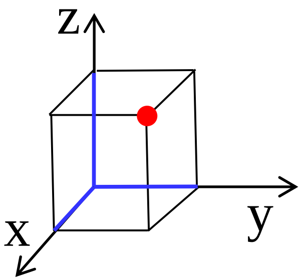
2. Coordinate Systems and Transformation

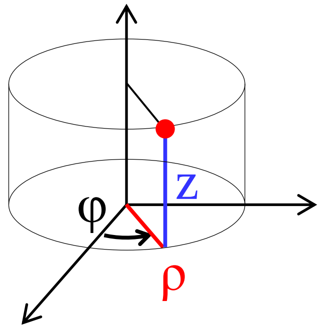
2.1 Introduction

직교좌표계 (Orthogonal Coordinate System):
좌표축이 서로 수직인 좌표계.

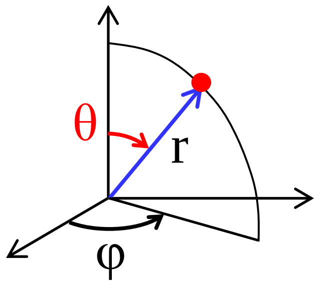
- 직각좌표계 (Cartesian Coordinate)

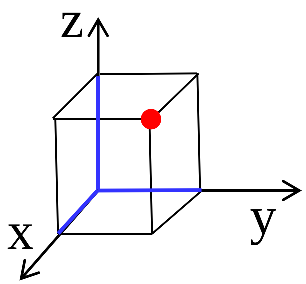


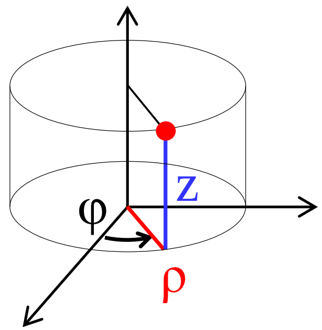
- 원통좌표계 (Cylindrical Coordinate)

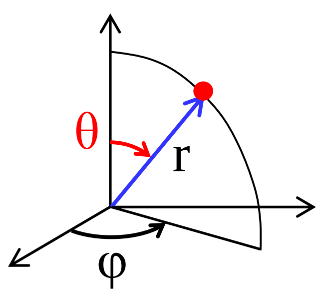


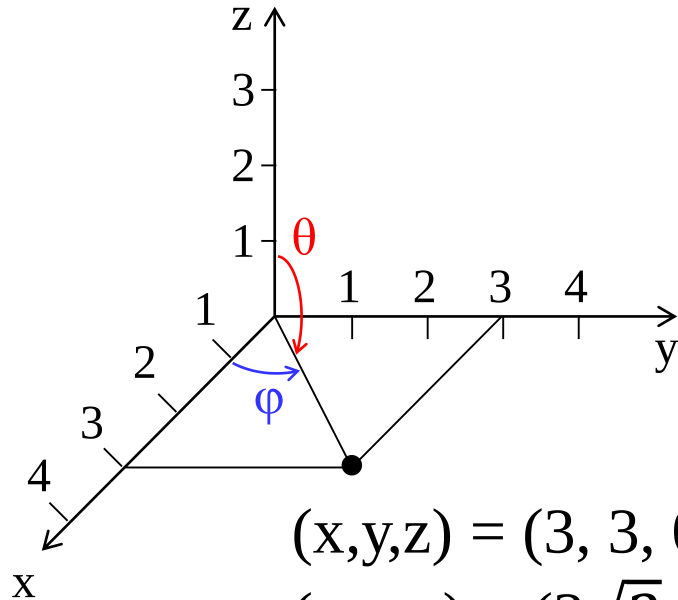
- 구좌표계 (Spherical Coordinate)
  극좌표계







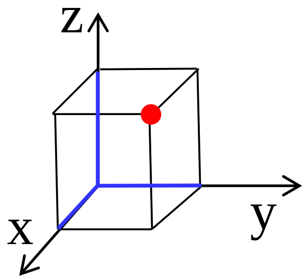


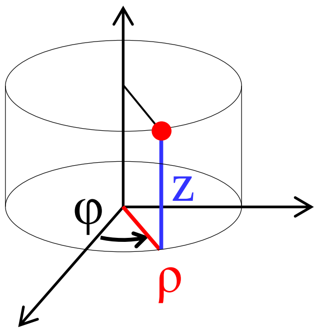


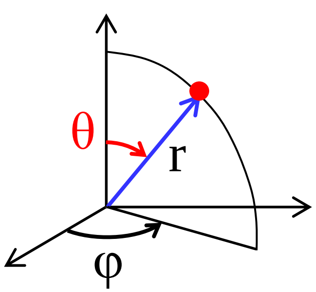
$$(x,y,z) = (3,\ 3,\ 0)$$

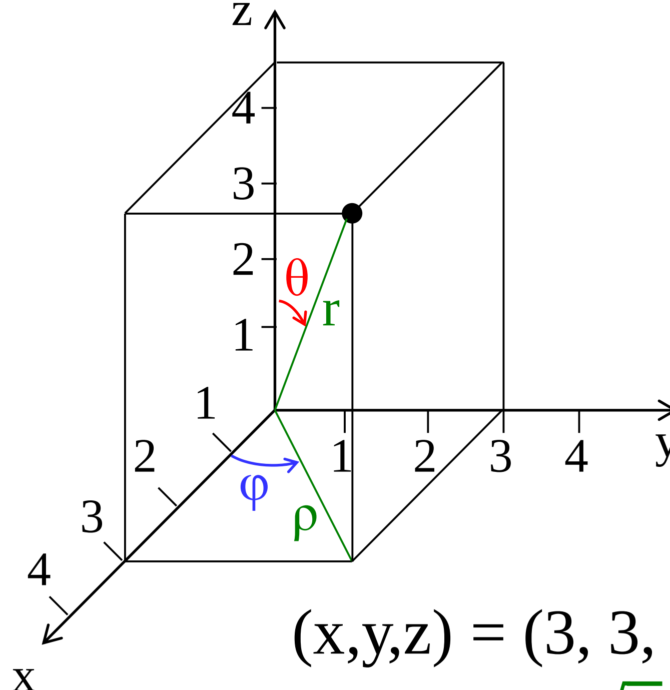
$$(\rho,\varphi,z) = (3\sqrt{2},\ \pi/4,\ 0)$$

$$(r,\theta,\varphi) = (3\sqrt{2},\ \pi/2,\ \pi/4)$$









$$(x,y,z) = (3,\ 3,\ 3\sqrt{2})$$

$$(\rho,\varphi,z) = (3\sqrt{2},\ \pi/4,\ 3\sqrt{2})$$

$$(r,\theta,\varphi) = (6,\ \pi/4,\ \pi/4)$$

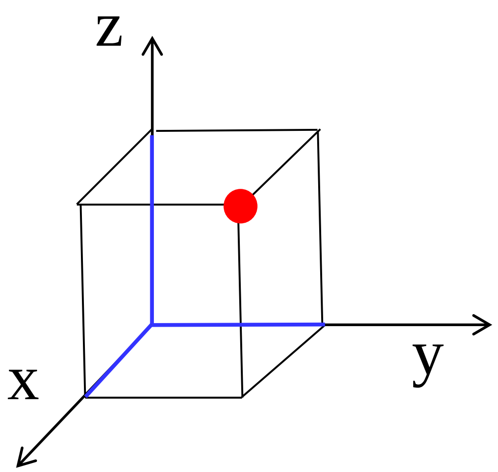
2.2 Cartesian Coordinates (x, y, z)

(x, y, z) 좌표의 범위

$$-\infty < x < \infty$$

$$-\infty < y < \infty \qquad (2.1)$$

$$-\infty < z < \infty$$



Vector $\vec{A}$ 의 표시 방법

$$\vec{A} = (A_x, A_y, A_z)$$

$$= A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \qquad (2.2)$$

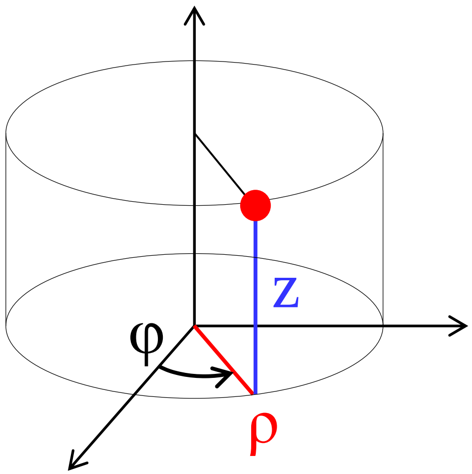
2.3 Circular Cylindrical Coordinate (ρ,φ,z)

(ρ,φ,z) 좌표의 범위

$$0 \leq \rho < \infty$$

$$0 \leq \varphi < 2\pi \qquad (2.3)$$

$$-\infty < z < \infty$$



Vector $\vec{A}$ 의 표시 방법

$$\vec{A} = (A_\rho, A_\varphi, A_z)$$

$$= A_\rho \vec{a}_\rho + A_\varphi \vec{a}_\varphi + A_z \vec{a}_z \qquad (2.4)$$

Vector $\vec{A}$ 의 크기

$$|\vec{A}| = ({A_\rho}^2 + {A_\varphi}^2 + {A_z}^2)^{1/2} \qquad (2.5)$$

단위 Vector 계산

$$\vec{A}\cdot\vec{B} \equiv AB\cos\theta_{AB}$$

$$\vec{a}_\rho\cdot\vec{a}_\rho = \vec{a}_\phi\cdot\vec{a}_\phi = \vec{a}_z\cdot\vec{a}_z = 1 \quad (2.6a)$$

$$\vec{a}_\rho\cdot\vec{a}_\phi = \vec{a}_\phi\cdot\vec{a}_z = \vec{a}_z\cdot\vec{a}_\rho = 0 \quad (2.6b)$$ ← 서로 수직

$$\vec{A}\times\vec{B} \equiv AB\sin\theta_{AB}\vec{a}_n$$

$$\vec{a}_\rho\times\vec{a}_\phi = \vec{a}_z \quad (2.6c)$$

$$\vec{a}_\phi\times\vec{a}_z = \vec{a}_\rho \quad (2.6d)$$

$$\vec{a}_z\times\vec{a}_\rho = \vec{a}_\phi \quad (2.6e)$$

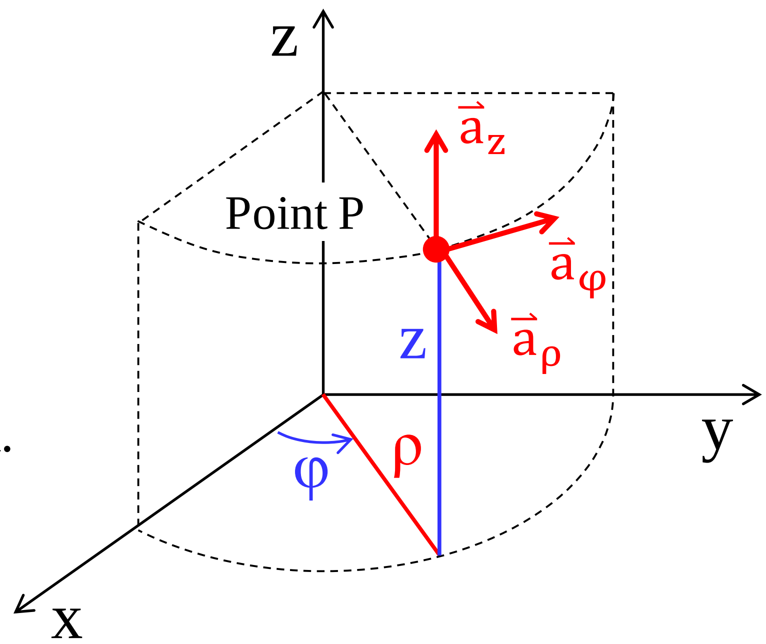


Fig 2.1 Point **P** and unit vectors in the cylindrical coordinate system.

직각좌표계와 원통 좌표계의 관계식

$$\begin{pmatrix} \rho = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}\dfrac{y}{x} \\ z = z \end{pmatrix} \quad (2.7)$$

$$\begin{pmatrix} x = \rho(a_x \cdot a_\rho) = \rho\cos\phi \\ y = \rho(a_y \cdot a_\rho) = \rho\sin\phi \\ z = z \end{pmatrix} \quad (2.8)$$

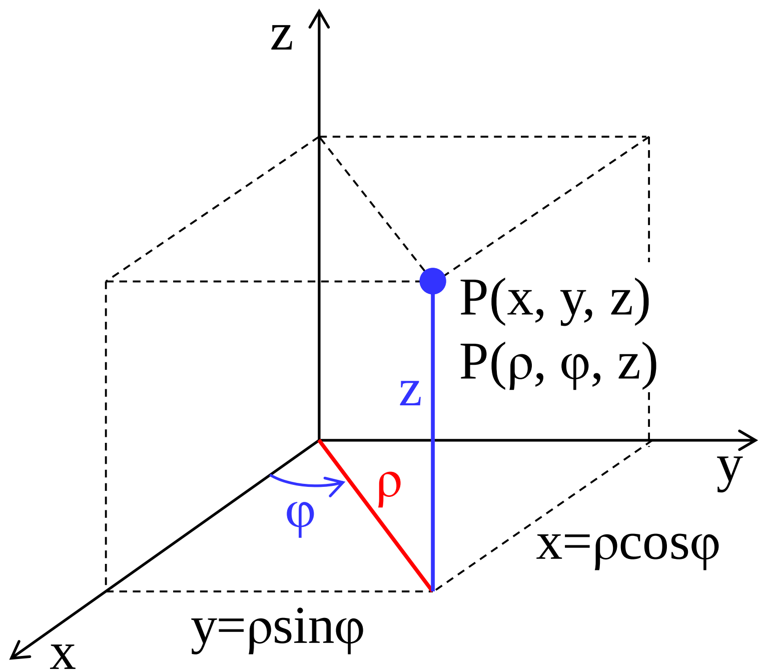
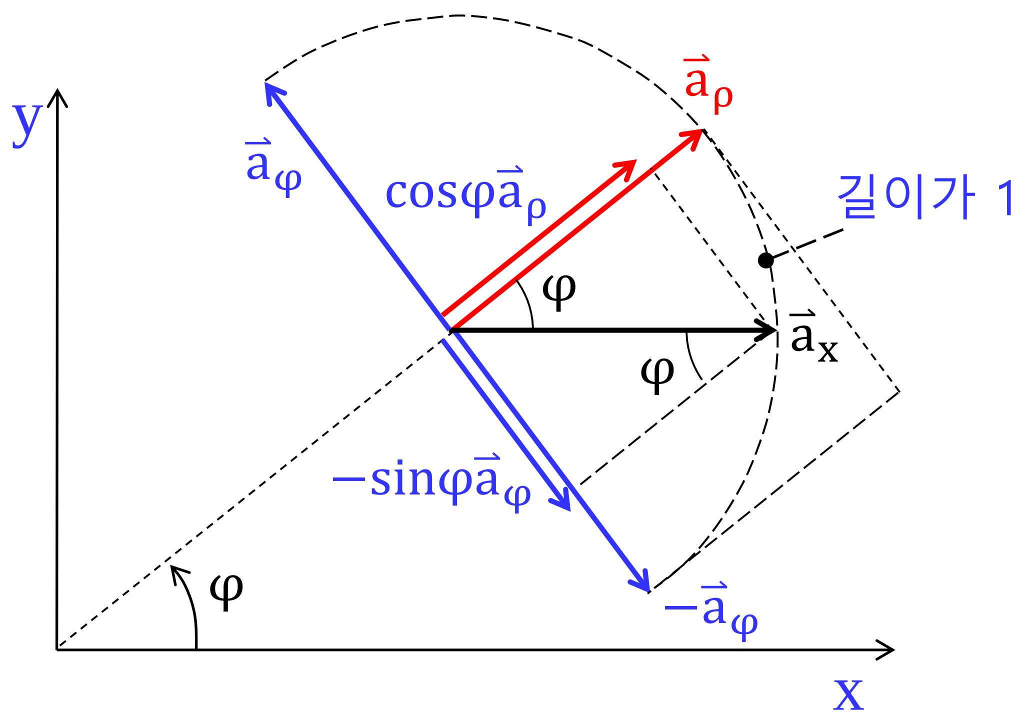


Fig 2.2

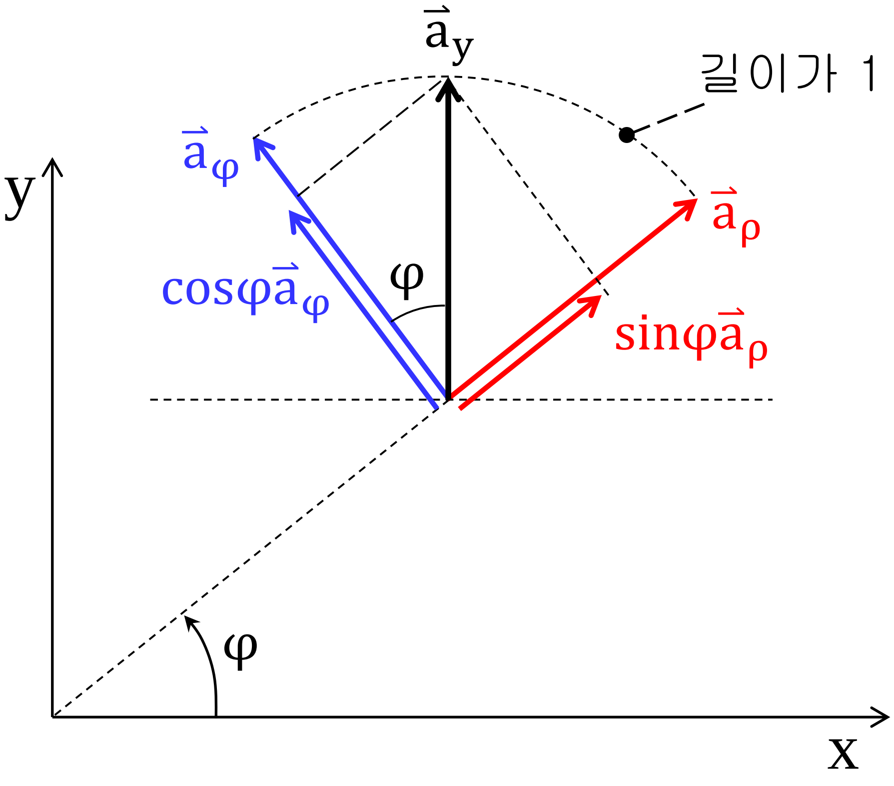
직각좌표계와 원통 좌표계의 관계식

$$\begin{cases} \boxed{\vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi} \\ \vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \vec{a}_z \end{cases} \qquad (2.9)$$



직각좌표계와 원통 좌표계의 관계식

$$\begin{pmatrix} \vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi \\ \boxed{\vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi} \\ \vec{a}_z = \vec{a}_z \end{pmatrix} \qquad (2.9)$$



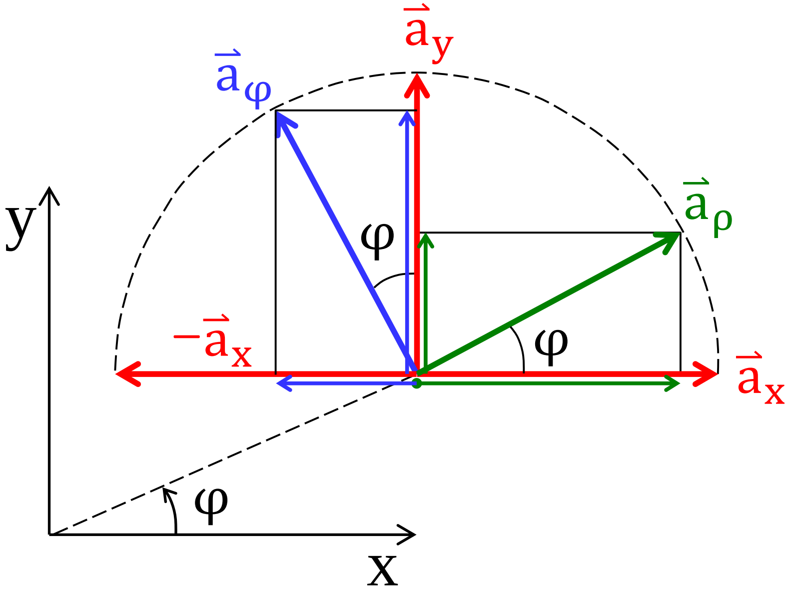
$$\begin{pmatrix} \vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi \\ \vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \vec{a}_z \end{pmatrix} \quad (2.9)$$

2원 1차 연립 방정식 $\begin{pmatrix} ax + by = c \\ dx + ey = f \end{pmatrix}$

$$\begin{pmatrix} \vec{a}_\rho = \cos\phi\,\vec{a}_x + \sin\phi\,\vec{a}_y \\ \vec{a}_\phi = -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y \\ \vec{a}_z = \vec{a}_z \end{pmatrix} \quad (2.10)$$

$$\vec{a}_\rho = \cos\phi\,\vec{a}_x + \sin\phi\,\vec{a}_y$$

$$\vec{a}_\phi = \cos\phi\,\vec{a}_y - \sin\phi\,\vec{a}_x$$



$$\begin{pmatrix} \vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi \\ \vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \vec{a}_z \end{pmatrix} \qquad (2.9)$$

대입

$$(2.2) \quad \vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\begin{aligned} \vec{A} &= (A_x\cos\phi + A_y\sin\phi)\vec{a}_\rho \\ &\quad + (-A_x\sin\phi + A_y\cos\phi)\vec{a}_\phi + A_z\vec{a}_z \\ &= A_\rho\vec{a}_\rho + A_\phi\vec{a}_\phi + A_z\vec{a}_z \qquad (2.11) \end{aligned}$$

$$\text{where} \begin{pmatrix} A_\rho = A_x\cos\phi + A_y\sin\phi \\ A_\phi = -A_x\sin\phi + A_y\cos\phi \\ A_z = A_z \end{pmatrix} \quad (2.12)$$

Matrix 곱하기

$$\underline{\underline{A}} = A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \qquad B_{ij} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} A_{ij}B_{jk} = A_{1j}B_{j1} &= A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} \\ &= 1*1 + 2*2 + 3*1 \\ &= 8 \end{aligned}}$$

$$A_{ij}B_{jk} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 5 \\ 3 & 4 & 5 \\ 9 & 5 & 4 \end{bmatrix}$$

$$\boxed{\begin{aligned} A_i &= C_{ij}B_j \\ &= C_{i1}B_1 + C_{i2}B_2 + C_{i3}B_3 \\[1em] A_1 &= C_{11}B_1 + C_{12}B_2 + C_{13}B_3 \\ A_2 &= C_{21}B_1 + C_{22}B_2 + C_{23}B_3 \\ A_3 &= C_{31}B_1 + C_{32}B_2 + C_{33}B_3 \\[1em] \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \end{aligned}}$$

$$\begin{cases} A_\rho = A_x \cos\phi + A_y \sin\phi \\ A_\phi = -A_x \sin\phi + A_y \cos\phi \\ A_z = A_z \end{cases} \qquad (2.12)$$

$$\begin{cases} A_\rho = \cos\phi A_x + \sin\phi A_y + 0 * A_z \\ A_\phi = -\sin\phi A_x + \cos\phi A_y + 0 * A_z \\ A_z = 0 * A_x + 0 * A_y + 1 * A_z \end{cases}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad (2.13)$$

Matrix

$(A_{ij}B_{jk})C_{km} = A_{ij}(B_{jk}C_{km})$ 결합 법칙

$A_{ij}(B_{jk} + C_{jk}) = A_{ij}B_{jk} + A_{ij}C_{jk}$ 분배 법칙

$(B_{ij} + C_{ij})A_{jk} = B_{ij}A_{jk} + C_{ij}A_{jk}$ 분배 법칙

$A_{ij}B_{jk} \neq B_{ij}A_{jk}$ 교환 법칙

$$A_i = C_{ij}B_j$$

$$B_k = D_{ki}A_i$$
$$= D_{ki}C_{ij}B_j$$
$$= E_{kj}B_j$$
$$= B_k$$

$$\boxed{E_{kj} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : \text{Unit Matrix}}$$

$D_{ij} = C_{ij}^{-1}$: Inverse Matrix of C_{ij}

$$C_{ij}^{-1}C_{jk} = E_{ik}$$

Inverse Matrix 구하기

$$F_1 = x + 2y + 3z$$
$$F_2 = -2x - 3y - 4z$$
$$F_3 = 2x + 2y + 4z$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 1 & +2 & +3 \\ -2 & -3 & -4 \\ 2 & +2 & +4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 + 2U_1 \\ U_3 - 2U_1 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}\begin{pmatrix} V_1 - 2V_2 \\ V_2 \\ V_3 + 2V_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} -3 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 - W_3/2 \\ W_2 - W_3 \\ W_3/2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

Inverse Matrix 구하기

$$A_{ij} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix}$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 + 2U_1 \\ U_3 - 2U_1 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \begin{pmatrix} V_1 - 2V_2 \\ V_2 \\ V_3 + 2V_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -3 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 - W_3/2 \\ W_2 - W_3 \\ W_3/2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

$$A_{IJ}^{-1} = \begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

Vector 좌표 변환

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad (2.13)$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \qquad (2.14)$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \qquad (2.15)$$

방정식 (2.16) 유도

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$
$$= A_\rho\vec{a}_\rho + A_\phi\vec{a}_\phi + A_z\vec{a}_z$$

$$A_x = \vec{a}_x \cdot \vec{A} = \vec{a}_x \cdot \vec{a}_\rho A_\rho + \vec{a}_x \cdot \vec{a}_\phi A_\phi + \vec{a}_x \cdot \vec{a}_z A_z$$
$$A_y = \vec{a}_y \cdot \vec{A} = \vec{a}_y \cdot \vec{a}_\rho A_\rho + \vec{a}_y \cdot \vec{a}_\phi A_\phi + \vec{a}_y \cdot \vec{a}_z A_z$$
$$A_z = \vec{a}_z \cdot \vec{A} = \vec{a}_z \cdot \vec{a}_\rho A_\rho + \vec{a}_z \cdot \vec{a}_\phi A_\phi + \vec{a}_z \cdot \vec{a}_z A_z$$

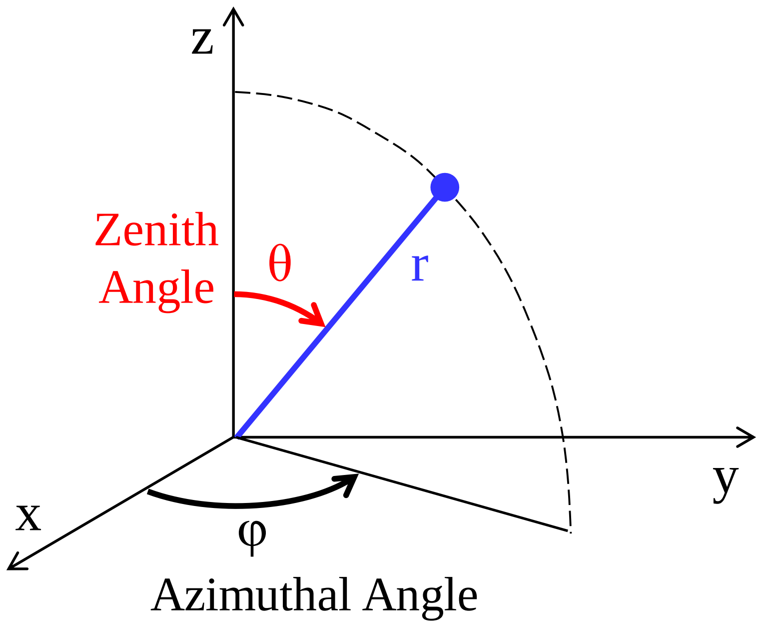
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \vec{a}_x \cdot \vec{a}_\rho & \vec{a}_x \cdot \vec{a}_\phi & \vec{a}_x \cdot \vec{a}_z \\ \vec{a}_y \cdot \vec{a}_\rho & \vec{a}_y \cdot \vec{a}_\phi & \vec{a}_y \cdot \vec{a}_z \\ \vec{a}_z \cdot \vec{a}_\rho & \vec{a}_z \cdot \vec{a}_\phi & \vec{a}_z \cdot \vec{a}_z \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \quad (2.16)$$

2.4 Spherical Coordinates (r, θ, φ)

$$\begin{pmatrix} 0 \le r < \infty \\ 0 \le \theta < \pi \\ 0 \le \phi < 2\pi \end{pmatrix} \qquad (2.17)$$

$$\vec{A} = (A_r, A_\theta, A_\phi)$$
$$= A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi \qquad (2.18)$$

$$|A| = (A_r^2 + A_\theta^2 + A_\phi^2)^{1/2} \qquad (2.19)$$

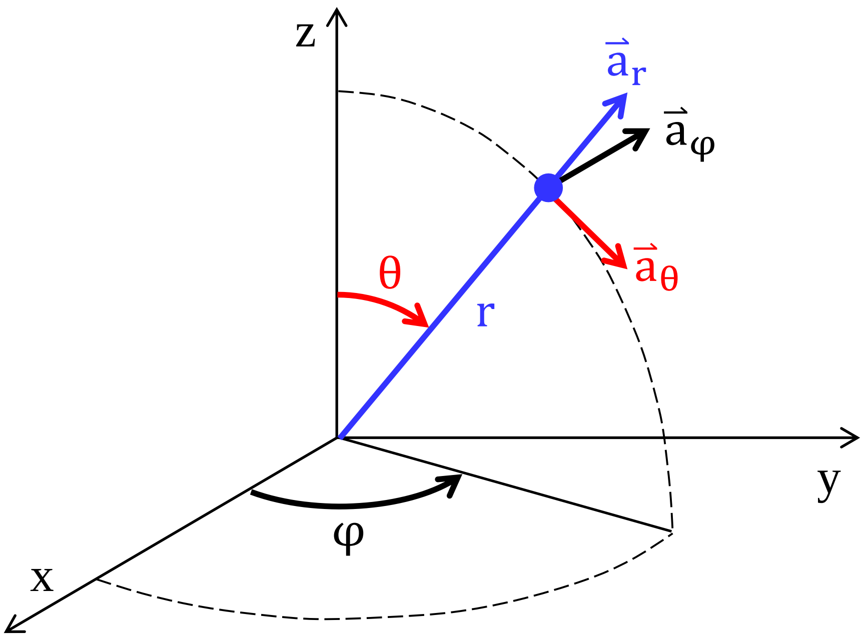


단위 Vector 계산

$$\vec{a}_r \cdot \vec{a}_r = \vec{a}_\theta \cdot \vec{a}_\theta = \vec{a}_\phi \cdot \vec{a}_\phi = 1$$

$$\vec{a}_r \cdot \vec{a}_\phi = \vec{a}_\theta \cdot \vec{a}_\phi = \vec{a}_\phi \cdot \vec{a}_r = 0$$

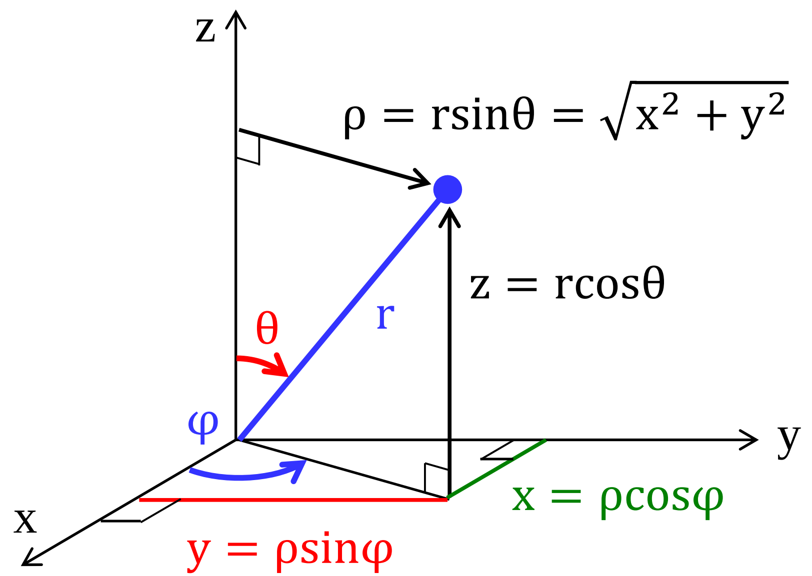
$$\begin{pmatrix} \vec{a}_r \times \vec{a}_\theta = \vec{a}_\phi \\ \vec{a}_\theta \times \vec{a}_\phi = \vec{a}_r \\ \vec{a}_\phi \times \vec{a}_r = \vec{a}_\theta \end{pmatrix} \qquad (2.20)$$



직각좌표계와 구좌표계의 관계식

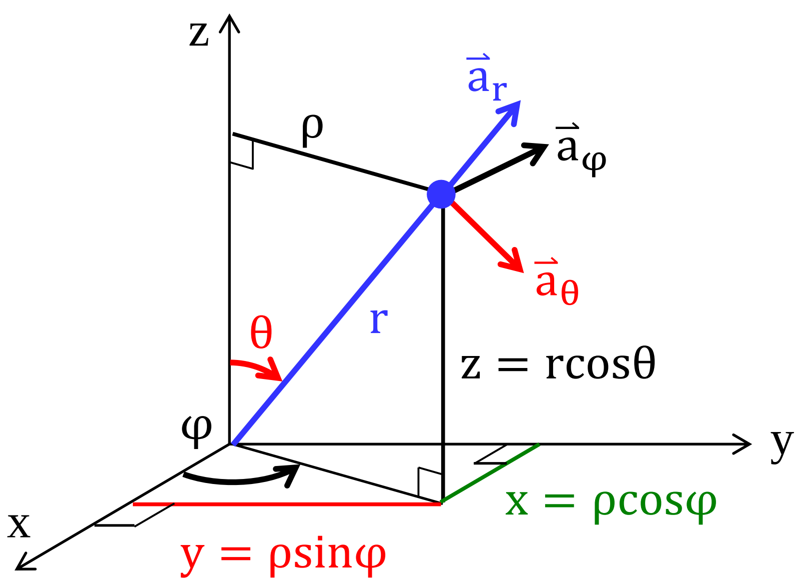
$$\begin{pmatrix} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1} \dfrac{\sqrt{x^2 + y^2}}{z} \\ \phi = \tan^{-1} \dfrac{y}{x} \end{pmatrix} \qquad (2.21)$$

$$\begin{pmatrix} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{pmatrix} \qquad (2.22)$$

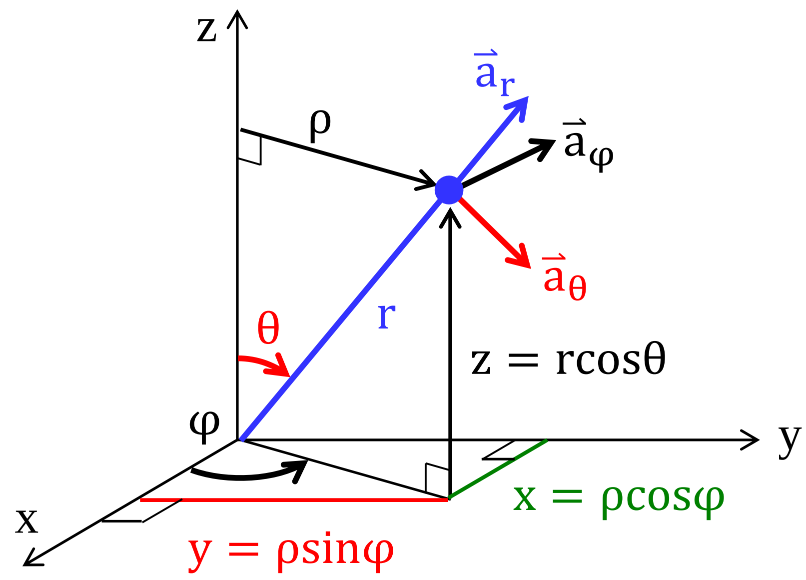


$$\left(\begin{aligned}
\vec{a}_x &= \sin\theta\cos\phi\,\vec{a}_r \\
&\quad + \cos\theta\cos\phi\,\vec{a}_\theta \\
&\quad - \sin\phi\, a_\phi \\
\vec{a}_y &= \sin\theta\sin\phi\,\vec{a}_r \\
&\quad + \cos\theta\sin\phi\,\vec{a}_\theta \\
&\quad + \cos\phi\,\vec{a}_\phi \\
\vec{a}_z &= \cos\vec{a}_r - \sin\vec{a}_\theta
\end{aligned}\right) \qquad (2.23)$$

$$\left(\begin{aligned}
\vec{a}_r &= \sin\theta\cos\phi\,\vec{a}_x \\
&\quad + \sin\theta\sin\phi\,\vec{a}_y \\
&\quad + \cos\theta\,\vec{a}_z \\
\vec{a}_\theta &= \cos\theta\cos\phi\,\vec{a}_x \\
&\quad + \cos\theta\sin\phi\,\vec{a}_y \\
&\quad - \sin\theta\,\vec{a}_z \\
\vec{a}_\phi &= -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y
\end{aligned}\right) \qquad (2.24)$$



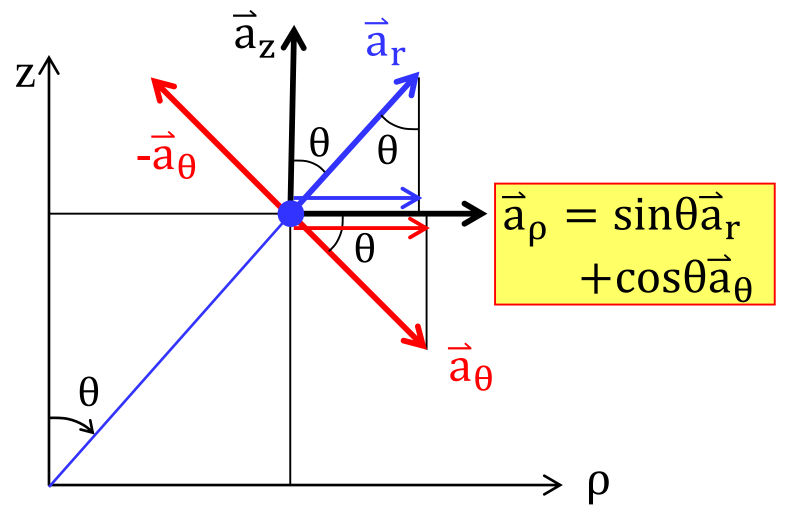
$$(2.23)\quad \begin{pmatrix} \boxed{\vec{a}_x = \sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\, a_\phi} \\ \vec{a}_y = \sin\theta\sin\phi\,\vec{a}_r + \cos\theta\sin\phi\,\vec{a}_\theta + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \cos\theta\,\vec{a}_r - \sin\theta\,\vec{a}_\theta \end{pmatrix}$$



$$(2.9)\quad \begin{pmatrix} \vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi \\ \vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \vec{a}_z \end{pmatrix}$$

$$\begin{aligned} \vec{a}_x &= \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi \\ &= \cos\phi(\sin\theta\,\vec{a}_r + \cos\theta\,\vec{a}_\theta) - \sin\phi\,\vec{a}_\phi \\ &= \cos\phi\sin\theta\,\vec{a}_r + \cos\phi\cos\theta\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi \end{aligned}$$

$$\boxed{\vec{a}_x = \sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi}$$



$$\begin{cases} \vec{a}_x = \sin\theta\cos\phi\,\vec{a}_r + \cos\theta\cos\phi\,\vec{a}_\theta - \sin\phi\,\vec{a}_\phi \\ \vec{a}_y = \sin\theta\sin\phi\,\vec{a}_r + \cos\theta\sin\phi\,\vec{a}_\theta + \cos\phi\,\vec{a}_\phi \\ \vec{a}_z = \cos\theta\,\vec{a}_r - \sin\theta\,\vec{a}_\theta \end{cases}$$

$$\begin{aligned} \vec{A} &= (A_x, A_y, A_z) \\ &= A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z \\ &= (A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta)\vec{a}_r \\ &+ (A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta)\vec{a}_\theta \qquad (2.25) \\ &+ (-A_x\sin\phi + A_y\cos\phi)\vec{a}_\phi \end{aligned}$$

$$\begin{aligned} \vec{A} &= (A_r, A_\theta, A_\phi) \\ &= A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi \end{aligned}$$

$$\begin{cases} A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta \\ A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta \qquad (2.26) \\ A_\phi = -A_x\sin\phi + A_y\cos\phi \end{cases}$$

$$\begin{aligned} A &= (A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta)\vec{a}_r \\ &+ (A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta)\vec{a}_\theta \\ &+ (-A_x \sin\phi + A_y \cos\phi)\vec{a}_\phi \end{aligned} \tag{2.25}$$

$$\begin{cases} A_r = A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta \\ A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta \\ A_\phi = -A_x \sin\phi + A_y \cos\phi \end{cases} \tag{2.26}$$

$$\Downarrow$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \tag{2.27}$$

$$\vec{a}_r = \sin\theta\cos\phi\,\vec{a}_x + \sin\theta\sin\phi\,\vec{a}_y + \cos\theta\,\vec{a}_z$$

$$\vec{a}_\theta = \cos\theta\cos\phi\,\vec{a}_x + \cos\theta\sin\phi\,\vec{a}_y - \sin\theta\,\vec{a}_z \qquad (2.24)$$

$$\vec{a}_\phi = -\sin\phi\,\vec{a}_x + \cos\phi\,\vec{a}_y$$

대입

$$\vec{A} = A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} \qquad (2.28)$$

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$
$$= A_r\vec{a}_r + A_\theta\vec{a}_\theta + A_\phi\vec{a}_\phi$$

$$A_r = \vec{a}_r \cdot \vec{A} = \vec{a}_r \cdot \vec{a}_x A_x + \vec{a}_r \cdot \vec{a}_y A_y + \vec{a}_r \cdot \vec{a}_z A_z$$
$$A_\theta = \vec{a}_\theta \cdot \vec{A} = \vec{a}_\theta \cdot \vec{a}_x A_x + \vec{a}_\theta \cdot \vec{a}_y A_y + \vec{a}_\theta \cdot \vec{a}_z A_z$$
$$A_\phi = \vec{a}_\phi \cdot \vec{A} = \vec{a}_\phi \cdot \vec{a}_x A_x + \vec{a}_\phi \cdot \vec{a}_y A_y + \vec{a}_\phi \cdot \vec{a}_z A_z$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \vec{a}_r \cdot \vec{a}_x & \vec{a}_r \cdot \vec{a}_y & \vec{a}_r \cdot \vec{a}_z \\ \vec{a}_\theta \cdot \vec{a}_x & \vec{a}_\theta \cdot \vec{a}_y & \vec{a}_\theta \cdot \vec{a}_z \\ \vec{a}_\phi \cdot \vec{a}_x & \vec{a}_\phi \cdot \vec{a}_y & \vec{a}_\phi \cdot \vec{a}_z \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \qquad (2.29)$$

** 원통 좌표계

$$\begin{pmatrix} \rho = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1} \dfrac{y}{x} \\ z = z \end{pmatrix} \quad (2.7)$$

$$\begin{pmatrix} x = \rho(\vec{a}_x \cdot \vec{a}_\rho) = \rho \cos \phi \\ y = \rho(\vec{a}_y \cdot \vec{a}_\rho) = \rho \sin \phi \\ z = z \end{pmatrix} \quad (2.8)$$

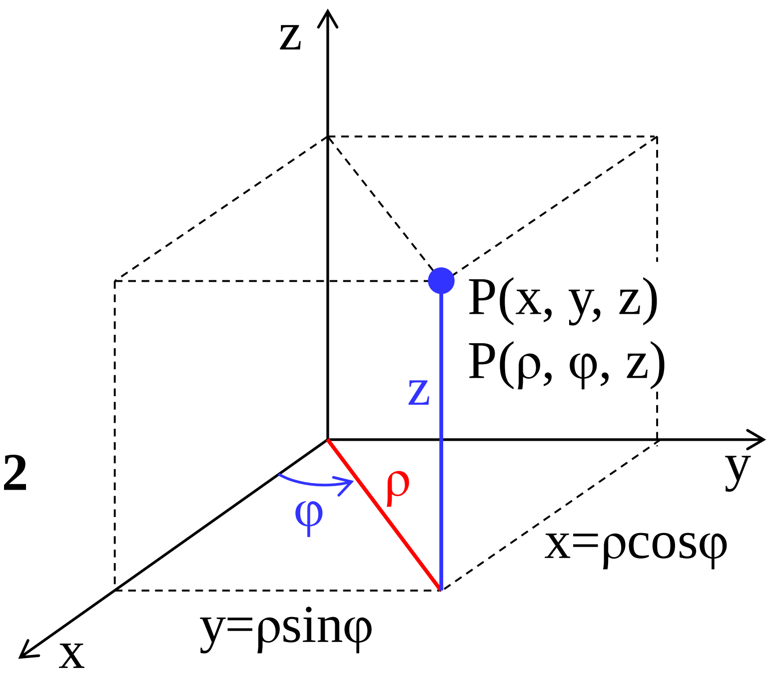


Fig 2.2

** 극 좌표계

$$r = \sqrt{x^2 + y^2 + z^2}$$

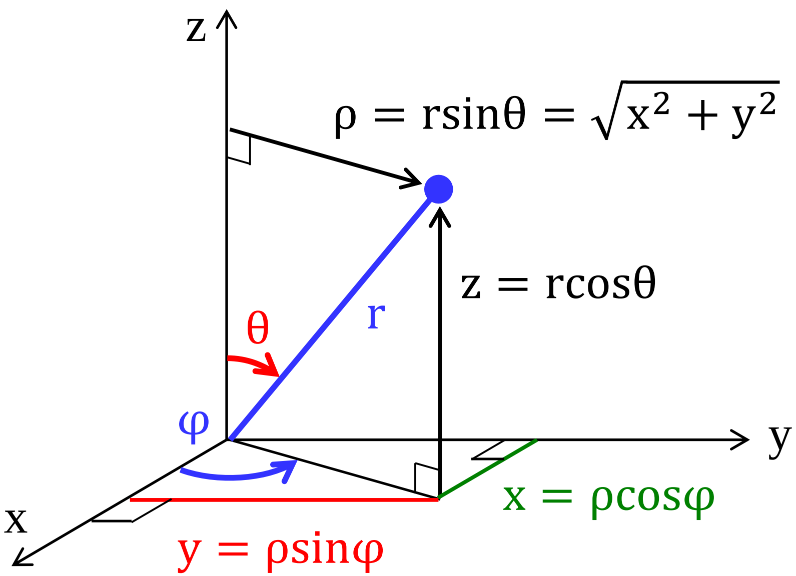
$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} \qquad (2.21)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi \qquad (2.22)$$

$$z = r\cos\theta$$



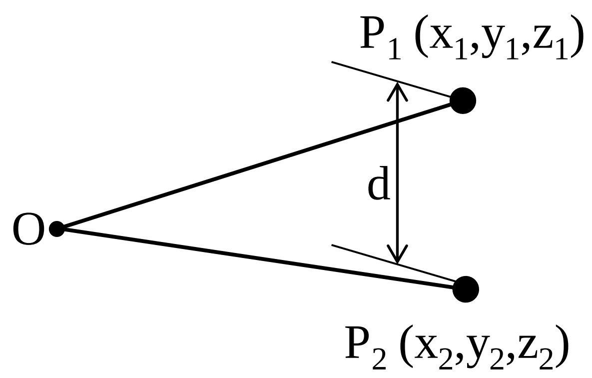
2 점 사이의 거리

Catesian coordinate

$$d = |\vec{r}_2 - \vec{r}_1| \qquad (2.30)$$

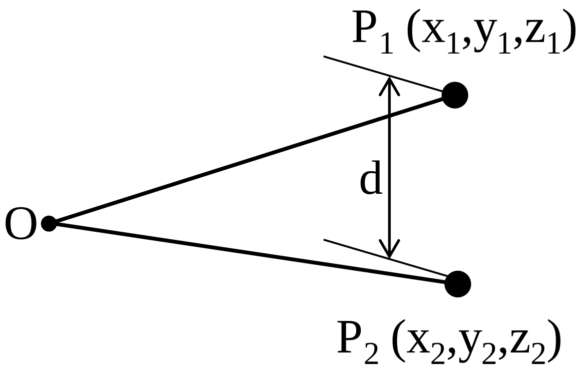
$$\vec{r}_2 - \vec{r}_1 = [(x_2 - x_1),\ (y_2 - y_1),\ (z_2 - z_1)]$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \qquad (2.31)$$



Cylindrical coord.

$$(2.8)\quad \begin{cases} x = \rho\cos\phi \\ y = \rho\sin\phi \\ z = z \end{cases}$$

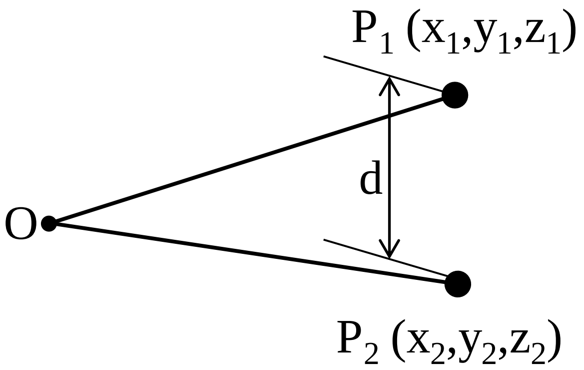


$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$
$$= (\rho_2\cos\phi_2 - \rho_1\cos\phi_1)^2$$
$$+ (\rho_2\sin\phi_2 - \rho_1\sin\phi_1)^2$$
$$+ (z_2 - z_1)^2$$

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\phi_2 - \phi_1) + (z_2 - z_1)^2 \qquad (2.32)$$

Spherical coord

$$(2.22)\quad \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

$$= (r_2\sin\theta_2\cos\phi_2 - r_1\sin\theta_1\cos\phi_1)^2$$

$$+ (r_2\sin\theta_2\sin\phi_2 - r_1\sin\theta_1\sin\phi_1)^2$$

$$+ (r_2\cos\theta_2 - r_1\cos\theta_1)^2$$

$$d^2 = r_2^2 + r_1^2 - 2r_1r_2\cos\theta_2\theta_1$$

$$- 2r_1r_2\sin\theta_2\sin\theta_1\cos(\phi_2 - \phi_1) \qquad (2.33)$$

Ex 2.1 $\vec{A} = y\vec{a}_x + (x+z)\vec{a}_y$를 원통좌표, 구좌표로 표현하라.
P(-2,6,3)와 $\vec{A}$ 를 원통좌표, 구좌표로 구하라.

$$(2.13)\quad \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} A_\rho = y\cos\phi + (x+y)\sin\phi \\ A_\phi = -y\sin\phi + (x+z)\cos\phi \\ A_z = 0 \end{pmatrix} \leftarrow \text{원통 좌표계}$$

$$(2.27)\quad \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} A_r = y\sin\theta\cos\phi + (x+z)\sin\theta\sin\phi \\ A_\theta = y\cos\theta\cos\phi + (x+z)\cos\theta\sin\phi \\ A_\phi = -y\sin\phi + (x+z)\cos\phi \end{pmatrix} \leftarrow \text{구좌표계}$$

Ex 2.1 P(-2,6,3), $\vec{A} = (y, x+z, 0)$ 일 때 P와 $\vec{A}$를 원통좌표, 구좌표로 구하라.

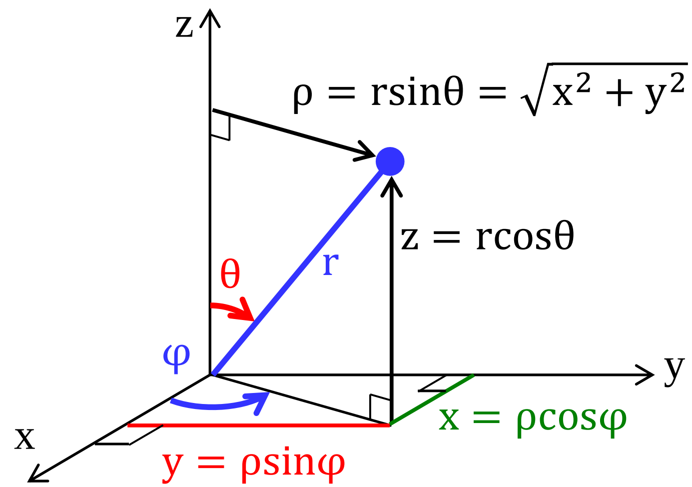
$$\rho = \sqrt{x^2 + y^2} = \sqrt{4+36} = 6.32$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{6}{-2} = 108.43^o$$

$$z = 3$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4+36+9} = 7$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1}\frac{\sqrt{40}}{3} = 64.62^o$$

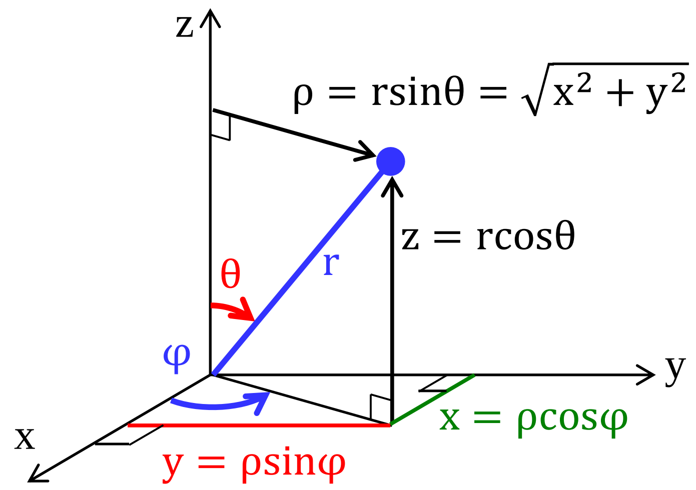


P(-2, 6, 3)

=P(6.32, 108.43º, 3) 원통좌표계

=P(7, 64.62º, 108.43º) 구좌표계

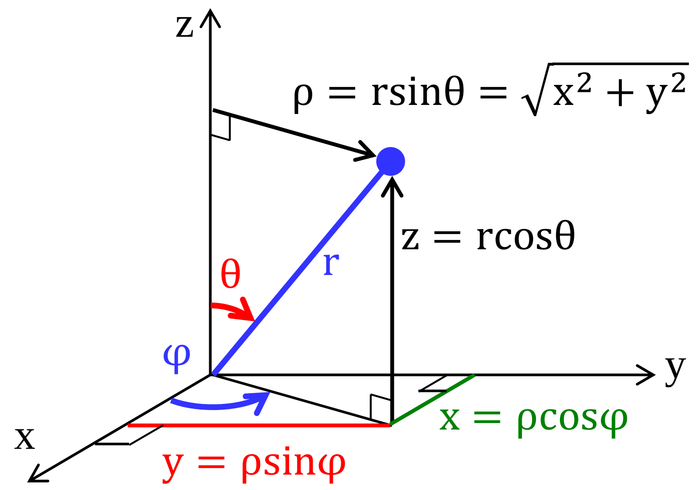
Ex 2.1 P(-2,6,3), $\vec{A} = (y, x+z, 0)$ 일 때 P와 $\vec{A}$를 원통좌표, 구좌표로 구하라.



$$\begin{cases} x = -2 \\ y = 6 \\ z = 3 \end{cases}$$

$$\begin{pmatrix} \rho = \sqrt{x^2+y^2} \\ \cos\phi = \dfrac{x}{\rho} \quad \sin\phi = \dfrac{y}{\rho} \\ r = \sqrt{x^2+y^2+z^2} \\ \cos\theta = \dfrac{z}{r} \quad \sin\theta = \dfrac{\rho}{r} \end{pmatrix} \rightarrow \begin{pmatrix} \rho = \sqrt{(-2)^2+6^2} = \sqrt{40} \\ \cos\phi = \dfrac{-2}{\sqrt{40}}, \quad \sin\phi = \dfrac{6}{\sqrt{40}} \\ r = \sqrt{(-2)^2+6^2+3^2} = 7 \\ \cos\theta = \dfrac{3}{7}, \qquad \sin\theta = \dfrac{\sqrt{40}}{7} \end{pmatrix}$$

$$\begin{pmatrix} x = -2 \\ y = 6 \\ z = 3 \end{pmatrix} \qquad \begin{pmatrix} \rho = \sqrt{(-2)^2 + 6^2} = \sqrt{40} \\ \cos\phi = \dfrac{-2}{\sqrt{40}}, \quad \sin\phi = \dfrac{6}{\sqrt{40}} \\ r = \sqrt{(-2)^2 + 6^2 + 3^2} = 7 \\ \cos\theta = \dfrac{3}{7}, \qquad \sin\theta = \dfrac{\sqrt{40}}{7} \end{pmatrix}$$



Cylindrical coord.

$$\begin{pmatrix} A_\rho = y\cos\phi + (x+y)\sin\phi \\ A_\phi = -y\sin\phi + (x+z)\cos\phi \\ A_z = 0 \end{pmatrix} \to \vec{A} = (-6/\sqrt{40},\ -38/\sqrt{40},\ 0)$$

Spherical coord.

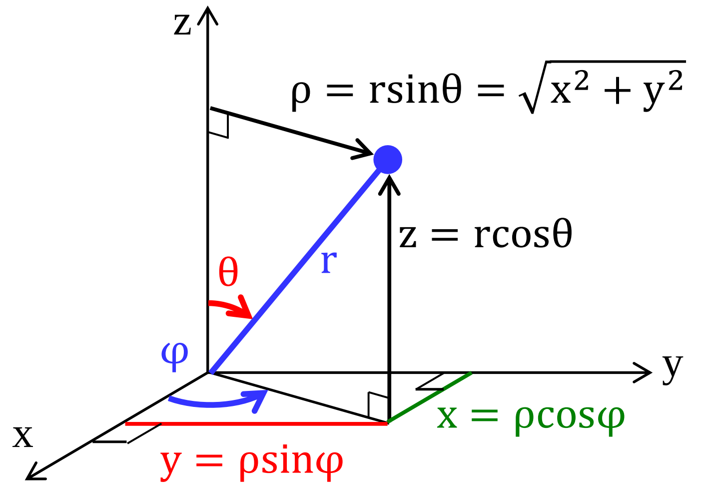
$$\begin{pmatrix} A_r = y\sin\theta\cos\phi + (x+y)\sin\theta\sin\phi \\ A_\theta = y\cos\theta\cos\phi + (x+y)\cos\theta\sin\phi \\ A_\phi = -y\sin\phi + (x+y)\cos\phi \end{pmatrix} \to \vec{A} = (-0.8571,\ -0.4066,\ -6.008)$$

Ex 2.2 $\vec{B} = 10\vec{a}_r/r + r\cos\theta\vec{a}_\theta + \vec{a}_\varphi$를 직각좌표계, 원통좌표계로 표현하라.
직각좌표계 Point (-3,4,0)일 때 $\vec{B}$ 를 직각좌표계로 원통좌표계 Point (5,π/2,-2)일 때 $\vec{B}$ 를 원통좌표계로 표시하라.

구좌표계→ 직각좌표계

$$(2.28)\quad \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

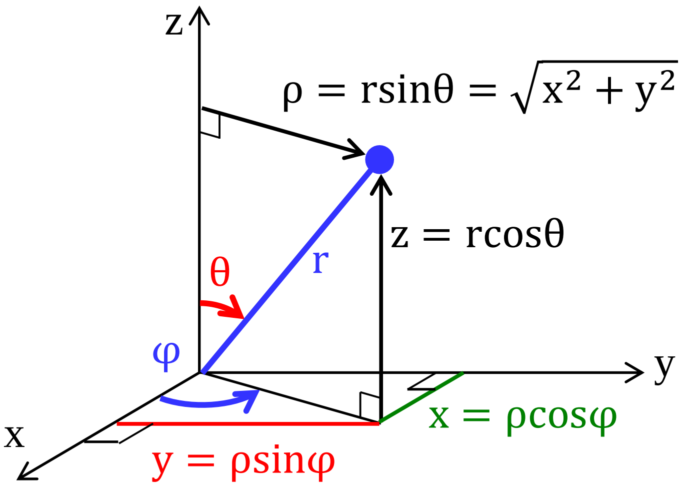
$$= \begin{bmatrix} (10/r)\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi \\ (10/r)\sin\theta\sin\phi + r\cos^2\theta\cos\phi + \cos\phi \\ (10/r)\cos\theta - r\cos\theta\sin\theta \end{bmatrix}$$



$$(2.21)\quad \begin{pmatrix} r = \sqrt{x^2+y^2+z^2} \\ \tan\theta = \dfrac{\sqrt{x^2+y^2}}{z} \\ \tan\phi = \dfrac{y}{x} \end{pmatrix} \rightarrow \begin{pmatrix} r = \sqrt{x^2+y^2+z^2} \\ \rho = \sqrt{x^2+y^2} \\ \cos\theta = \dfrac{z}{r},\ \sin\theta = \dfrac{\rho}{r} \\ \tan\phi = \dfrac{y}{x},\ \cos\phi = \dfrac{x}{\rho},\ \sin\phi = \dfrac{y}{\rho} \end{pmatrix}$$

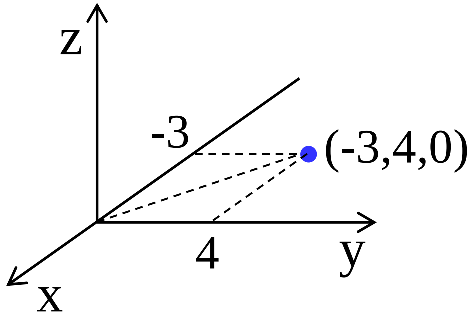
$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} (10/r)\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi \\ (10/r)\sin\theta\sin\phi + r\cos^2\theta\cos\phi + \cos\phi \\ (10/r)\cos\theta - r\cos\theta\sin\theta \end{bmatrix}$$

$$\left(\begin{array}{l} r = \sqrt{x^2 + y^2 + z^2} \\ \rho = \sqrt{x^2 + y^2} \\ \cos\theta = \dfrac{z}{r},\ \sin\theta = \dfrac{\rho}{r} \\ \tan\phi = \dfrac{y}{x},\ \cos\phi = \dfrac{x}{\rho},\ \sin\phi = \dfrac{y}{\rho} \end{array} \right) \rightarrow$$



$$\left(\begin{array}{l} r = \sqrt{(-3)^2 + 4^2 + 0^2} = 5 \\ \theta = \tan^{-1}\dfrac{\sqrt{(-3)^2 + 4^2}}{0} = \dfrac{\pi}{2} \\ \qquad \cos\theta = 0,\ \sin\theta = 1 \\ \tan\phi = \dfrac{4}{3},\ \cos\phi = -\dfrac{3}{5},\ \sin\phi = \dfrac{4}{5} \end{array} \right) \Rightarrow \boxed{\vec{B} = (-2, 1, 0)}$$

직각좌표계



Ex 2.2 $\vec{B} = 10\vec{a}_r/r + r\cos\theta\vec{a}_\theta + \vec{a}_\varphi$를 직각좌표계, 원통좌표계로 표현하라.
직각좌표계 Point (-3,4,0)일 때 $\vec{B}$ 를 직각좌표계로 원통좌표계 Point (5,π/2,-2)일 때 $\vec{B}$ 를 원통좌표계로 표시하라.

구좌표계 → 원통좌표계

$$(2.28)\quad \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$(2.13)\quad \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

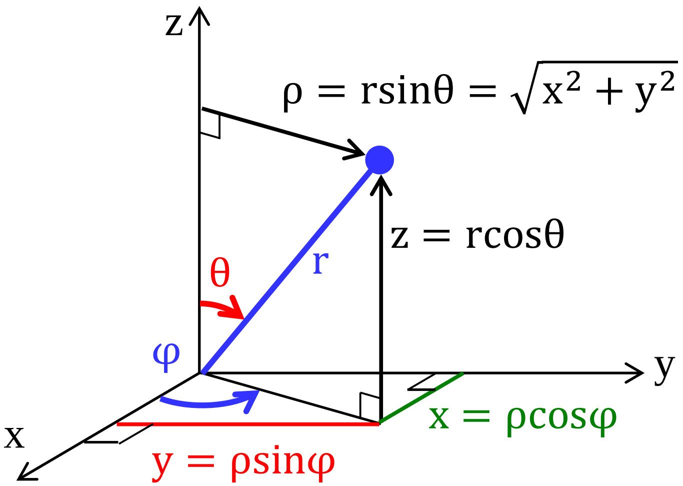
$$= \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \boxed{\begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}}$$

원통좌표계

$$\begin{pmatrix} \rho = 5 \\ \varphi = \pi/2 \\ z = -2 \end{pmatrix}$$

$$\begin{pmatrix} r = \sqrt{\rho^2 + z^2} \\ \cos\theta = \dfrac{z}{r}, \ \sin\theta = \dfrac{\rho}{r} \\ \cos\phi = \dfrac{x}{\rho}, \ \sin\phi = \dfrac{y}{\rho} \end{pmatrix} \rightarrow \begin{pmatrix} r = \sqrt{5^2 + (-2)^2} = \sqrt{29} \\ \cos\theta = -2/\sqrt{29}, \ \sin\theta = 5/\sqrt{29} \\ \cos\phi = 0, \ \sin\phi = 1 \end{pmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix} = \boxed{\begin{pmatrix} 2.67 \\ 1 \\ 1.167 \end{pmatrix}}$$



2.5 Constant-Coordinate Surfaces

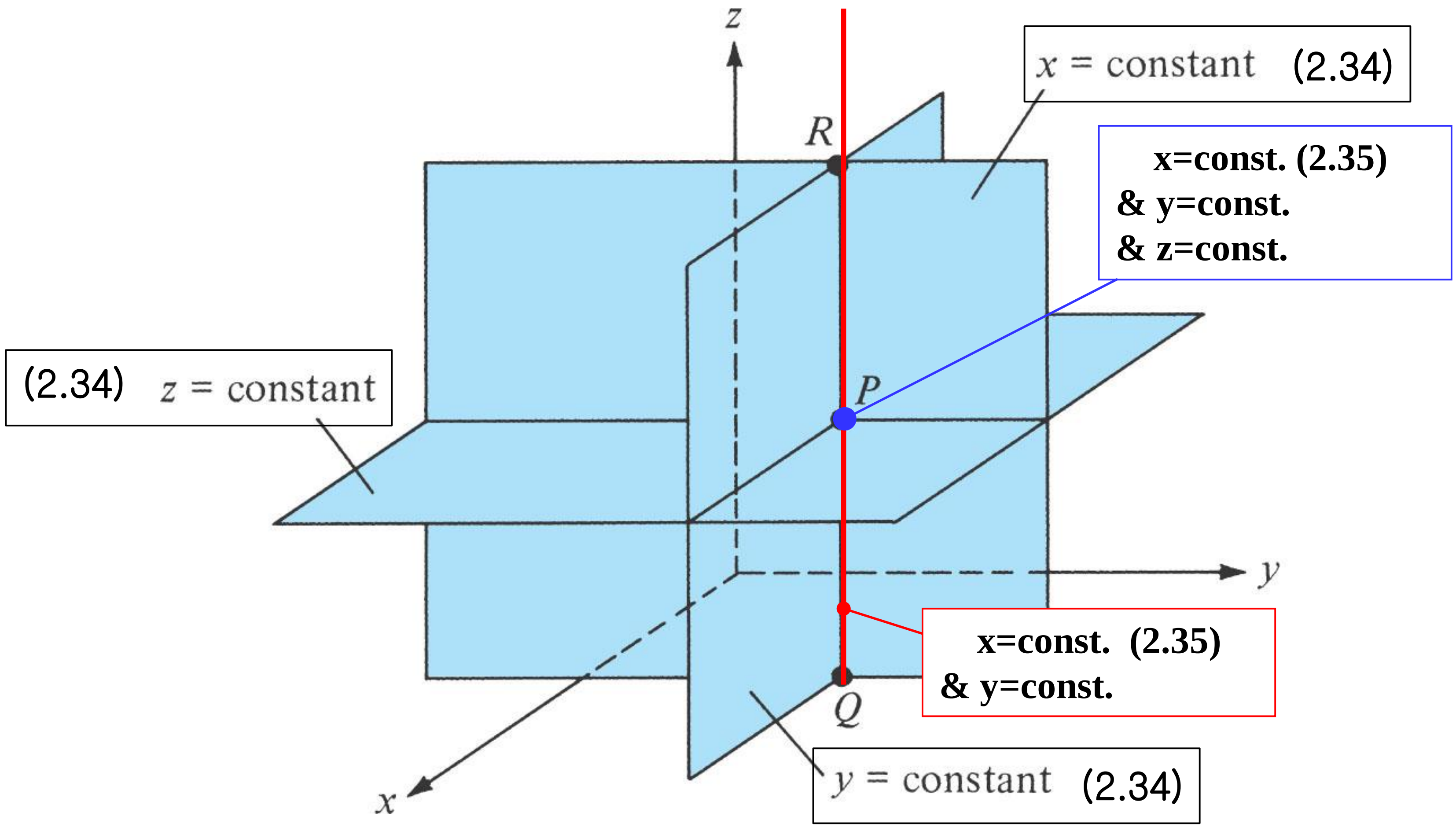


Figure 2.7 Constant *x*, *y*, and *z* surfaces.

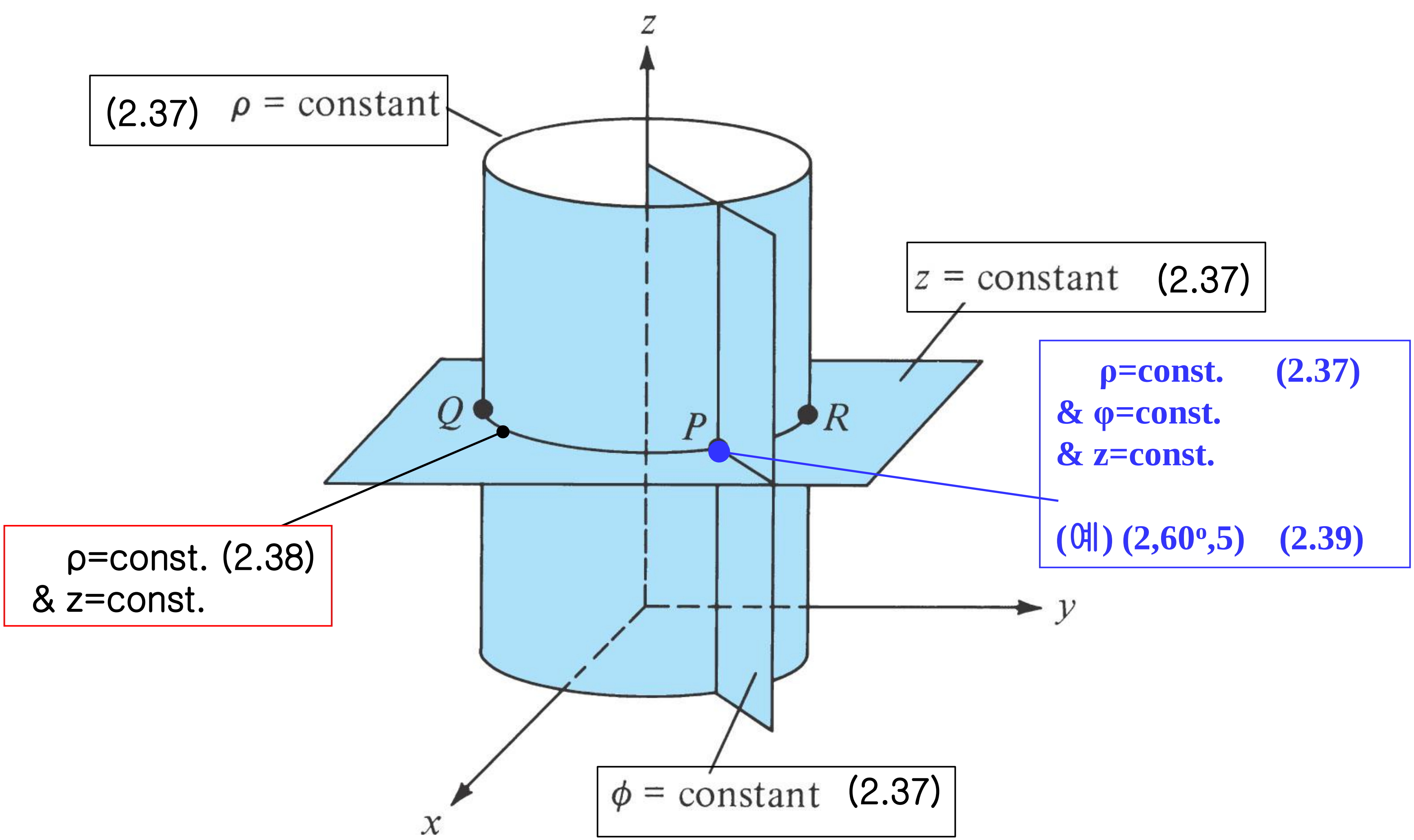


Figure 2.8 Constant ρ, ϕ, and z surfaces.

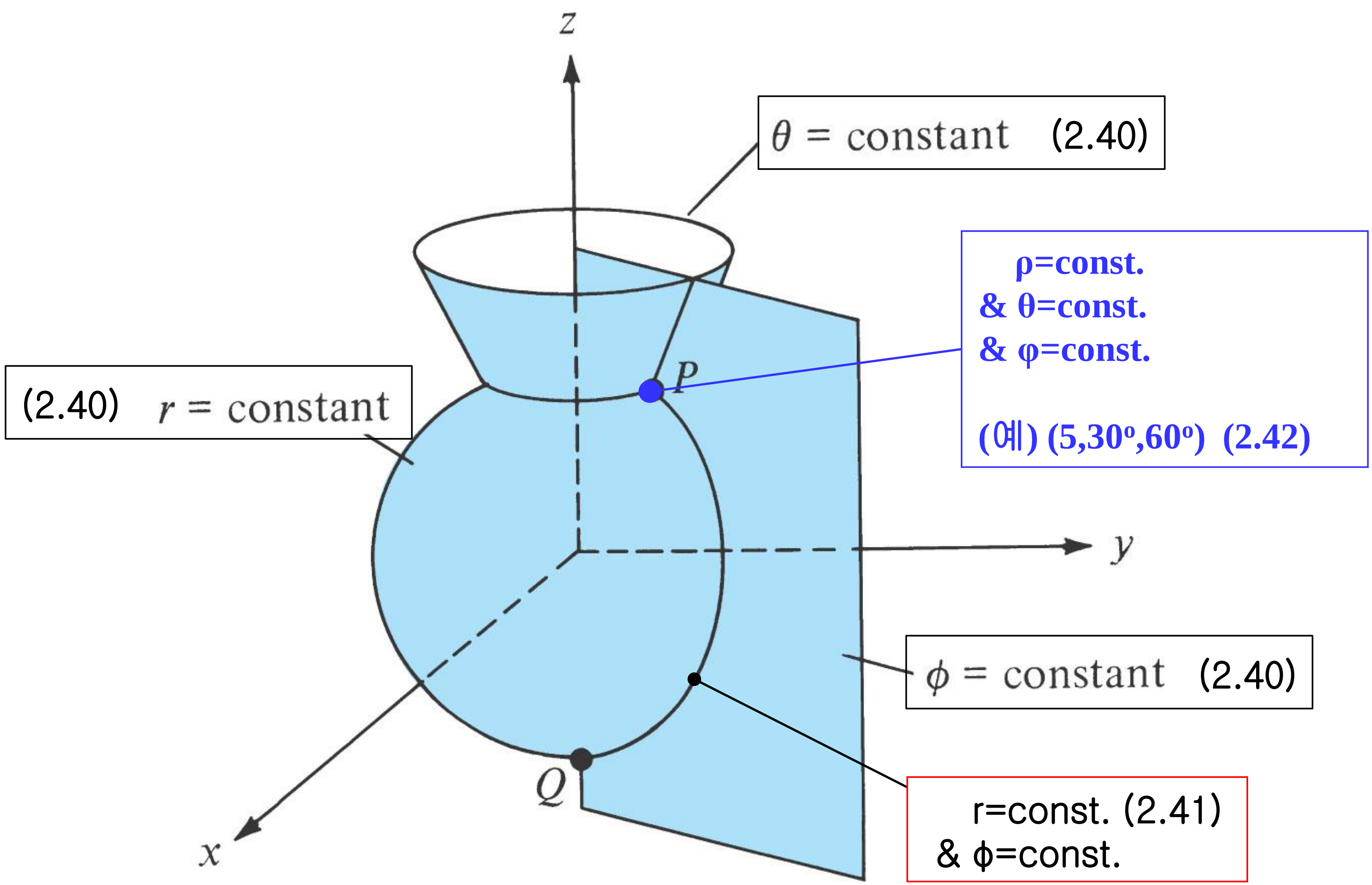


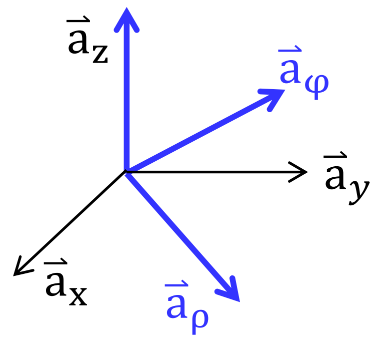
Figure 2.9 Constant r, θ, and ϕ surfaces.

Ex 2.3 $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\varphi + 3\vec{a}_z$, $\vec{F} = \vec{a}_\rho + 2\vec{a}_\varphi - 6\vec{a}_z$ 일 때

(a) $|\vec{E} \times \vec{F}|$를 구하라.

$(\vec{a}_\rho, \vec{a}_\phi, \vec{a}_z)$

$$\begin{cases} \vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z \\ \vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z \end{cases}$$



$(\vec{a}_x, \vec{a}_y, \vec{a}_z)$

$$\begin{cases} \vec{E} = -5\vec{a}_x + 10\vec{a}_y + 3\vec{a}_z \\ \vec{F} = \vec{a}_x + 2\vec{a}_y - 6\vec{a}_z \end{cases}$$

$$|\vec{E} \times \vec{F}| = \left\| \begin{matrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ -5 & 10 & 3 \\ 1 & 2 & -6 \end{matrix} \right\|$$

$$= |-66\vec{a}_\rho \ -27\vec{a}_\phi \ -20\vec{a}_z|$$

$$= \sqrt{66^2 + 27^2 + 20^2}$$

$$= 74.06$$

Ex 2.3 $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\varphi + 3\vec{a}_z, \vec{F} = \vec{a}_\rho + 2\vec{a}_\varphi - 6\vec{a}_z$ 일 때

(b) Point P(5, π/2, 3) 에서 Line (x=2, z=3)에 평행한 $\vec{E}$의 성분.

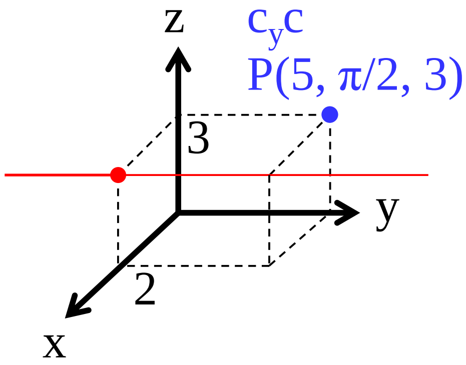
Line (x=2, z=3)는 y축에 평행.

y축에 평행한 $\vec{E}$의 성분: $(\vec{E}\cdot\vec{a}_y)\vec{a}_y = (\vec{E}\cdot\vec{a}_\rho)\vec{a}_\rho = -5\vec{a}_\rho$

$$\vec{a}_y = \sin\phi\,\vec{a}_\rho + \cos\phi\,\vec{a}_\phi \quad (2.9)$$

$$= \sin\frac{\pi}{2}\vec{a}_\rho + \cos\frac{\pi}{2}\vec{a}_\phi$$

$$= \vec{a}_\rho$$



Ex 2.3 $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\varphi + 3\vec{a}_z, \vec{F} = \vec{a}_\rho + 2\vec{a}_\varphi - 6\vec{a}_z$ 일 때

(c) Point P(5, π/2, 3) 에서 Surface (z=3) 와 이루는 각.

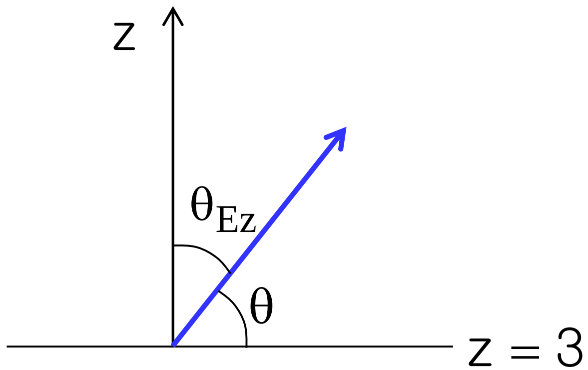
$\vec{a}_z$ 는 Surface **z =3** 과 수직.

$$\vec{E} \cdot \vec{a}_z = E\cos\theta_{Ez}$$

$$(-5, 10, 3) \cdot (0, 0, 1) = \sqrt{134}\cos\theta_{Ez}$$

$$\theta_{Ez} = 74.98^o$$

$$\theta = 90^o - \theta_{Ez} = 15.2^o$$



Ex 2.4 $\vec{D} = r\sin\varphi\vec{a}_r - \sin\theta\cos\varphi\vec{a}_\theta/r + r^2\vec{a}_\varphi$ 일 때

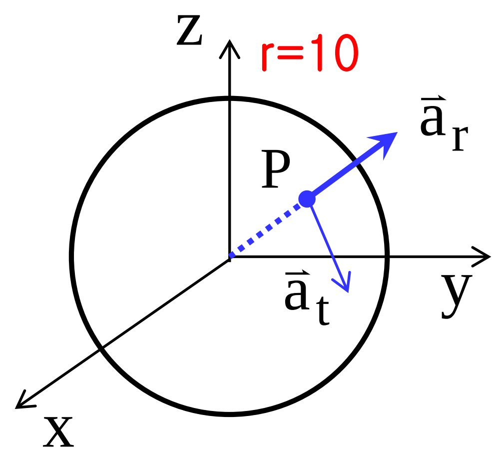
(a) Point P(10,150°,330°) 에서 $\vec{D}$

(b) Point P에서 Surface r=10에 접하는 $\vec{D}$의 Vector 성분

(a)

$$\vec{D} = 10\sin(330^\circ)\,\vec{a}_r - \frac{1}{10}\sin(150^\circ)\cos(330^\circ)\,\vec{a}_\theta + 10^2\vec{a}_\varphi$$

$$= (-5,\ 0.043,\ 100)$$



(b)

$\vec{a}_r$ 은 Surface **r =10** 에 수직.

$\vec{D}$ 의 수직인 성분: $\vec{D}_n = (\vec{D}\cdot\vec{a}_r)\vec{a}_r = (-5, 0, 0)$

$\vec{D}$ 의 평행인 성분: $\vec{D}_t = \vec{D} - \vec{D}_n = (0, 0.043,\ \ 100)$

Ex 2.4 $\vec{D} = r\sin\varphi\vec{a}_r - \sin\theta\cos\varphi\vec{a}_\theta/r + r^2\vec{a}_\varphi$ 일 때

(c) Point P(10,150°,330°) 에서 $\vec{D}$에 수직이고 Cone θ=150°에 접하는 Unit Vector

$$\vec{D}\times\vec{a}_\theta = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\phi \\ -5 & 0.043 & 100 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= -100\vec{a}_r - 5\vec{a}_\phi$$

$$\vec{a} = \frac{-100\vec{a}_r - 5\vec{a}_\phi}{\sqrt{100^2+5^2}}$$

$$= -0.9988\vec{a}_r - 0.0499\vec{a}_\phi$$

$\vec{a}_\theta$ 성분을 제외해야 하는데
$\vec{a}_\theta$ 성분 없음 → 그대로 답