \\ \sin\dfrac{\pi}{6} = \dfrac{1}{2} \\ \cos\dfrac{\pi}{3} = \dfrac{1}{2} \\ \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2} \end{cases}$$



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사람의 절대속도

$$\vec{u}_b + \vec{u}_m = (7.071,\ -7.071) + (-1.414,\ -1.414)$$

$$= (5.657,\ -8.485)$$

$$\left|\vec{u}_b + \vec{u}_m\right| = 10.2\ \mathrm{km/hr}$$

실제 가는 방향 $\theta = ?$

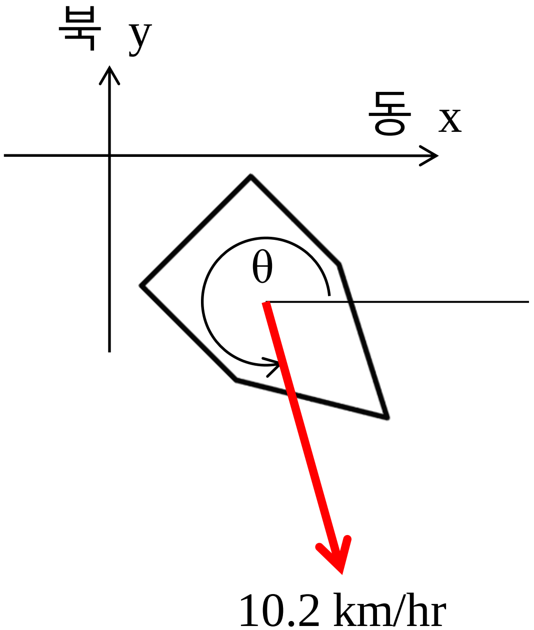
$$\tan\theta = \frac{-8.485}{5.657}$$

$$= -1.499$$

$$\theta = \tan^{-1}(-1.499)$$

$$= -\tan^{-1}(1.499)$$

$$= -56.3^{\circ}$$



1.7 Vector Multiplication

1. Scalar (or dot) product : $\vec{A} \cdot \vec{B}$

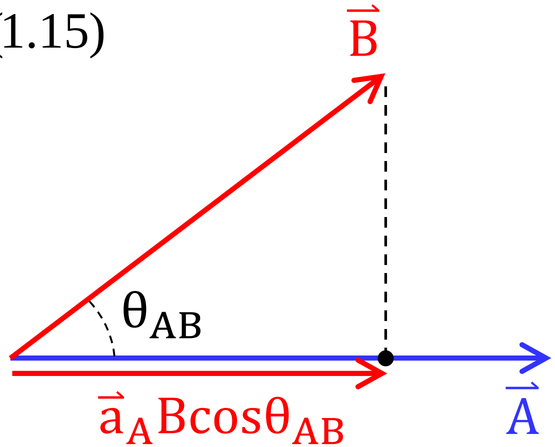
2. Vector (or cross) product : $\vec{A} \times \vec{B}$

3. Scalar triple product : $\vec{A} \cdot (\vec{A} \times \vec{C})$

4. Vector triple product : $\vec{A} \times (\vec{A} \times \vec{C})$

A. Dot Product

$$\vec{A}\cdot\vec{B} \equiv AB\cos\theta_{AB} \qquad (1.15)$$



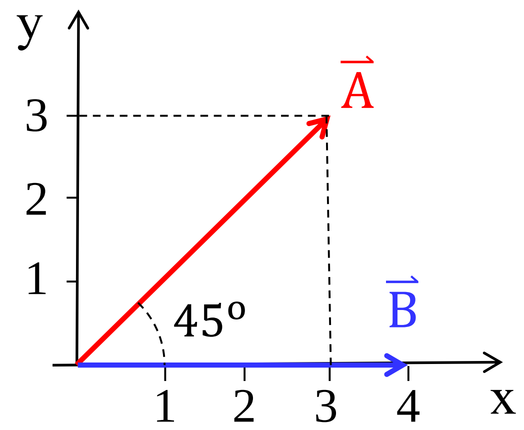
$$\begin{pmatrix} \vec{A} = (A_x, A_y, A_z) \\ \vec{B} = (B_x, B_y, B_z) \end{pmatrix} \rightarrow \vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad (1.16)$$

$$\vec{A} = (3,3), \quad \vec{B} = (4,0)$$

$$A = 3\sqrt{2}$$

$$B = 4$$

$$\cos(45^o) = 1/\sqrt{2}$$



$$\vec{A} \cdot \vec{B} = AB\cos\theta_{AB}$$

$$= 3\sqrt{2} \times 4 \times \frac{1}{\sqrt{2}}$$

$$= 12$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$= 3 \times 4 + 3 \times 0$$

$$= 12$$

(i) Commutative law :

$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A} \qquad (1.17)$$

(ii) Distributive law :

$$\vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C} \qquad (1.18)$$

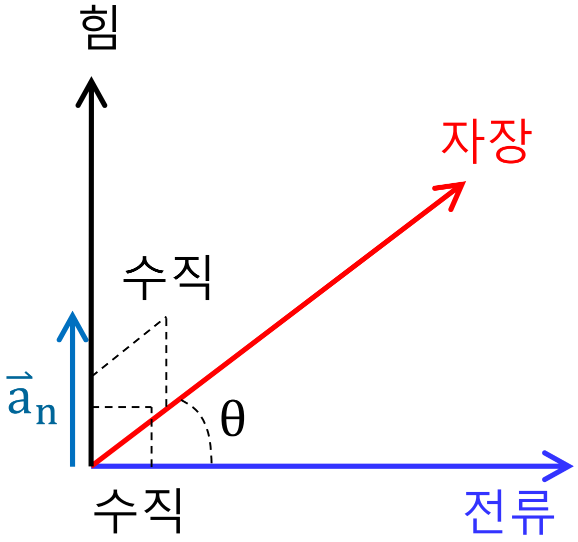
$$\vec{A}\cdot\vec{A}=|\vec{A}|^2=A^2 \qquad (1.19)$$

(iii) note :

$$\vec{a}_x\cdot\vec{a}_y=\vec{a}_y\cdot\vec{a}_z=\vec{a}_z\cdot\vec{a}_x=0 \qquad (1.20a)$$

$$\vec{a}_x\cdot\vec{a}_x=\vec{a}_y\cdot\vec{a}_y=\vec{a}_z\cdot\vec{a}_z=1 \qquad (1.20b)$$

B. Cross Product



$$m\vec{a} = \vec{J} \times \vec{B}$$

$$\vec{J} \times \vec{B} \equiv JB \sin\theta \vec{a}_n$$

$$\vec{A} \times \vec{B} \equiv AB \sin\theta_{AB} \vec{a}_n \qquad (1.21)$$

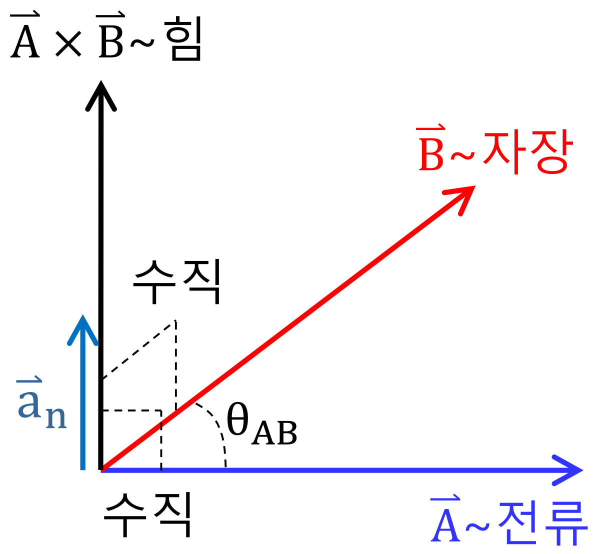


Fig. 1.7 The cross product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$\begin{pmatrix} \vec{A} = (A_x, A_y, A_z) \\ \vec{B} = (B_x, B_y, B_z) \end{pmatrix} \rightarrow$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad (1.22a)$$

$$\begin{aligned} &= (A_y B_z - A_z B_y)\vec{a}_x \\ &- (A_x B_z - A_z B_x)\vec{a}_y \qquad (1.22b) \\ &+ (A_x B_y - A_y B_x)\vec{a}_z \end{aligned}$$

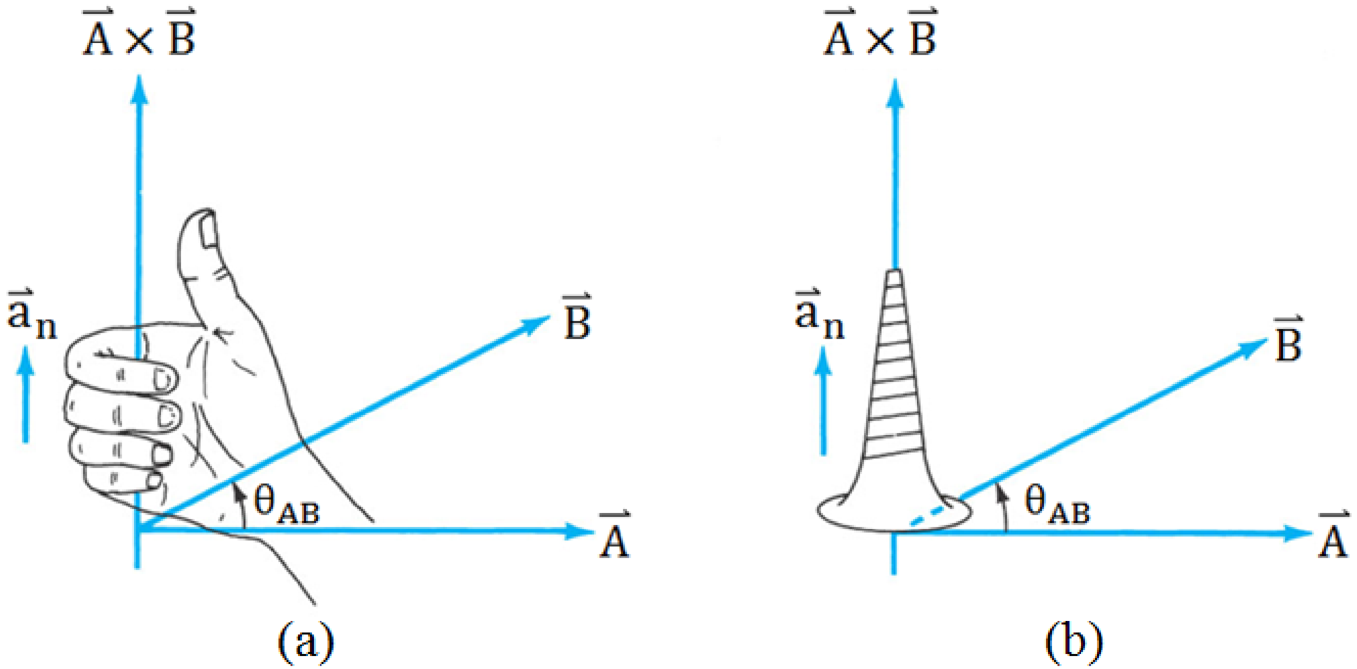


Fig. 1.8 Direction of $\vec{\mathbf{A}}, \vec{\mathbf{B}}$ and $\vec{\mathbf{a}}_{\mathbf{n}}$ using
(a) the right-hand rule and
(b) the right-handed-screw rule.

$$\vec{A}\times\vec{B} \equiv AB\sin\theta_{AB}\vec{a}_n$$

$$A = 3\sqrt{2}$$

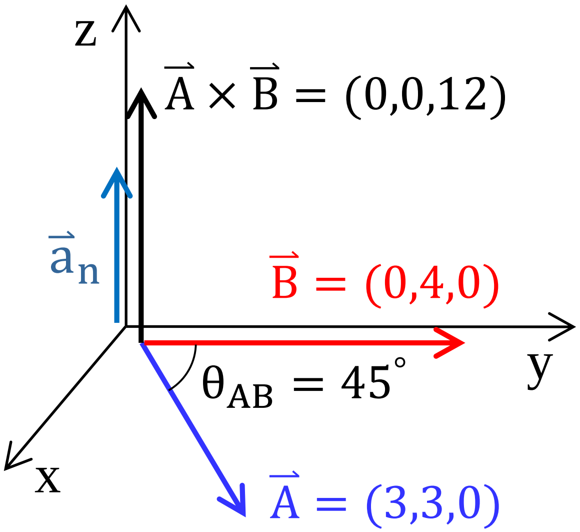
$$B = 4$$

$$\theta_{AB} = 45^o$$

$$\left|\vec{A}\times\vec{B}\right| = 3\sqrt{2}\times 4\times \sin 45^o$$

$$= 12$$

$$\vec{A}\times\vec{B} = \boxed{(0,0,12)}$$



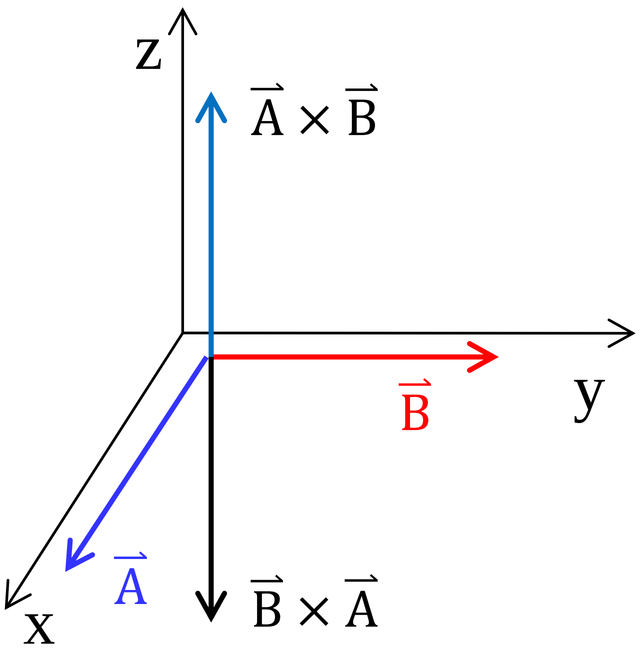
$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 3 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$= (0)\vec{a}_x - (0)\vec{a}_y + (12)\vec{a}_z$$

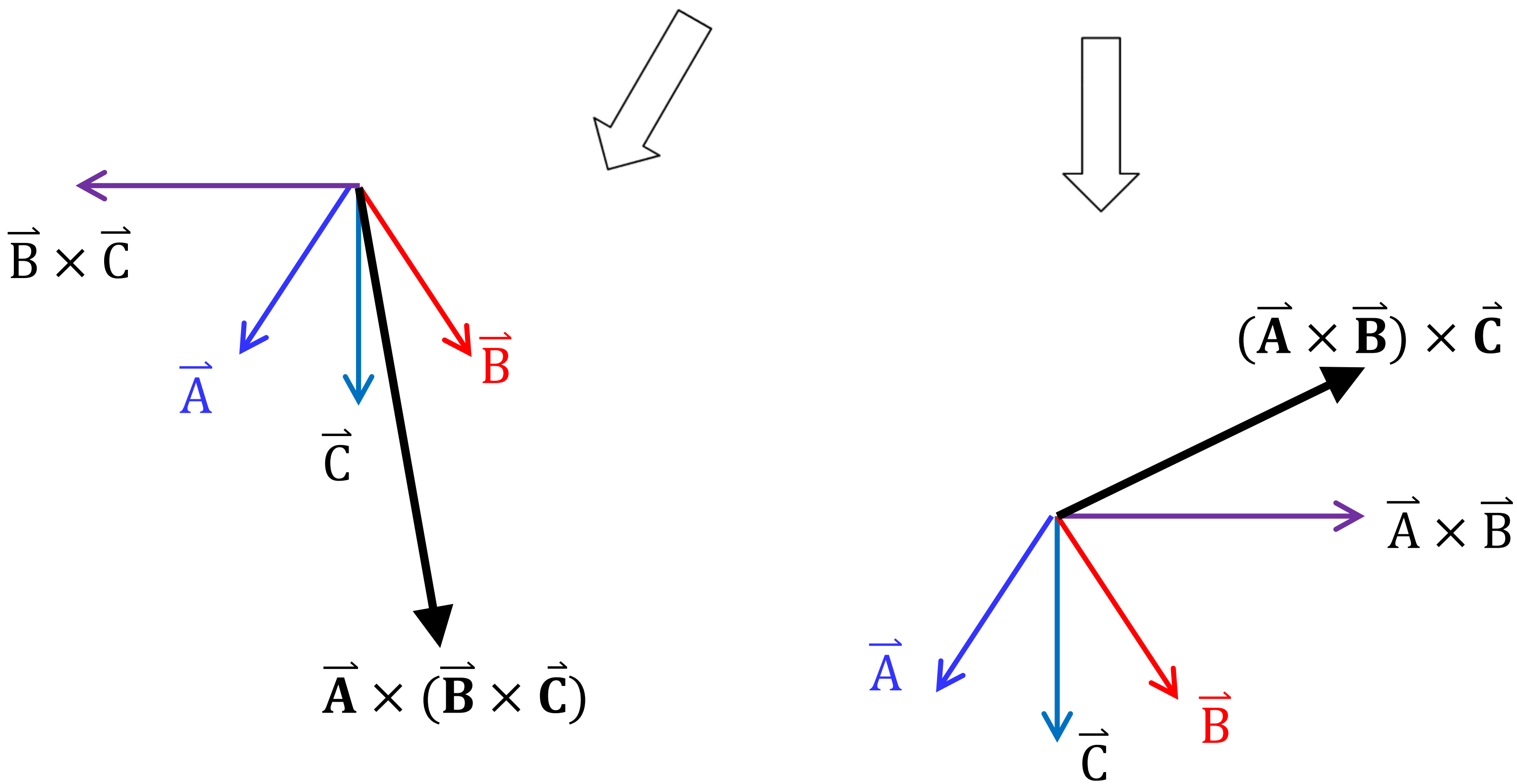
$$= \boxed{(0,0,12)}$$

(i) It is not commutative : $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ (1.23a)

It is anti − commutative: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (1.23b)



(ii) It is not associative: $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$ (1.24)



* $\vec{A}, \vec{B}, \vec{C}$가 동일 평면에 있을 경우

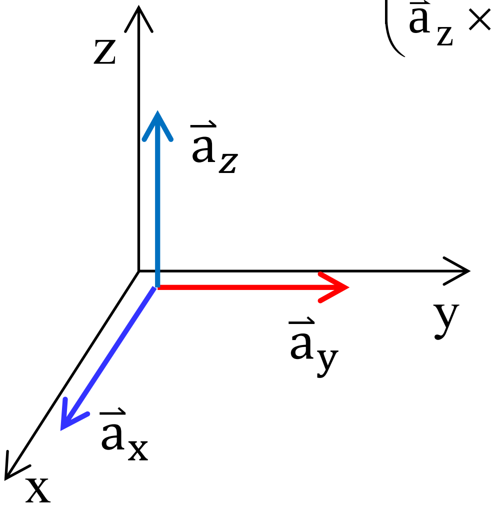
(iii) It is distributive:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad (1.25)$$

(iv) Note:

$$\vec{A} \times \vec{A} = 0 \quad (1.26)$$

$$\begin{cases} \vec{a}_x \times \vec{a}_y = \vec{a}_z \\ \vec{a}_y \times \vec{a}_z = \vec{a}_x \\ \vec{a}_z \times \vec{a}_x = \vec{a}_y \end{cases} \quad (1.27)$$



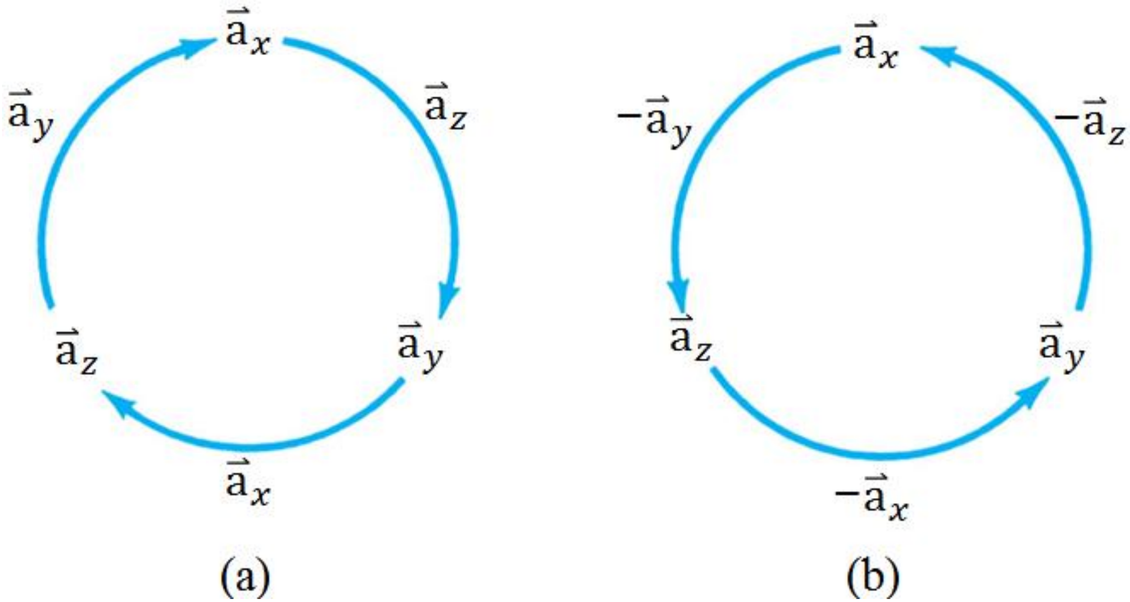


Fig. 1.9 Cross product using cyclic permutation
(a) Moving clockwise leads to positive results.
(b) Moving counterclockwise leads to negative results.

C. Scalar Triple Product

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B}) \qquad (1.28)$$

$$\begin{pmatrix} \vec{A}=(A_{x,}A_y,A_z) \\ \vec{B}=(B_x,B_y,B_z) \\ \vec{C}=(C_x,C_y,C_z) \end{pmatrix}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{A}\cdot\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$=(A_y.\vec{a}_x+A_y.\vec{a}_y+A_z\vec{a}_z)\cdot\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$=\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_Z \\ C_x & C_y & C_z \end{vmatrix} \qquad (1.29)$$

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})$$

$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} B_x & B_y & B_z \\ C_x & C_y & C_z \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} C_x & C_y & C_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

D. Vector Triple Product

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \qquad (1.30)$$

$$(\vec{A} \cdot \vec{B})\vec{C} \neq \vec{A}(\vec{B} \cdot \vec{C}) \qquad (1.31)$$

$$(\vec{A} \cdot \vec{B})\vec{C} = \vec{C}(\vec{A} \cdot \vec{B}) \qquad (1.32)$$

1.8 Component of a Vector

$$A_B \equiv A\cos\theta_{AB}$$

$$= |\vec{A}||\vec{a}_B|\cos\theta_{AB}$$

$$A_B = \vec{A}\cdot\vec{a}_B \qquad (1.33)$$

$$\vec{A}_B = A_B\vec{a}_B = (\vec{A}\cdot\vec{a}_B)\vec{a}_B \qquad (1.34)$$

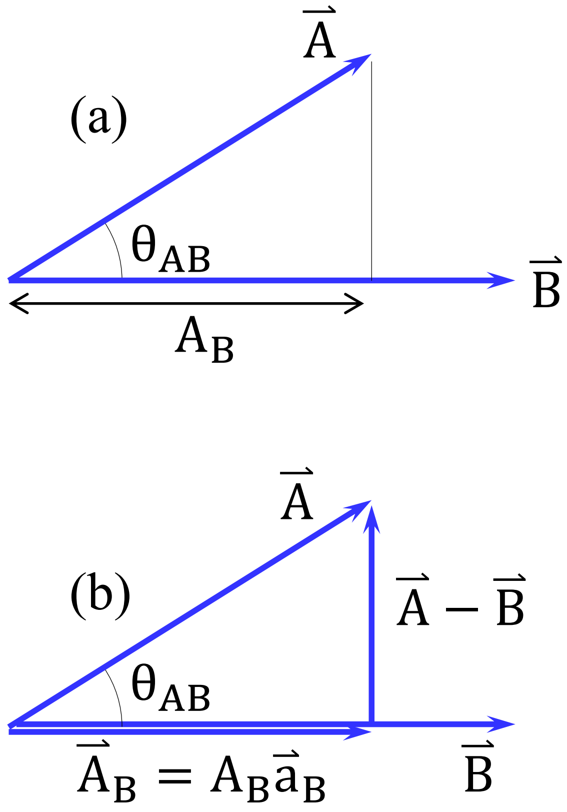


Figure 1.10 Components of $\vec{A}$ along $\vec{B}$:
(a) scalar component A_B,
(b) vector component $\vec{A}_B$.

예제 1.4 Vector $\vec{\mathbf{A}}$=(3,4,1), $\vec{\mathbf{B}}$=(0,2,-5) 일 때 Vector 사이의 각도를 구하라.

$$\cos\theta_{AB} = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$= \frac{(3,4,1)\cdot(0,2,-5)}{\sqrt{3^2+4^2+1^2}\sqrt{0^2+2^2+(-5)^2}}$$

$$= 0.1092$$

$$\cos^{-1}(0.1092) = 83.73^{o}$$

예제 1.5 Vector $\vec{P}$=(2,0,-1), $\vec{Q}$=(2,-1,2), $\vec{R}$=(2,-3,1) 일 때 다음을 구하라.

(a) $(\vec{P}+\vec{Q})\times(\vec{P}-\vec{Q})$
(b) $\vec{Q}\cdot\vec{R}\times\vec{P}$
(c) $\vec{P}\cdot\vec{Q}\times\vec{R}$
(d) $\sin\theta_{QR}$
(e) $\vec{P}\times(\vec{Q}\times\vec{R})$
(f) $\vec{Q}$와 $\vec{R}$에 수직인 unit vector
(g) $\vec{P}_Q$

(a) $(\vec{P}+\vec{Q})\times(\vec{P}-\vec{Q})$

$$
\begin{aligned}
&= \vec{P}\times(\vec{P}-\vec{Q})+\vec{Q}\times(\vec{P}-\vec{Q}) \\
&= \vec{P}\times\vec{P}-\vec{P}\times\vec{Q}+\vec{Q}\times\vec{P}-\vec{Q}\times\vec{Q} \\
&= 0+\vec{Q}\times\vec{P}+\vec{Q}\times\vec{P}-0 \\
&= 2\vec{Q}\times\vec{P} = 2\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 2 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} \\
&= 2(-1\times-1-2\times0\ ,\ -2\times-1+2\times2\ ,\ 2\times0- -1\times2) \\
&= 2(1,\ 6,\ 2) \\
&= (2,\ 12,\ 4)
\end{aligned}
$$

예제 1.5 Vector $\vec{P}=(2,0,-1)$, $\vec{Q}=(2,-1,2)$, $\vec{R}=(2,-3,1)$ 일 때 다음을 구하라.

(b) $\vec{Q}\cdot\vec{R}\times\vec{P}$

$$\vec{Q}\cdot\vec{R}\times\vec{P}=\begin{vmatrix}2 & -1 & 2\\ 2 & -3 & 1\\ 2 & 0 & -1\end{vmatrix}=\begin{vmatrix}2 & -1 & 2\\ 2 & -3 & 1\\ 2 & 0 & -1\\ 2 & -1 & 2\\ 2 & -3 & 1\end{vmatrix}$$

-12 -, 0 -, 2 -

+ 6, + 0, + -2

$$=(+6+0-2)-(-12+0+2)$$

$$=14$$

$$\vec{Q}\cdot\vec{R}\times\vec{P}=\begin{vmatrix}2 & -1 & 2\\ 2 & -3 & 1\\ 2 & 0 & -1\end{vmatrix}=\left(\begin{array}{l}+(2)\times(-3\times-1 \;-\; 1\times 0)\\ -(-1)\times(2\times-1 \;-\; 1\times 2)\\ +(2)\times(2\times 0 \;-\; -3\times 2)\end{array}\right.=\left(\begin{array}{l}+(2)\times(3)\\ -(-1)\times(-4)\\ +(2)\times(6)\end{array}\right.$$

$$=+6-4+12$$

$$=14$$

예제 1.5 Vector $\vec{P}$=(2,0,-1), $\vec{Q}$=(2,-1,2), $\vec{R}$=(2,-3,1) 일 때 다음을 구하라.
(c) $\vec{P} \cdot \vec{Q} \times \vec{R}$

$$\vec{Q} \cdot \vec{R} \times \vec{P} = \begin{vmatrix} 2 & -1 & 2 \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 14$$

$$\vec{P} \cdot \vec{Q} \times \vec{R} = \begin{vmatrix} 2 & 0 & -1 \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 2 \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 14$$

$$10 \quad 0 \quad 4 \qquad\qquad 6 \quad -4 \quad 12$$

(cf)

(1.28) $\vec{A} \cdot (\vec{B} \times \vec{C})$

$= \vec{B} \cdot (\vec{C} \times \vec{A})$

$= \vec{C} \cdot (\vec{A} \times \vec{B})$

예제 1.5 Vector $\vec{P}$=(2,0,-1), $\vec{Q}$=(2,-1,2), $\vec{R}$=(2,-3,1) 일 때 다음을 구하라.

(d) $\sin\theta_{QR}$

(e) $\vec{P} \times (\vec{Q} \times \vec{R})$

$$\text{(d)}\quad \sin\theta_{\vec{Q}\vec{R}} = \frac{|\vec{Q}\times\vec{R}|}{|\vec{Q}||\vec{R}|} = \left|\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 2 & -1 & 2 \\ 2 & -3 & 1 \end{vmatrix}\right| / \sqrt{9\times 14} = 0.5976$$

(e)

$$\text{Eq (1.30)}\quad \vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) \quad \rightarrow$$

$$\vec{P}\times(\vec{Q}\times\vec{R}) = \vec{Q}(\vec{P}\cdot\vec{R}) - \vec{R}(\vec{P}\cdot\vec{Q})$$

$$= (2,-1,2)(4+0-1) - (2,-3,1)(4+0-2)$$

$$= (2,3,4)$$

예제 1.5 Vector $\vec{P}$=(2,0,-1), $\vec{Q}$=(2,-1,2), $\vec{R}$=(2,-3,1) 일 때 다음을 구하라.
(f) $\vec{Q}$와 $\vec{R}$에 수직인 unit vector

$$(f)\ \vec{a} = \pm\frac{\vec{Q}\times\vec{R}}{\left|\vec{Q}\times\vec{R}\right|}$$

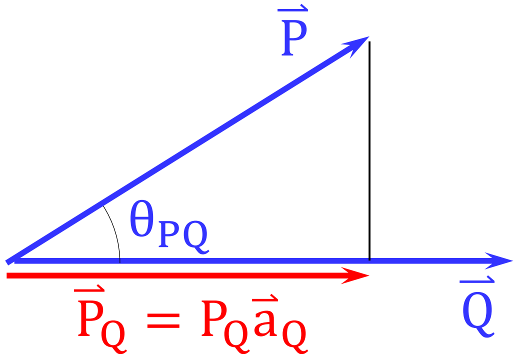
$$= \pm\frac{(5,\ 2,\ -4)}{\left|(5,\ 2,\ -4)\right|}$$

$$= \pm\frac{(5,\ 2,\ -4)}{\sqrt{5^2+2^2+(-4)^2}}$$

$$= \pm(0.745, 0.298, -0.596)$$

예제 1.5 Vector $\vec{P}$=(2,0,-1), $\vec{Q}$=(2,-1,2), $\vec{R}$=(2,-3,1) 일 때 다음을 구하라.
(g) $\vec{P}_Q$

$$\text{(g) } \vec{P}_Q = |\vec{P}|\cos\theta_{PQ}\vec{a}_Q$$
$$= (\vec{P}\cdot\vec{a}_Q)\vec{a}_Q$$
$$= \left(\vec{P}\cdot\frac{\vec{Q}}{|\vec{Q}|}\right)\frac{\vec{Q}}{|\vec{Q}|}$$
$$= \frac{(\vec{P}\cdot\vec{Q})\vec{Q}}{|\vec{Q}|^2}$$
$$= \frac{(4+0-2)(2,-1,2)}{(2^2+(-1)^2+2^2)}$$
$$= \frac{2}{9}(2,-1,2)$$



예제 1.6 다음을 유도 하라

(a) cosine 공식: $a^2 = b^2 + c^2 - 2bc\cos A$

(b) sine 공식: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

(a) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$\vec{a} \cdot \vec{a} = (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c})$

$= \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} + 2\vec{b} \cdot \vec{c}$

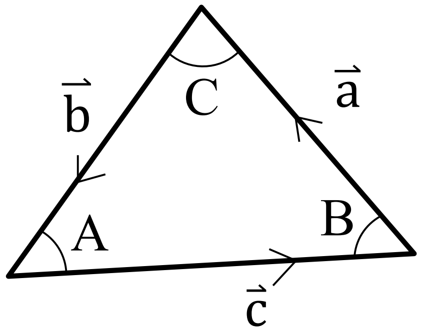
$a^2 = b^2 + c^2 - 2bc\cos A$

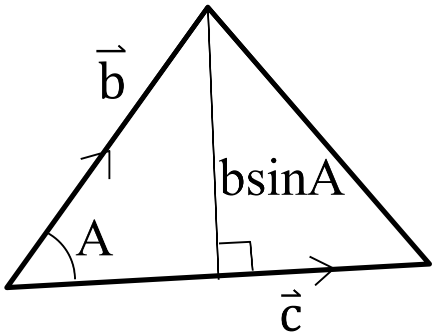
(b) 삼각형 면적

$$\left|\frac{1}{2}\vec{a} \times \vec{b}\right| = \left|\frac{1}{2}\vec{b} \times \vec{c}\right| = \left|\frac{1}{2}\vec{c} \times \vec{a}\right|$$

$$\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$$

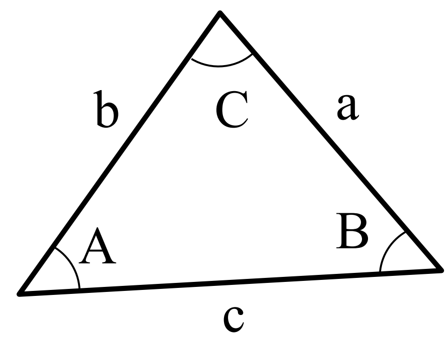
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$





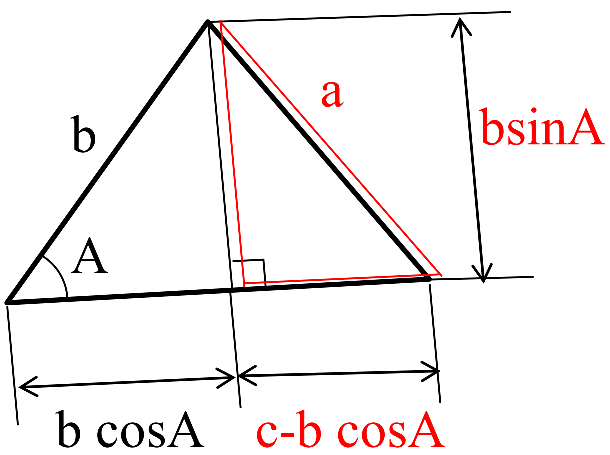
예제 1.6 다음을 유도 하라

(a) cosine 공식: $a^2 = b^2 + c^2 - 2b\cos A$



* 빨간 삼각형의 Pythagoras 정리

$$
\begin{aligned}
a^2 &= (b\sin A)^2 + (c - b\cos A)^2 \\
&= b^2\sin^2 A + c^2 - 2bc\cos A + b^2\cos^2 A \\
&= b^2\sin^2 A + b^2\cos^2 A + c^2 - 2bc\cos A \\
&= b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A \\
&= b^2 + c^2 - 2bc\cos A
\end{aligned}
$$



예제 1.7
(a) Point P_1, P_2, P_3 가 동일 직선에 있음을 보여라.
(b) P_4 (3,-1,0)와 직선 사이의 최단 거리.

(a) $\vec{r}_{12} = \vec{r}_{P_2} - \vec{r}_{P_1}$

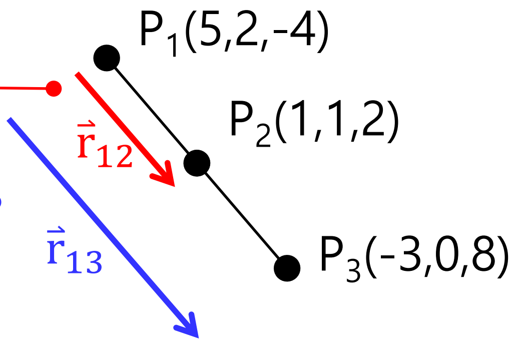
$= (1,1,2) - (5,2,-4)$

$= (-4,-1,6)$

$\vec{r}_{13} = \vec{r}_{P_3} - \vec{r}_{P_1}$

$= (-3,0,8) - (5,2,-4)$

$= (-2,-3,4)$



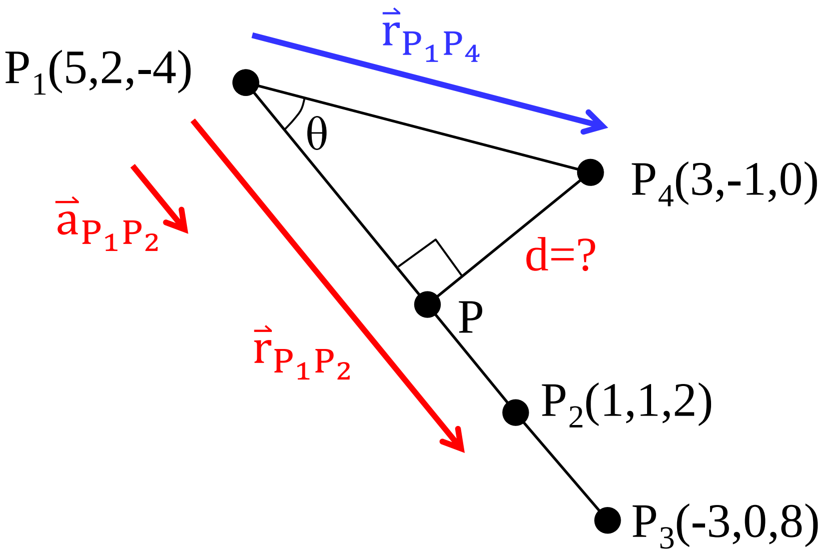
$$\vec{r}_{12} \times \vec{r}_{13} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ -4 & -1 & 6 \\ -8 & -2 & 12 \end{vmatrix} = (0,0,0) \rightarrow \text{동일 직선}$$

예제 1.7 (b) P_4 (3,-1,0)와 직선 사이의 최단 거리.

$$d = r_{P_1P_4}\sin\theta = \left|\vec{r}_{P_1P_4} \times \vec{a}_{P_1P_2}\right|$$

$$= \left|(-2,-3,4) \times \frac{(-4,-1,6)}{\left|(-4,-1,6)\right|}\right|$$

$$= 2.426$$



P_1, P_2를 잇는 직선의 방정식

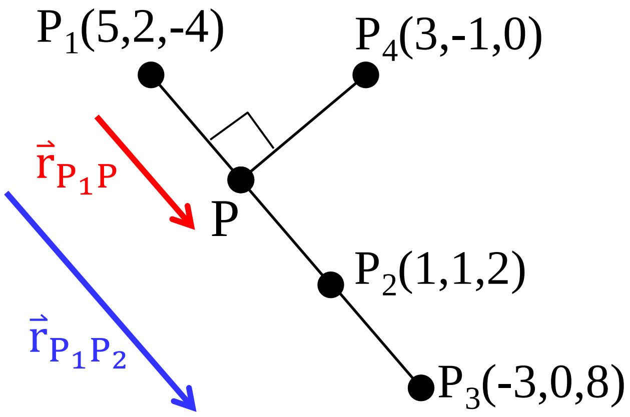
$$\vec{r}_{P_1P} = \vec{r}_P - \vec{r}_{P_1}$$

$$= \lambda \vec{r}_{P_1P_2} = \lambda(\vec{r}_{P_2} - \vec{r}_{P_1})$$

$$\vec{r}_P = \vec{r}_{P_1} + \lambda(\vec{r}_{P_2} - \vec{r}_{P_1})$$

$$= (5-4\lambda,\ 2-\lambda,\ -4+6\lambda)$$

$* \lambda = 2$ 일 때 P_3



$$\begin{pmatrix} x = 5-4\lambda \\ y = 2-\lambda \\ z = -4+6\lambda \end{pmatrix} \leftarrow 2 \text{ 개의 방정식}$$