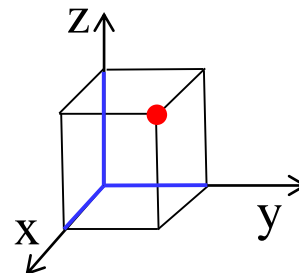


## **2. Coordinate Systems and Transformation**

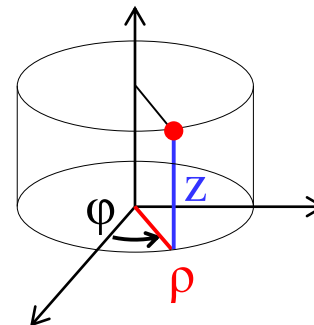
# 2.1 Introduction

직교좌표계 (Orthogonal Coordinate System):  
좌표축이 서로 수직인 좌표계.

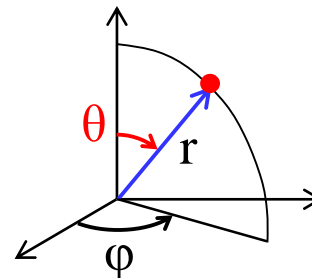
- 직각좌표계 (**Cartesian Coordinate**)

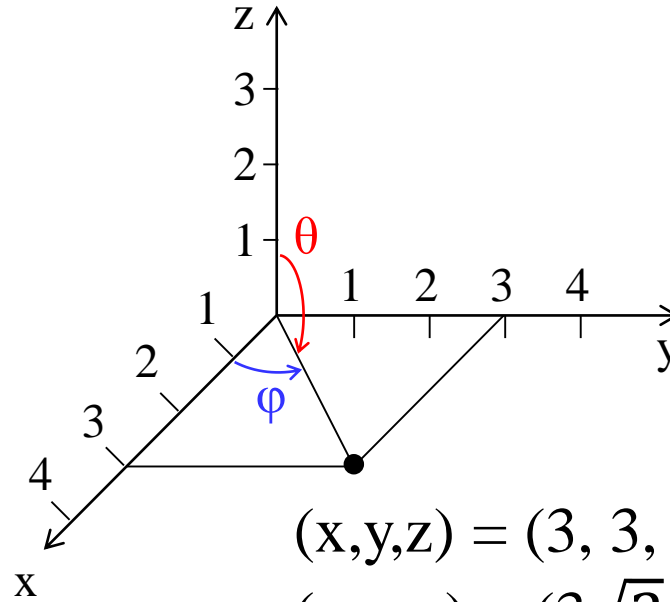
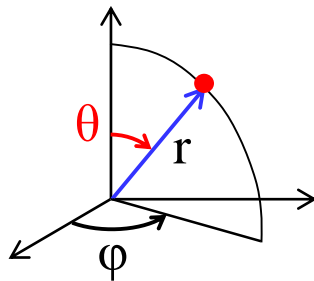
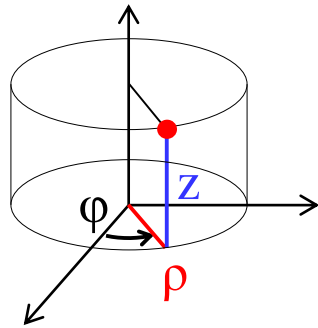
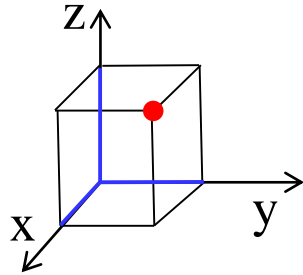


- 원통좌표계 (Cylindrical Coordinate)



- 구좌표계 (Spherical Coordinate)  
극좌표계

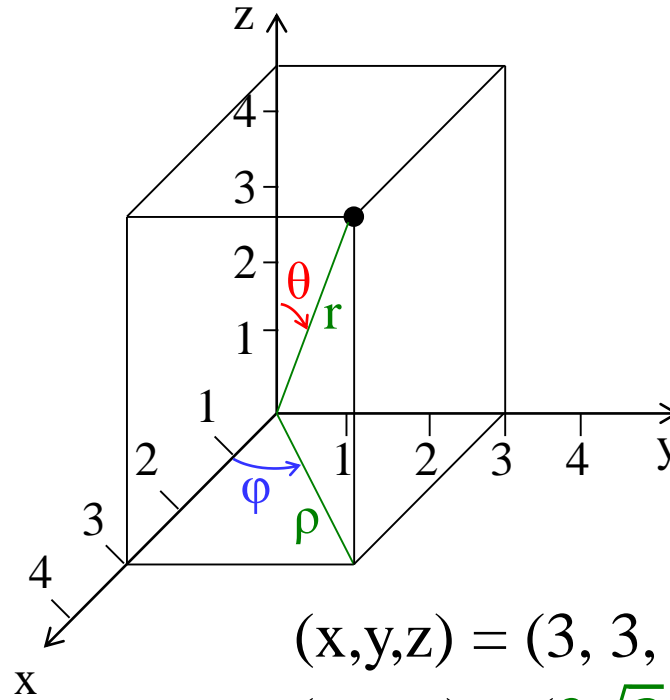
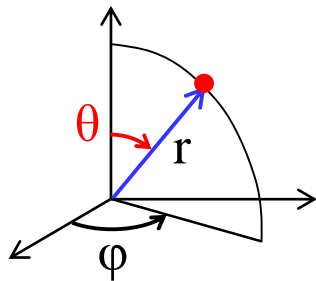
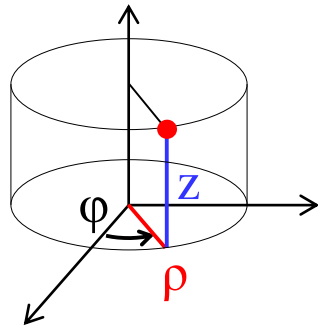
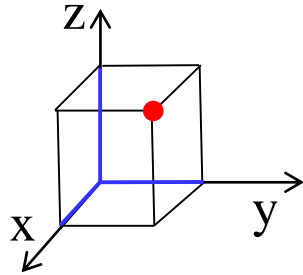




$$(x, y, z) = (3, 3, 0)$$

$$(\rho, \phi, z) = (3\sqrt{2}, \pi/4, 0)$$

$$(r, \theta, \phi) = (3\sqrt{2}, \pi/2, \pi/4)$$



$$(x, y, z) = (3, 3, 3\sqrt{2})$$

$$(\rho, \phi, z) = (3\sqrt{2}, \pi/4, 3\sqrt{2})$$

$$(r, \theta, \phi) = (6, \pi/4, \pi/4)$$

## 2.2 Cartesian Coordinates (x, y, z)

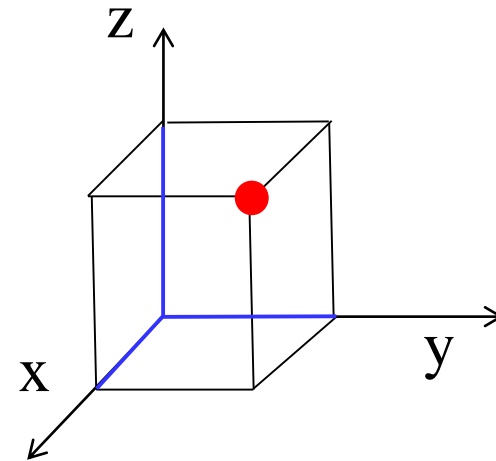
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(x, y, z) 좌표의 범위

$$-\infty < x < \infty$$

$$-\infty < y < \infty \quad (2.1)$$

$$-\infty < z < \infty$$



Vector  $\vec{A}$  의 표시 방법

$$\vec{A} = (A_x, A_y, A_z)$$

$$= A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z \quad (2.2)$$

## 2.3 Circular Cylindrical Coordinate ( $\rho, \varphi, z$ )

( $\rho, \varphi, z$ ) 좌표의 범위

$$0 \leq \rho < \infty$$

$$0 \leq \varphi < 2\pi \quad (2.3)$$

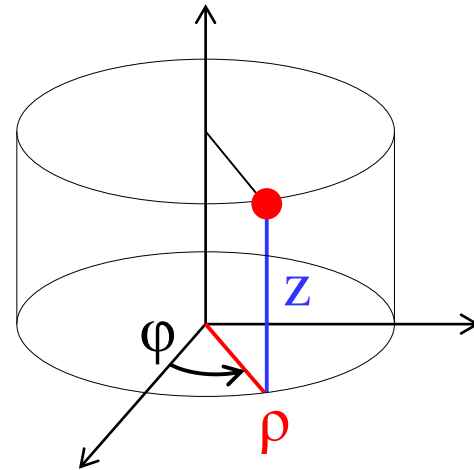
$$-\infty < z < \infty$$

Vector  $\vec{A}$  의 표시 방법

$$\begin{aligned} \vec{A} &= (A_\rho, A_\varphi, A_z) \\ &= A_\rho \vec{a}_\rho + A_\varphi \vec{a}_\varphi + A_z \vec{a}_z \end{aligned} \quad (2.4)$$

Vector  $\vec{A}$  의 크기

$$|\vec{A}| = (A_\rho^2 + A_\varphi^2 + A_z^2)^{1/2} \quad (2.5)$$



## 단위 Vector 계산

$$\vec{A} \cdot \vec{B} \equiv AB \cos \theta_{AB}$$

$$\vec{a}_\rho \cdot \vec{a}_\rho = \vec{a}_\phi \cdot \vec{a}_\phi = \vec{a}_z \cdot \vec{a}_z = 1 \quad (2.6a)$$

$$\vec{a}_\rho \cdot \vec{a}_\phi = \vec{a}_\phi \cdot \vec{a}_z = \vec{a}_z \cdot \vec{a}_\rho = 0 \quad (2.6b) \quad \leftarrow \text{서로 수직}$$

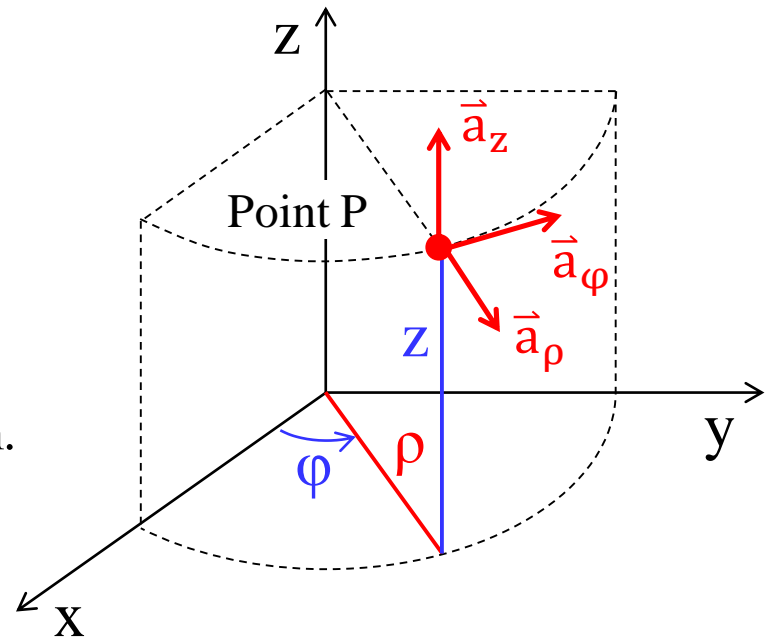
$$\vec{A} \times \vec{B} \equiv AB \sin \theta_{AB} \vec{a}_n$$

$$\vec{a}_\rho \times \vec{a}_\phi = \vec{a}_z \quad (2.6c)$$

$$\vec{a}_\phi \times \vec{a}_z = \vec{a}_\rho \quad (2.6d)$$

$$\vec{a}_z \times \vec{a}_\rho = \vec{a}_\phi \quad (2.6e)$$

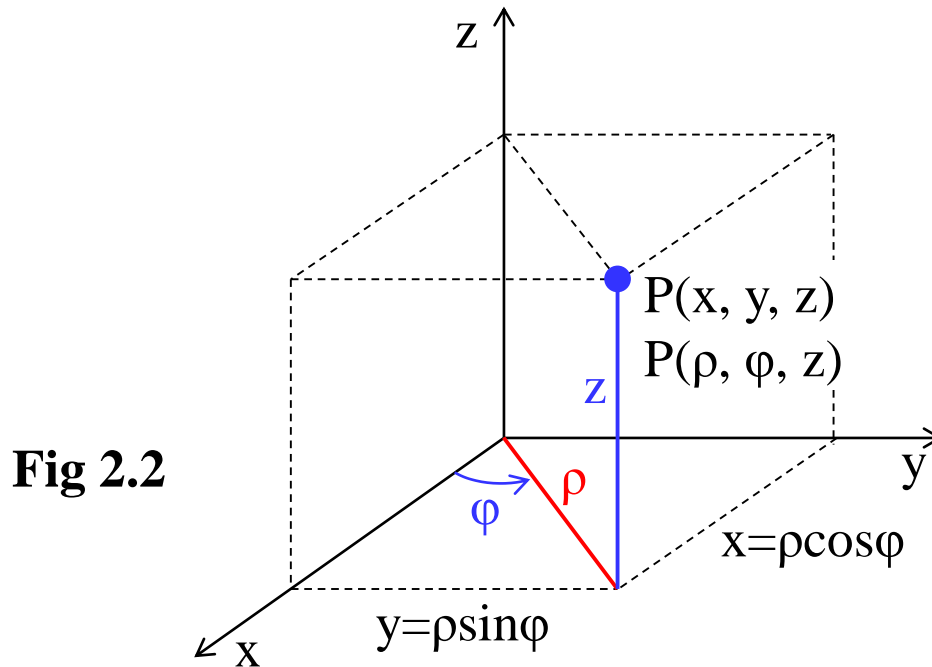
**Fig 2.1** Point **P** and unit vectors in the cylindrical coordinate system.



## 직각좌표계와 원통 좌표계의 관계식

$$\begin{pmatrix} \rho = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1} \frac{y}{x} \\ z = z \end{pmatrix} \quad (2.7)$$

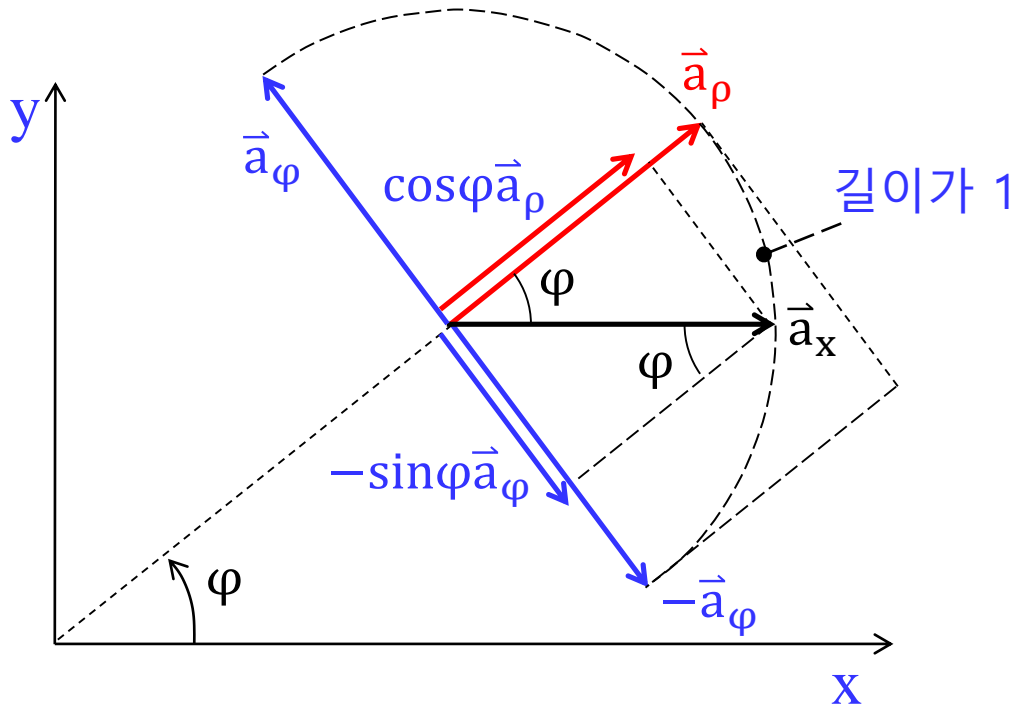
$$\begin{pmatrix} x = \rho(a_x \cdot a_\rho) = \rho \cos \phi \\ y = \rho(a_y \cdot a_\rho) = \rho \sin \phi \\ z = z \end{pmatrix} \quad (2.8)$$





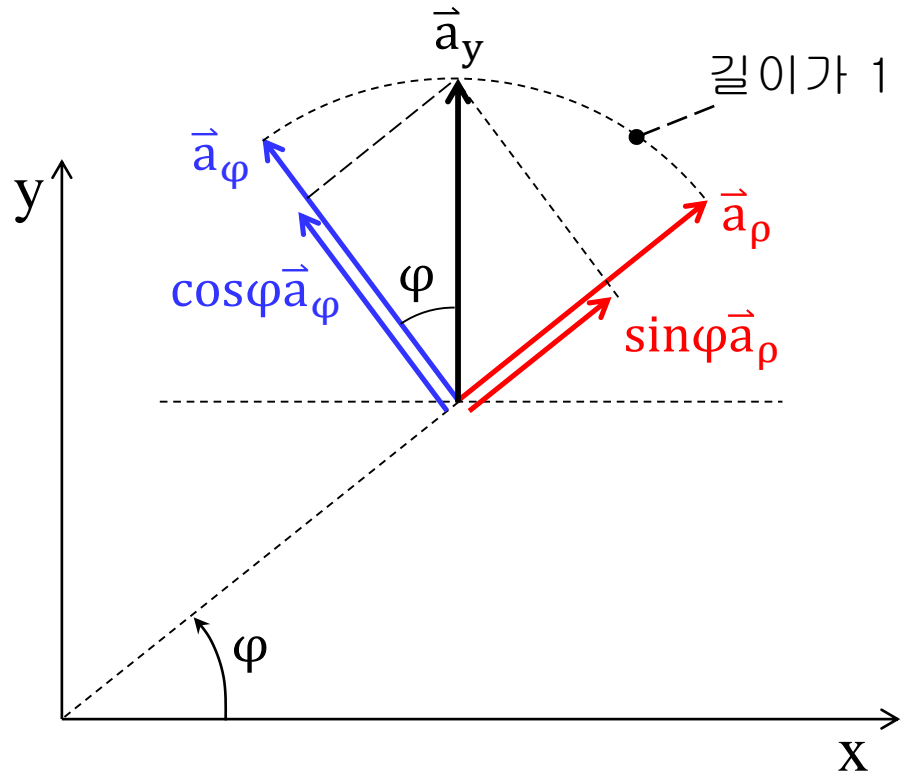
## 직각좌표계와 원통 좌표계의 관계식

$$\begin{cases} \bar{a}_x = \cos\phi \bar{a}_\rho - \sin\phi \bar{a}_\phi \\ \bar{a}_y = \sin\phi \bar{a}_\rho + \cos\phi \bar{a}_\phi \\ \bar{a}_z = \bar{a}_z \end{cases} \quad (2.9)$$




## 직각좌표계와 원통 좌표계의 관계식

$$\begin{pmatrix} \bar{a}_x = \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \phi \bar{a}_\rho + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \bar{a}_z \end{pmatrix} \quad (2.9)$$



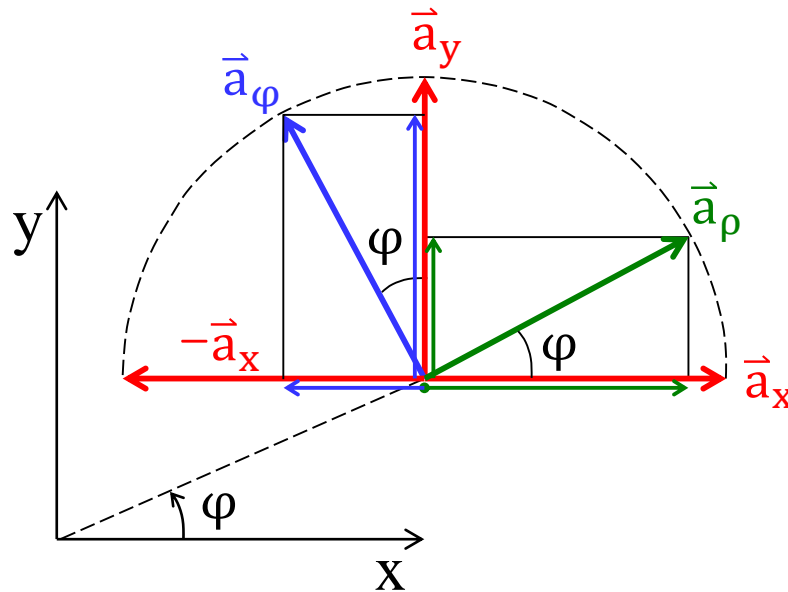
$$\begin{pmatrix} \bar{a}_x = \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \phi \bar{a}_\rho + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \bar{a}_z \end{pmatrix} \quad (2.9)$$

 2원 1차 연립 방정식  $\begin{pmatrix} ax + by = c \\ dx + ey = f \end{pmatrix}$

$$\begin{pmatrix} \bar{a}_\rho = \cos \phi \bar{a}_x + \sin \phi \bar{a}_y \\ \bar{a}_\phi = -\sin \phi \bar{a}_x + \cos \phi \bar{a}_y \\ \bar{a}_z = \bar{a}_z \end{pmatrix} \quad (2.10)$$

$$\vec{a}_\rho = \cos \phi \vec{a}_x + \sin \phi \vec{a}_y$$

$$\vec{a}_\phi = \cos \phi \vec{a}_y - \sin \phi \vec{a}_x$$



$$\begin{pmatrix} \bar{a}_x = \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \phi \bar{a}_\rho + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \bar{a}_z \end{pmatrix} \quad (2.9)$$

대입

$$(2.2) \quad \bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$$

$$\begin{aligned} \bar{A} &= (A_x \cos \phi + A_y \sin \phi) \bar{a}_\rho \\ &\quad + (-A_x \sin \phi + A_y \cos \phi) \bar{a}_\phi + A_z \bar{a}_z \\ &= A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z \end{aligned} \quad (2.11)$$

$$\text{where } \begin{pmatrix} A_\rho = A_x \cos \phi + A_y \sin \phi \\ A_\phi = -A_x \sin \phi + A_y \cos \phi \\ A_z = A_z \end{pmatrix} \quad (2.12)$$

# Matrix 곱하기

$$\underline{\underline{A}} = A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \quad B_{ij} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{aligned} A_{ij}B_{jk} &= A_{1j}B_{j1} = A_{11}B_{11} + A_{12}B_{21} + A_{13}B_{31} \\ &= 1*1 + 2*2 + 3*1 \\ &= 8 \end{aligned}$$

$$A_{ij}B_{jk} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 5 \\ 3 & 4 & 5 \\ 9 & 5 & 4 \end{bmatrix}$$

$$\begin{aligned}
 A_i &= C_{ij}B_j \\
 &= C_{i1}B_1 + C_{i2}B_2 + C_{i3}B_3
 \end{aligned}$$

$$\begin{aligned}
 A_1 &= C_{11}B_1 + C_{12}B_2 + C_{13}B_3 \\
 A_2 &= C_{21}B_1 + C_{22}B_2 + C_{23}B_3 \\
 A_3 &= C_{31}B_1 + C_{32}B_2 + C_{33}B_3
 \end{aligned}$$

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}$$

$$\begin{cases}
 A_\rho = A_x \cos \phi + A_y \sin \phi \\
 A_\phi = -A_x \sin \phi + A_y \cos \phi \\
 A_z = A_z
 \end{cases} \quad (2.12)$$

$$\begin{cases}
 A_\rho = \cos \phi A_x + \sin \phi A_y + 0^* A_z \\
 A_\phi = -\sin \phi A_x + \cos \phi A_y + 0^* A_z \\
 A_z = 0^* A_x + 0^* A_y + 1^* A_z
 \end{cases}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (2.13)$$

# Matrix

$$(A_{ij}B_{jk})C_{km} = A_{ij}(B_{jk}C_{km})$$

결합 법칙

$$A_{ij}(B_{jk} + C_{jk}) = A_{ij}B_{jk} + A_{ij}C_{jk}$$

분배 법칙

$$(B_{ij} + C_{ij})A_{jk} = B_{ij}A_{jk} + C_{ij}A_{jk}$$

분배 법칙

$$A_{ij}B_{jk} \neq B_{ij}A_{jk}$$

교환 법칙

$$A_i = C_{ij}B_j$$

$$B_k = D_{ki}A_i$$

$$= D_{ki}C_{ij}B_j$$

$$= E_{kj}B_j$$

$$= B_k$$

$$E_{kj} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} : \text{Unit Matrix}$$

$D_{ij} = C_{ij}^{-1}$  : Inverse Matrix of  $C_{ij}$

$$C_{ij}^{-1}C_{jk} = E_{ik}$$



# Inverse Matrix 구하기

$$F_1 = x + 2y + 3z$$

$$F_2 = -2x - 3y - 4z$$

$$F_3 = 2x + 2y + 4z$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 1 & +2 & +3 \\ -2 & -3 & -4 \\ 2 & +2 & +4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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$$\begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

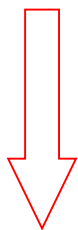
$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 + 2U_1 \\ U_3 - 2U_1 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \begin{pmatrix} V_1 - 2V_2 \\ V_2 \\ V_3 + 2V_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -3 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 - W_3/2 \\ W_2 - W_3 \\ W_3/2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

# Inverse Matrix 구하기

$$A_{ij} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix}$$



$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 + 2U_1 \\ U_3 - 2U_1 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} \begin{pmatrix} V_1 - 2V_2 \\ V_2 \\ V_3 + 2V_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -3 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A_{ij}^{-1} = \begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

$$\begin{pmatrix} W_1 - W_3/2 \\ W_2 - W_3 \\ W_3/2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 & 1/2 \\ 2 & -1 & -1 \\ 1 & 1 & 1/2 \end{bmatrix}$$

## Vector 좌표 변환

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (2.13)$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \quad (2.14)$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \quad (2.15)$$

## 방정식 (2.16) 유도

$$\begin{aligned}\tilde{\mathbf{A}} &= A_x \bar{\mathbf{a}}_x + A_y \bar{\mathbf{a}}_y + A_z \bar{\mathbf{a}}_z \\ &= A_\rho \bar{\mathbf{a}}_\rho + A_\phi \bar{\mathbf{a}}_\phi + A_z \bar{\mathbf{a}}_z\end{aligned}$$

$$A_x = \bar{\mathbf{a}}_x \cdot \tilde{\mathbf{A}} = \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_\rho A_\rho + \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_\phi A_\phi + \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_z A_z$$

$$A_y = \bar{\mathbf{a}}_y \cdot \tilde{\mathbf{A}} = \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_\rho A_\rho + \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_\phi A_\phi + \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_z A_z$$

$$A_z = \bar{\mathbf{a}}_z \cdot \tilde{\mathbf{A}} = \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_\rho A_\rho + \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_\phi A_\phi + \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_z A_z$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_\rho & \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_\phi & \bar{\mathbf{a}}_x \cdot \bar{\mathbf{a}}_z \\ \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_\rho & \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_\phi & \bar{\mathbf{a}}_y \cdot \bar{\mathbf{a}}_z \\ \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_\rho & \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_\phi & \bar{\mathbf{a}}_z \cdot \bar{\mathbf{a}}_z \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} \quad (2.16)$$

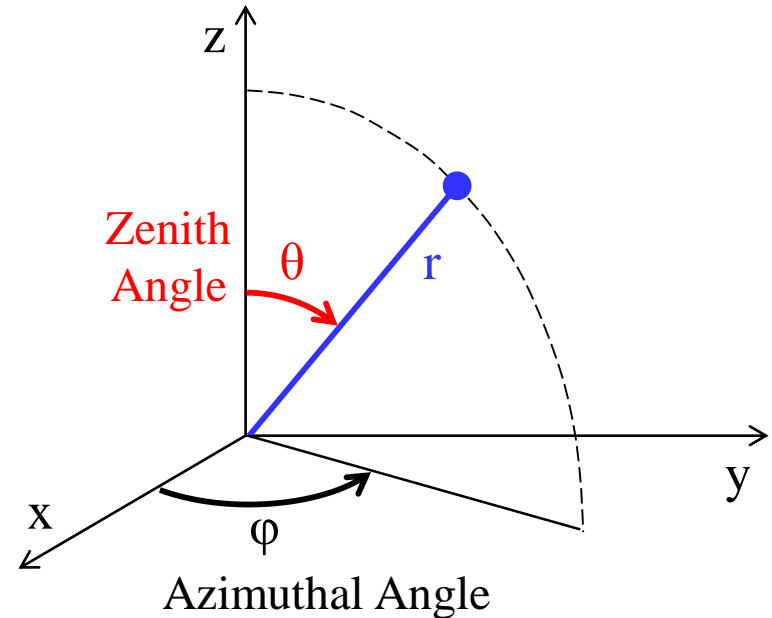
## 2.4 Spherical Coordinates ( $r, \theta, \phi$ )

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$$\left( \begin{array}{l} 0 \leq r < \infty \\ 0 \leq \theta < \pi \\ 0 \leq \phi < 2\pi \end{array} \right) \quad (2.17)$$

$$\begin{aligned} \vec{A} &= (A_r, A_\theta, A_\phi) \\ &= A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi \end{aligned} \quad (2.18)$$

$$|\vec{A}| = (A_r^2 + A_\theta^2 + A_\phi^2)^{1/2} \quad (2.19)$$

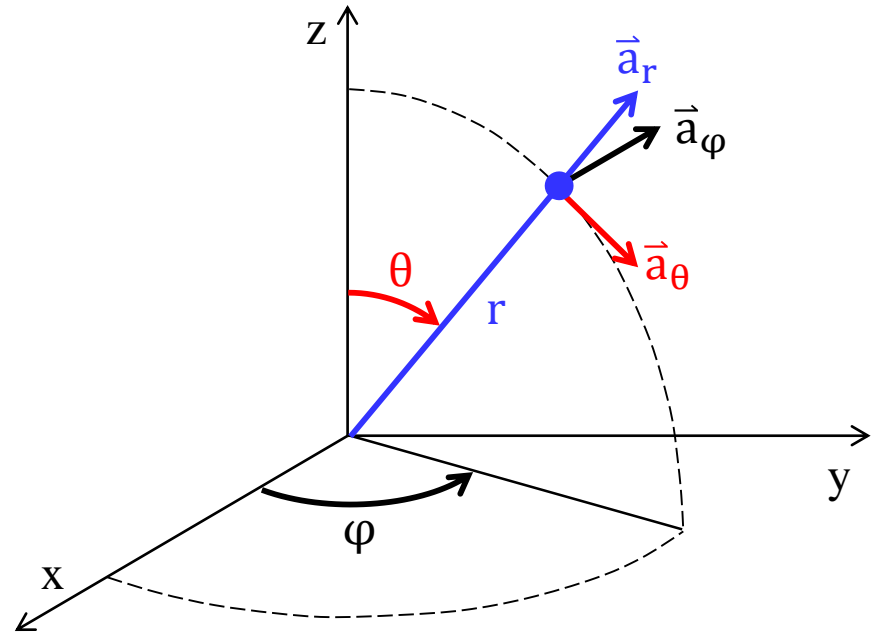


## 단위 Vector 계산

$$\bar{a}_r \cdot \bar{a}_r = \bar{a}_\theta \cdot \bar{a}_\theta = \bar{a}_\phi \cdot \bar{a}_\phi = 1$$

$$\bar{a}_r \cdot \bar{a}_\phi = \bar{a}_\theta \cdot \bar{a}_\phi = \bar{a}_\phi \cdot \bar{a}_r = 0$$

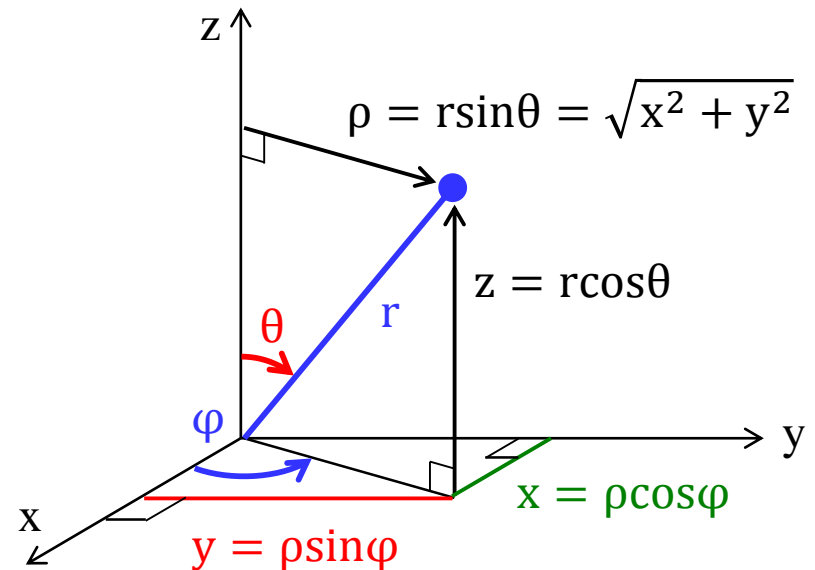
$$\left( \begin{array}{l} \bar{a}_r \times \bar{a}_\theta = \bar{a}_\phi \\ \bar{a}_\theta \times \bar{a}_\phi = \bar{a}_r \\ \bar{a}_\phi \times \bar{a}_r = \bar{a}_\theta \end{array} \right) \quad (2.20)$$



## 직각좌표계와 구좌표계의 관계식

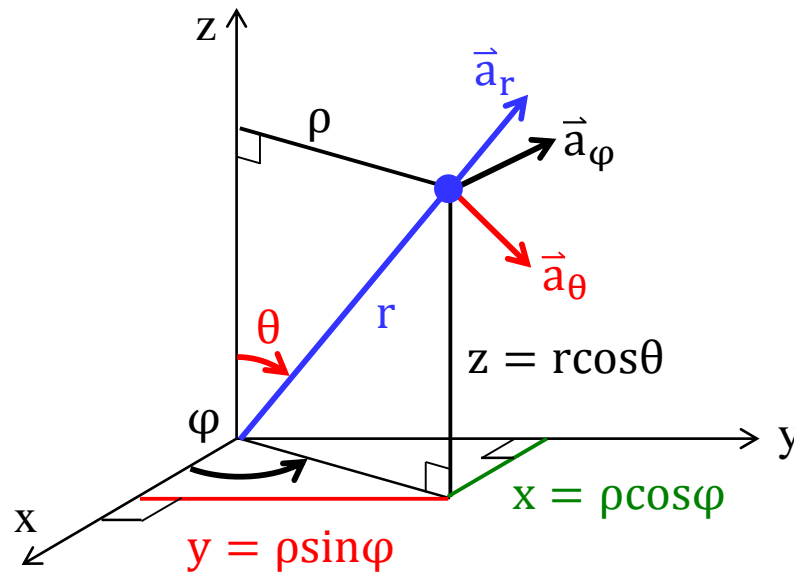
$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} \\ \phi = \tan^{-1} \frac{y}{x} \end{cases} \quad (2.21)$$

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases} \quad (2.22)$$



$$\left( \begin{array}{l} \bar{a}_x = \sin \theta \cos \phi \bar{a}_r \\ \quad + \cos \theta \cos \phi \bar{a}_\theta \\ \quad - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \theta \sin \phi \bar{a}_r \\ \quad + \cos \theta \sin \phi \bar{a}_\theta \\ \quad + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \cos \theta \bar{a}_r - \sin \theta \bar{a}_\theta \end{array} \right) \quad (2.23)$$

$$\left( \begin{array}{l} \bar{a}_r = \sin \theta \cos \phi \bar{a}_x \\ \quad + \sin \theta \sin \phi \bar{a}_y \\ \quad + \cos \theta \bar{a}_z \\ \bar{a}_\theta = \cos \theta \cos \phi \bar{a}_x \\ \quad + \cos \theta \sin \phi \bar{a}_y \\ \quad - \sin \theta \bar{a}_z \\ \bar{a}_\phi = -\sin \phi \bar{a}_x + \cos \phi \bar{a}_y \end{array} \right) \quad (2.24)$$

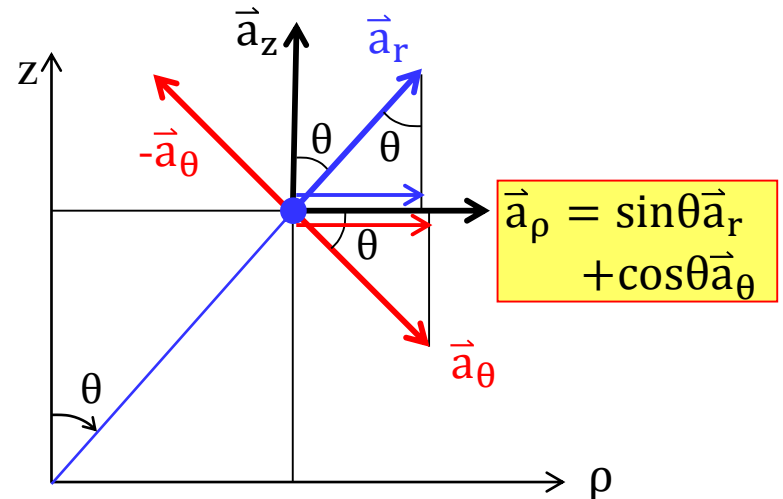
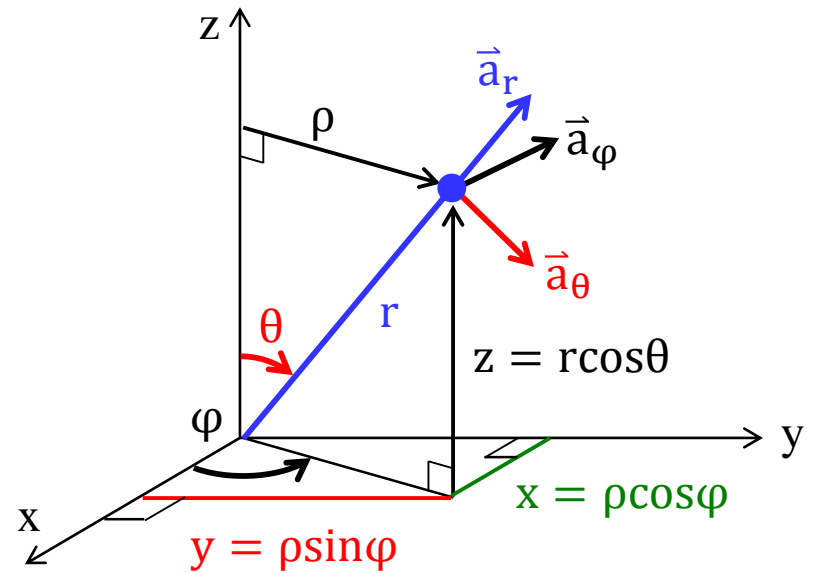




$$(2.23) \begin{cases} \bar{a}_x = \sin \theta \cos \phi \bar{a}_r + \cos \theta \cos \phi \bar{a}_\theta - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \theta \sin \phi \bar{a}_r + \cos \theta \sin \phi \bar{a}_\theta + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \cos \theta \bar{a}_r - \sin \theta \bar{a}_\theta \end{cases}$$

$$(2.9) \begin{cases} \bar{a}_x = \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \phi \bar{a}_\rho + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \bar{a}_z \end{cases}$$

$$\begin{aligned} \bar{a}_x &= \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi \\ &= \cos \phi (\sin \theta \bar{a}_r + \cos \theta \bar{a}_\theta) - \sin \phi \bar{a}_\phi \\ &= \cos \phi \sin \theta \bar{a}_r + \cos \phi \cos \theta \bar{a}_\theta - \sin \phi \bar{a}_\phi \\ \bar{a}_x &= \sin \theta \cos \phi \bar{a}_r + \cos \theta \cos \phi \bar{a}_\theta - \sin \phi \bar{a}_\phi \end{aligned}$$



$$\left( \begin{array}{l} \bar{a}_x = \sin \theta \cos \phi \bar{a}_r + \cos \theta \cos \phi \bar{a}_\theta - \sin \phi \bar{a}_\phi \\ \bar{a}_y = \sin \theta \sin \phi \bar{a}_r + \cos \theta \sin \phi \bar{a}_\theta + \cos \phi \bar{a}_\phi \\ \bar{a}_z = \cos \theta \bar{a}_r - \sin \theta \bar{a}_\theta \end{array} \right)$$

$$\begin{aligned} \bar{A} &= (A_x, A_y, A_z) \\ &= A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z \\ &= (A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta) \bar{a}_r \\ &\quad + (A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta) \bar{a}_\theta \quad (2.25) \\ &\quad + (-A_x \sin \phi + A_y \cos \phi) \bar{a}_\phi \end{aligned}$$

$$\begin{aligned} \bar{A} &= (A_r, A_\theta, A_\phi) \\ &= A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi \\ \left( \begin{array}{l} A_r = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta \\ A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta \\ A_\phi = -A_x \sin \phi + A_y \cos \phi \end{array} \right) \quad (2.26) \end{aligned}$$

$$\begin{aligned}
\mathbf{A} &= (\mathbf{A}_x \sin \theta \cos \phi + \mathbf{A}_y \sin \theta \sin \phi + \mathbf{A}_z \cos \theta) \bar{\mathbf{a}}_r \\
&+ (\mathbf{A}_x \cos \theta \cos \phi + \mathbf{A}_y \cos \theta \sin \phi - \mathbf{A}_z \sin \theta) \bar{\mathbf{a}}_\theta \\
&+ (-\mathbf{A}_x \sin \phi + \mathbf{A}_y \cos \phi) \bar{\mathbf{a}}_\phi
\end{aligned} \tag{2.25}$$

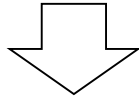
$$\left( \begin{array}{l}
\mathbf{A}_r = \mathbf{A}_x \sin \theta \cos \phi + \mathbf{A}_y \sin \theta \sin \phi + \mathbf{A}_z \cos \theta \\
\mathbf{A}_\theta = \mathbf{A}_x \cos \theta \cos \phi + \mathbf{A}_y \cos \theta \sin \phi - \mathbf{A}_z \sin \theta \\
\mathbf{A}_\phi = -\mathbf{A}_x \sin \phi + \mathbf{A}_y \cos \phi
\end{array} \right. \tag{2.26}$$

⇓

$$\begin{bmatrix} \mathbf{A}_r \\ \mathbf{A}_\theta \\ \mathbf{A}_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_x \\ \mathbf{A}_y \\ \mathbf{A}_z \end{bmatrix} \tag{2.27}$$

$$\vec{A} = A_r \vec{a}_r + A_\theta \vec{a}_\theta + A_\phi \vec{a}_\phi \quad \leftarrow \text{대입}$$

$$\begin{aligned} \vec{a}_r &= \sin \theta \cos \phi \vec{a}_x \\ &+ \sin \theta \sin \phi \vec{a}_y \\ &+ \cos \theta \vec{a}_z \\ \vec{a}_\theta &= \cos \theta \cos \phi \vec{a}_x \\ &+ \cos \theta \sin \phi \vec{a}_y \\ &- \sin \theta \vec{a}_z \\ \vec{a}_\phi &= -\sin \phi \vec{a}_x + \cos \phi \vec{a}_y \end{aligned} \quad (2.24)$$



$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} \quad (2.28)$$

$$\begin{aligned}\bar{\mathbf{A}} &= A_x \bar{\mathbf{a}}_x + A_y \bar{\mathbf{a}}_y + A_z \bar{\mathbf{a}}_z \\ &= A_r \bar{\mathbf{a}}_r + A_\theta \bar{\mathbf{a}}_\theta + A_\phi \bar{\mathbf{a}}_\phi\end{aligned}$$

$$A_r = \bar{\mathbf{a}}_r \cdot \bar{\mathbf{A}} = \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_x A_x + \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_y A_y + \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_z A_z$$

$$A_\theta = \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{A}} = \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_x A_x + \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_y A_y + \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_z A_z$$

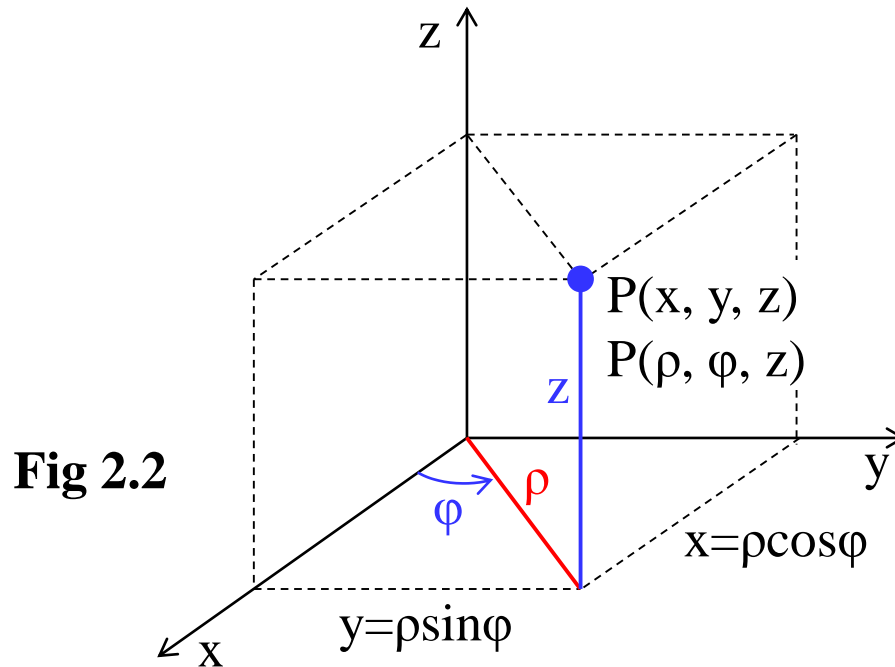
$$A_\phi = \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{A}} = \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_x A_x + \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_y A_y + \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_z A_z$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_x & \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_y & \bar{\mathbf{a}}_r \cdot \bar{\mathbf{a}}_z \\ \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_x & \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_y & \bar{\mathbf{a}}_\theta \cdot \bar{\mathbf{a}}_z \\ \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_x & \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_y & \bar{\mathbf{a}}_\phi \cdot \bar{\mathbf{a}}_z \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad (2.29)$$

## \*\* 원통 좌표계

$$\begin{pmatrix} \rho = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1} \frac{y}{x} \\ z = z \end{pmatrix} \quad (2.7)$$

$$\begin{pmatrix} x = \rho(\bar{a}_x \cdot \bar{a}_\rho) = \rho \cos \phi \\ y = \rho(\bar{a}_y \cdot \bar{a}_\rho) = \rho \sin \phi \\ z = z \end{pmatrix} \quad (2.8)$$

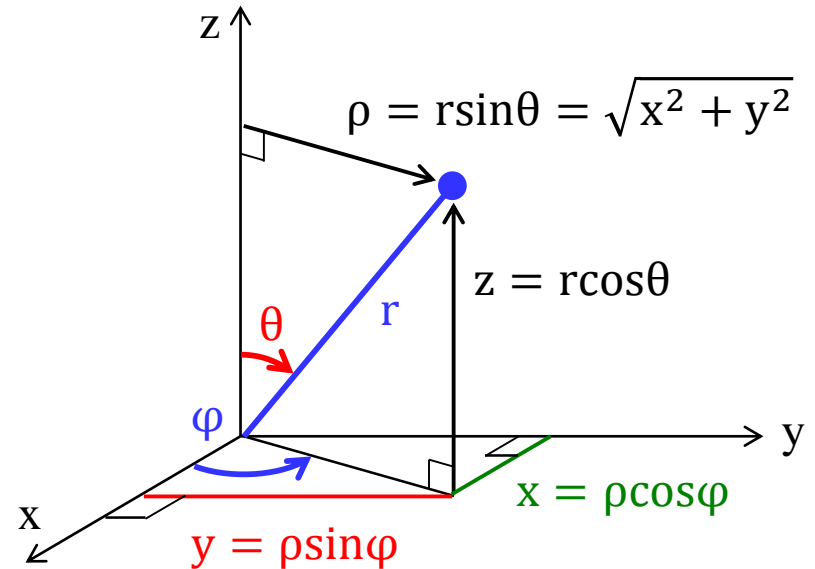


## \*\* 극 좌표계

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} \quad (2.21)$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$x = r \sin \theta \cos \phi$$
$$y = r \sin \theta \sin \phi \quad (2.22)$$
$$z = r \cos \theta$$



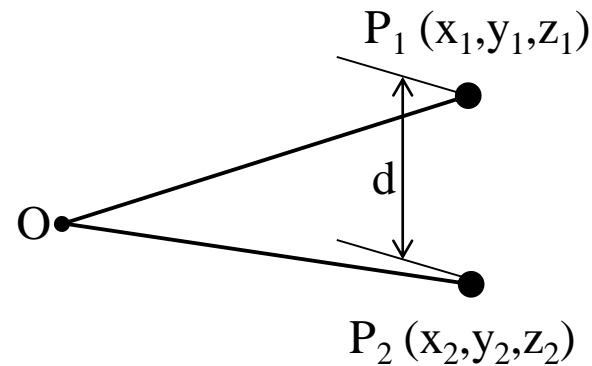
## 2 점 사이의 거리

Catesian coordinate

$$d = |\vec{r}_2 - \vec{r}_1| \quad (2.30)$$

$$\vec{r}_2 - \vec{r}_1 = [(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)]$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad (2.31)$$



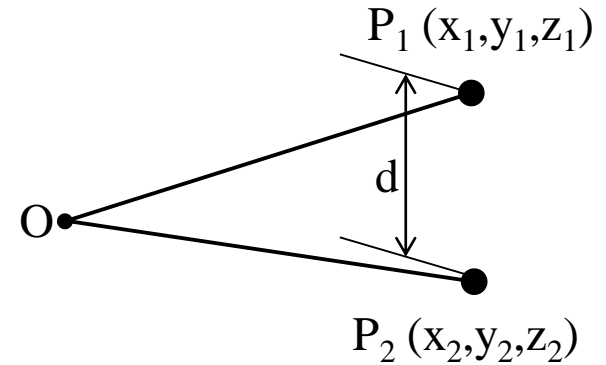


## Cylindrical coord.

$$(2.8) \begin{cases} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{cases}$$

$$\begin{aligned} d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \\ &= (\rho_2 \cos \phi_2 - \rho_1 \cos \phi_1)^2 \\ &\quad + (\rho_2 \sin \phi_2 - \rho_1 \sin \phi_1)^2 \\ &\quad + (z_2 - z_1)^2 \end{aligned}$$

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2 \quad (2.32)$$

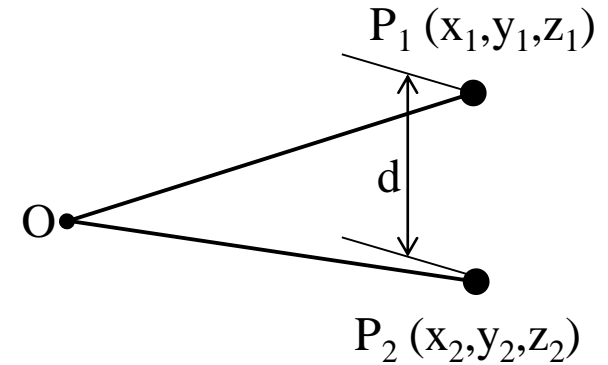


## Spherical coord

$$(2.22) \begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

$$\begin{aligned} d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \\ &= (r_2 \sin \theta_2 \cos \phi_2 - r_1 \sin \theta_1 \cos \phi_1)^2 \\ &\quad + (r_2 \sin \theta_2 \sin \phi_2 - r_1 \sin \theta_1 \sin \phi_1)^2 \\ &\quad + (r_2 \cos \theta_2 - r_1 \cos \theta_1)^2 \end{aligned}$$

$$\begin{aligned} d^2 &= r_2^2 + r_1^2 - 2r_1r_2 \cos \theta_2 \theta_1 \\ &\quad - 2r_1r_2 \sin \theta_2 \sin \theta_1 \cos(\phi_2 - \phi_1) \quad (2.33) \end{aligned}$$



**Ex 2.1**  $\vec{A} = y\vec{a}_x + (x + z)\vec{a}_y$ 를 원통좌표, 구좌표로 표현하라.  
 $P(-2,6,3)$ 와  $\vec{A}$ 를 원통좌표, 구좌표로 구하라.

$$(2.13) \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$$\left( \begin{array}{l} A_\rho = y \cos\phi + (x + z) \sin\phi \\ A_\phi = -y \sin\phi + (x + z) \cos\phi \\ A_z = 0 \end{array} \right) \leftarrow \text{원통 좌표계}$$

$$(2.27) \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$$\left( \begin{array}{l} A_r = y \sin\theta \cos\phi + (x + z) \sin\theta \sin\phi \\ A_\theta = y \cos\theta \cos\phi + (x + z) \cos\theta \sin\phi \\ A_\phi = -y \sin\phi + (x + z) \cos\phi \end{array} \right) \leftarrow \text{구좌표계}$$

**Ex 2.1**  $P(-2,6,3), \vec{A} = (y, x+z, 0)$  일 때  $P$ 와  $\vec{A}$ 를 원통좌표, 구좌표로 구하라.

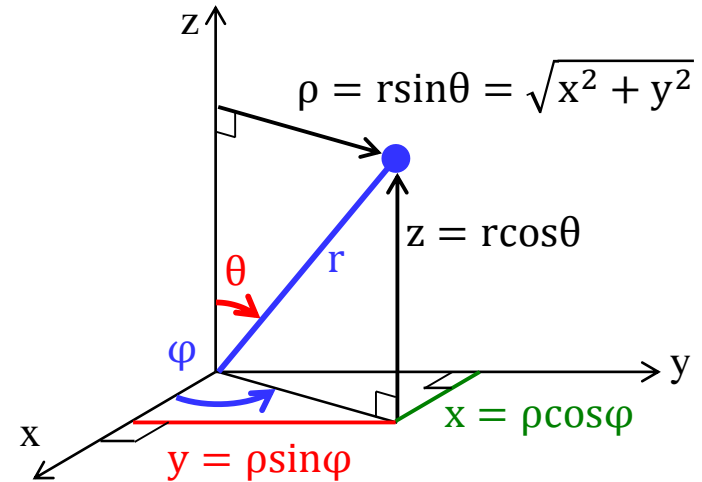
$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.32$$

$$\phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{6}{-2} = 108.43^\circ$$

$$z = 3$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4 + 36 + 9} = 7$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1} \frac{\sqrt{40}}{3} = 64.62^\circ$$



$$P(-2, 6, 3)$$

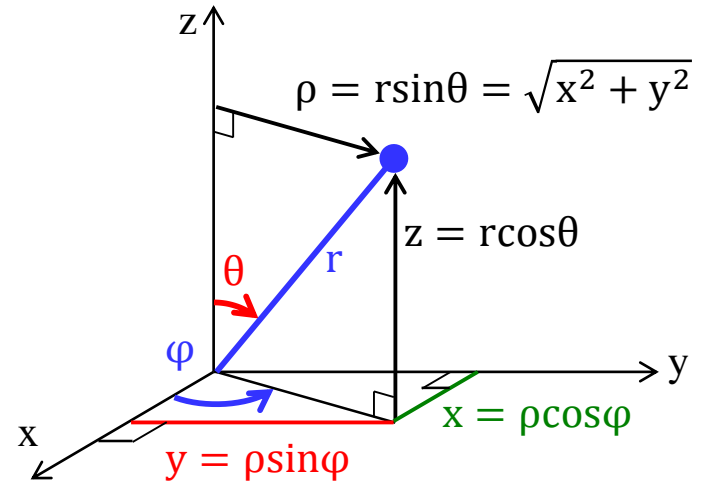
$$= P(6.32, 108.43^\circ, 3) \quad \text{원통좌표계}$$

$$= P(7, 64.62^\circ, 108.43^\circ) \quad \text{구좌표계}$$

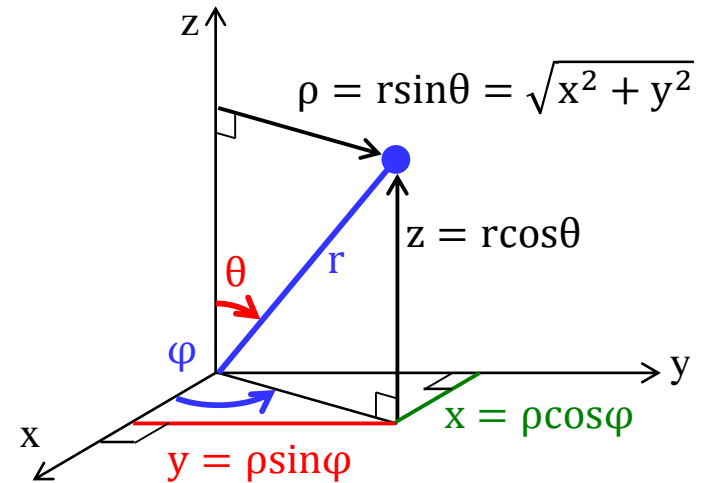
**Ex 2.1**  $P(-2,6,3), \vec{A} = (y, x+z, 0)$  일 때  $P$ 와  $\vec{A}$ 를 원통좌표, 구좌표로 구하라.

$$\begin{cases} x = -2 \\ y = 6 \\ z = 3 \end{cases}$$

$$\begin{pmatrix} \rho = \sqrt{x^2 + y^2} \\ \cos \phi = \frac{x}{\rho} \quad \sin \phi = \frac{y}{\rho} \\ r = \sqrt{x^2 + y^2 + z^2} \\ \cos \theta = \frac{z}{r} \quad \sin \theta = \frac{\rho}{r} \end{pmatrix} \rightarrow \begin{pmatrix} \rho = \sqrt{(-2)^2 + 6^2} = \sqrt{40} \\ \cos \phi = \frac{-2}{\sqrt{40}}, \quad \sin \phi = \frac{6}{\sqrt{40}} \\ r = \sqrt{(-2)^2 + 6^2 + 3^2} = 7 \\ \cos \theta = \frac{3}{7}, \quad \sin \theta = \frac{\sqrt{40}}{7} \end{pmatrix}$$



$$\left( \begin{array}{l} x = -2 \\ y = 6 \\ z = 3 \end{array} \right) \left( \begin{array}{l} \rho = \sqrt{(-2)^2 + 6^2} = \sqrt{40} \\ \cos \phi = \frac{-2}{\sqrt{40}}, \quad \sin \phi = \frac{6}{\sqrt{40}} \\ r = \sqrt{(-2)^2 + 6^2 + 3^2} = 7 \\ \cos \theta = \frac{3}{7}, \quad \sin \theta = \frac{\sqrt{40}}{7} \end{array} \right)$$



Cylindrical coord.

$$\left( \begin{array}{l} A_\rho = y \cos \phi + (x + y) \sin \phi \\ A_\phi = -y \sin \phi + (x + z) \cos \phi \\ A_z = 0 \end{array} \right) \rightarrow \vec{A} = (-6/\sqrt{40}, -38/\sqrt{40}, 0)$$

Spherical coord.

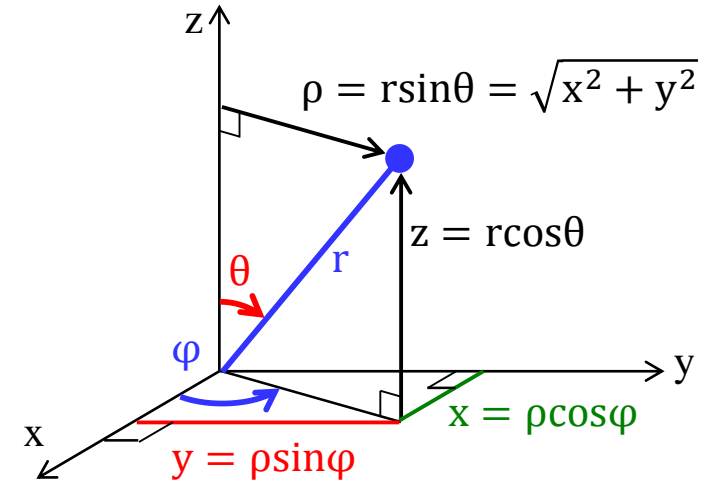
$$\left( \begin{array}{l} A_r = y \sin \theta \cos \phi + (x + y) \sin \theta \sin \phi \\ A_\theta = y \cos \theta \cos \phi + (x + y) \cos \theta \sin \phi \\ A_\phi = -y \sin \phi + (x + y) \cos \phi \end{array} \right) \rightarrow \vec{A} = (-0.8571, -0.4066, -6.008)$$

**Ex 2.2**  $\vec{B} = 10\vec{a}_r/r + r\cos\theta\vec{a}_\theta + \vec{a}_\phi$ 를 직각좌표계, 원통좌표계로 표현하라.  
 직각좌표계 Point (-3,4,0)일 때  $\vec{B}$ 를 직각좌표계로 원통좌표계 Point  $(5, \pi/2, -2)$ 일 때  $\vec{B}$ 를 원통좌표계로 표시하라.

구좌표계 → 직각좌표계

$$(2.28) \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

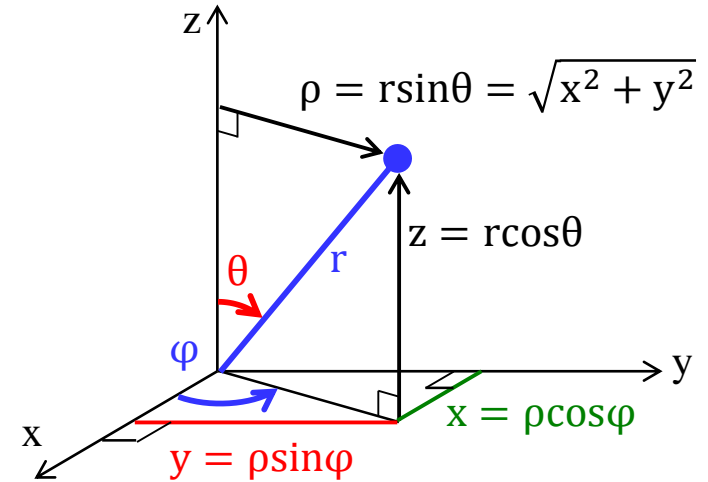
$$= \begin{bmatrix} (10/r)\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi \\ (10/r)\sin\theta\sin\phi + r\cos^2\theta\cos\phi + \cos\phi \\ (10/r)\cos\theta - r\cos\theta\sin\theta \end{bmatrix}$$



$$(2.21) \left( \begin{array}{l} r = \sqrt{x^2 + y^2 + z^2} \\ \tan\theta = \frac{\sqrt{x^2 + y^2}}{z} \\ \tan\phi = \frac{y}{x} \end{array} \right) \rightarrow \left( \begin{array}{l} r = \sqrt{x^2 + y^2 + z^2} \\ \rho = \sqrt{x^2 + y^2} \\ \cos\theta = \frac{z}{r}, \sin\theta = \frac{\rho}{r} \\ \tan\phi = \frac{y}{x}, \cos\phi = \frac{x}{\rho}, \sin\phi = \frac{y}{\rho} \end{array} \right)$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} (10/r)\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi \\ (10/r)\sin\theta\sin\phi + r\cos^2\theta\cos\phi + \cos\phi \\ (10/r)\cos\theta - r\cos\theta\sin\theta \end{bmatrix}$$

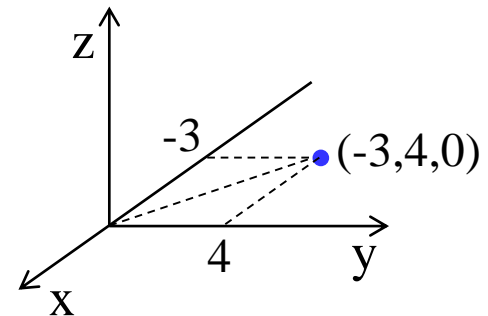
$$\left( \begin{array}{l} r = \sqrt{x^2 + y^2 + z^2} \\ \rho = \sqrt{x^2 + y^2} \\ \cos\theta = \frac{z}{r}, \sin\theta = \frac{\rho}{r} \\ \tan\phi = \frac{y}{x}, \cos\phi = \frac{x}{\rho}, \sin\phi = \frac{y}{\rho} \end{array} \right) \rightarrow$$



$$\left( \begin{array}{l} r = \sqrt{(-3)^2 + 4^2 + 0^2} = 5 \\ \theta = \tan^{-1} \frac{\sqrt{(-3)^2 + 4^2}}{0} = \frac{\pi}{2} \\ \cos\theta = 0, \sin\theta = 1 \\ \tan\phi = \frac{4}{-3}, \cos\phi = -\frac{3}{5}, \sin\phi = \frac{4}{5} \end{array} \right)$$

직각좌표계

$$\Rightarrow \bar{B} = (-2, 1, 0)$$





**Ex 2.2**  $\vec{B} = 10\vec{a}_r/r + r\cos\theta\vec{a}_\theta + \vec{a}_\phi$ 를 직각좌표계, 원통좌표계로 표현하라.  
 직각좌표계 Point (-3,4,0)일 때  $\vec{B}$ 를 직각좌표계로 원통좌표계 Point  $(5, \pi/2, -2)$ 일 때  $\vec{B}$ 를 원통좌표계로 표시하라.

구좌표계  $\rightarrow$  원통좌표계

$$(2.28) \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$(2.13) \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

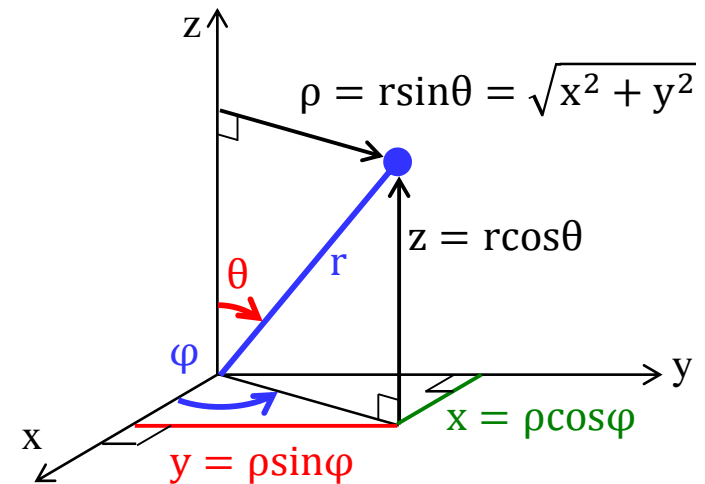
$$= \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r\cos\theta \\ 1 \end{bmatrix}$$

# 원통좌표계

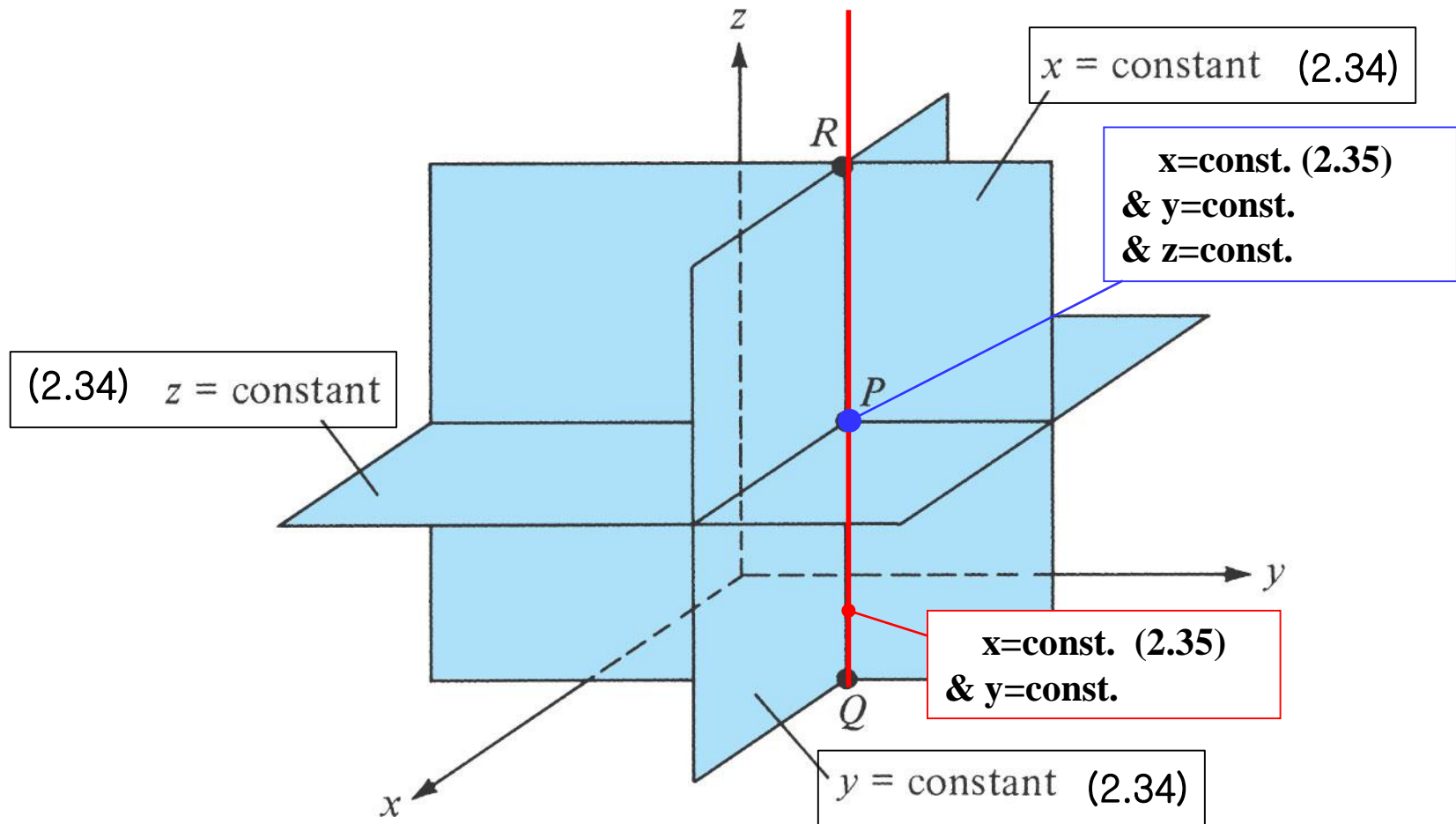
$$\begin{pmatrix} \rho = 5 \\ \varphi = \pi/2 \\ z = -2 \end{pmatrix}$$

$$\begin{pmatrix} r = \sqrt{\rho^2 + z^2} \\ \cos \theta = \frac{z}{r}, \sin \theta = \frac{\rho}{r} \\ \cos \phi = \frac{x}{\rho}, \sin \phi = \frac{y}{\rho} \end{pmatrix} \rightarrow \begin{pmatrix} r = \sqrt{5^2 + (-2)^2} = \sqrt{29} \\ \cos \theta = -2/\sqrt{29}, \sin \theta = 5/\sqrt{29} \\ \cos \phi = 0, \sin \phi = 1 \end{pmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} 10/r \\ r \cos \theta \\ 1 \end{bmatrix} = \begin{pmatrix} 2.67 \\ 1 \\ 1.167 \end{pmatrix}$$



## 2.5 Constant-Coordinate Surfaces



**Figure 2.7** Constant  $x$ ,  $y$ , and  $z$  surfaces.

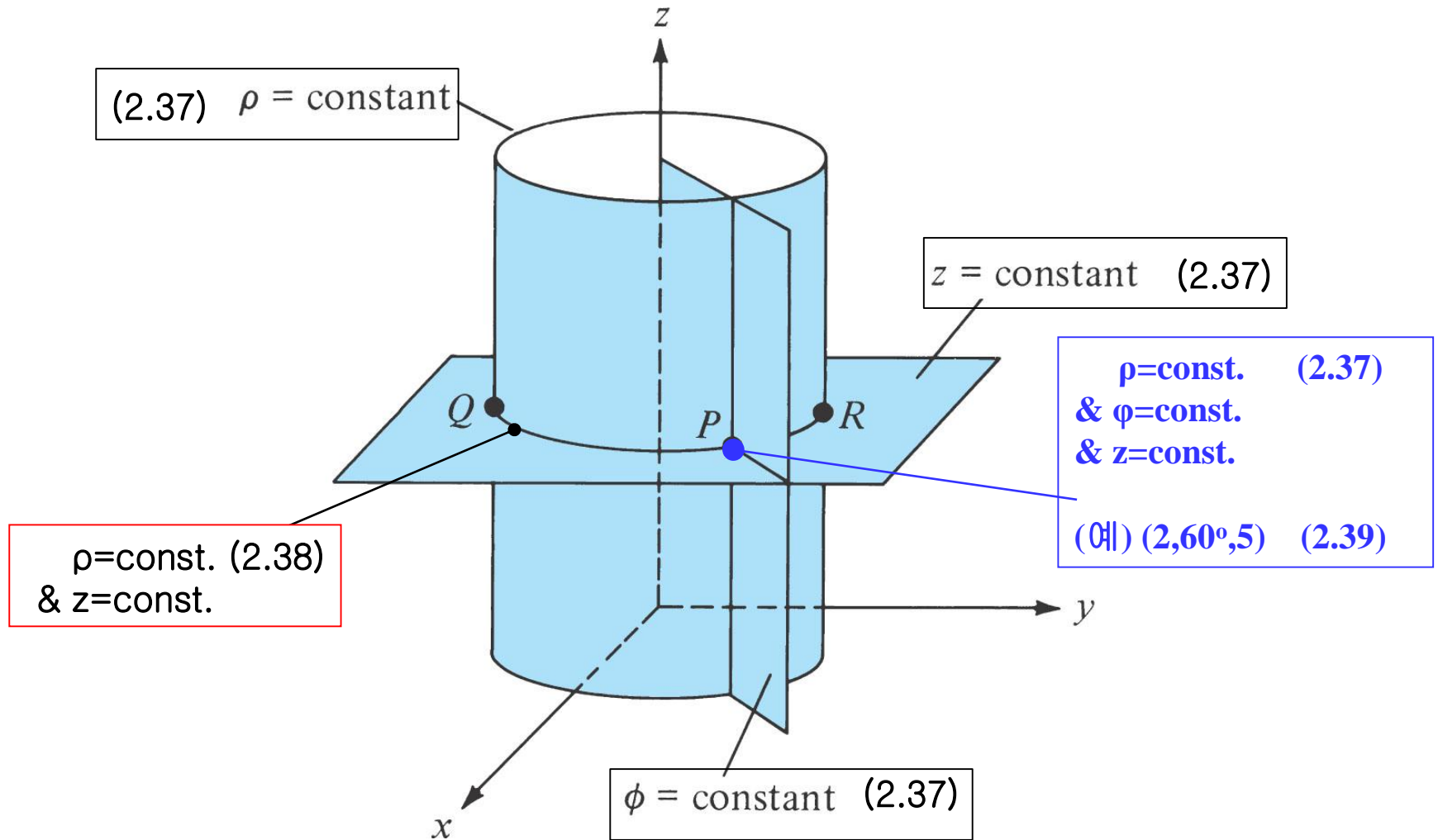


Figure 2.8 Constant  $\rho$ ,  $\phi$ , and  $z$  surfaces.

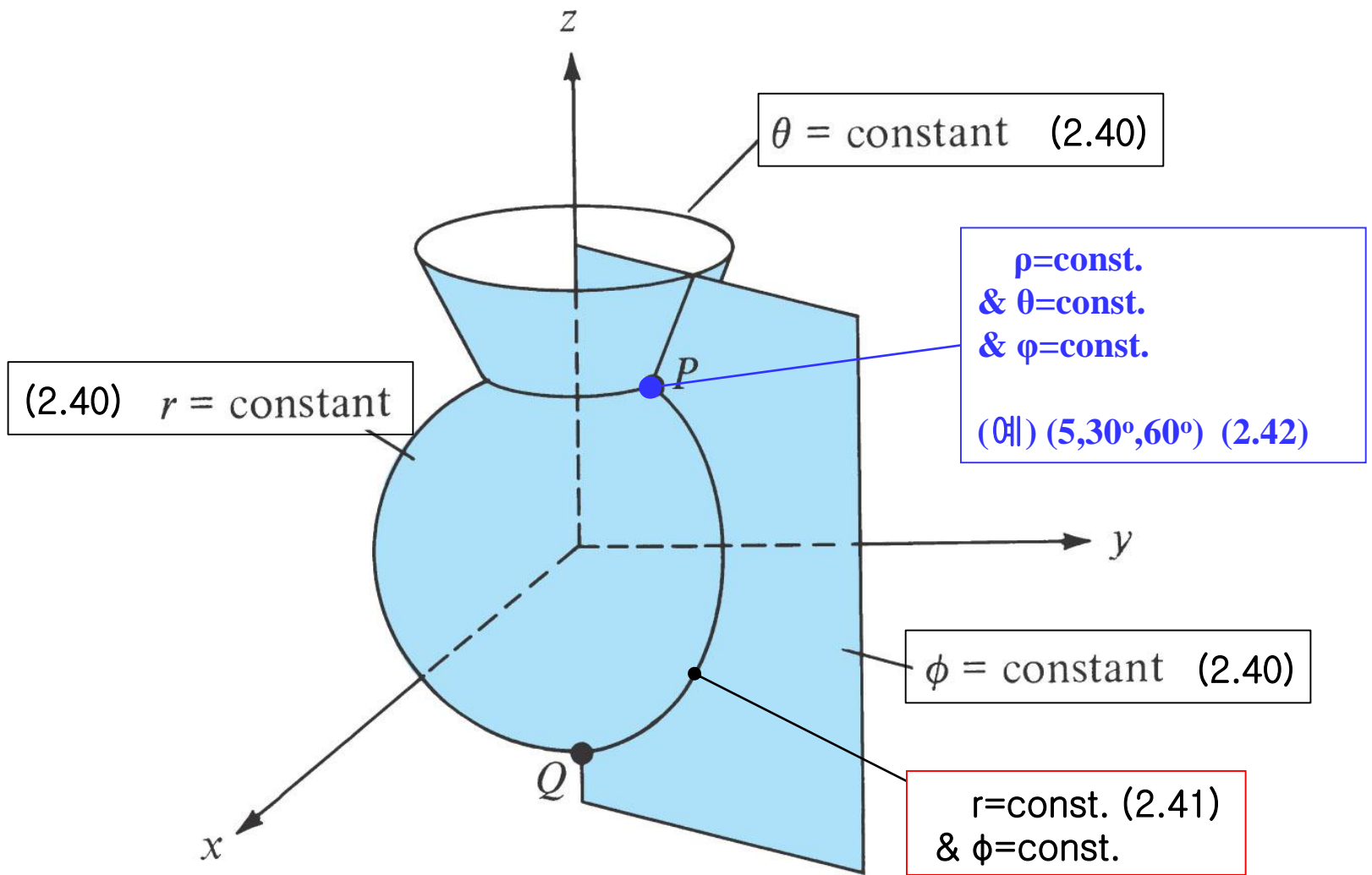
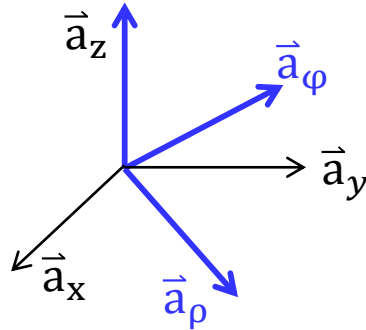


Figure 2.9 Constant  $r$ ,  $\theta$ , and  $\phi$  surfaces.

**Ex 2.3**  $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z$ ,  $\vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z$  일 때  
 (a)  $|\vec{E} \times \vec{F}|$ 를 구하라.

$(\vec{a}_\rho, \vec{a}_\phi, \vec{a}_z)$

$$\begin{cases} \vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z \\ \vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z \end{cases}$$



$(\vec{a}_x, \vec{a}_y, \vec{a}_z)$

$$\begin{cases} \vec{E} = -5\vec{a}_x + 10\vec{a}_y + 3\vec{a}_z \\ \vec{F} = \vec{a}_x + 2\vec{a}_y - 6\vec{a}_z \end{cases}$$

$$\begin{aligned} |\vec{E} \times \vec{F}| &= \left| \begin{vmatrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ -5 & 10 & 3 \\ 1 & 2 & -6 \end{vmatrix} \right| \\ &= \left| -66\vec{a}_\rho - 27\vec{a}_\phi - 20\vec{a}_z \right| \\ &= \sqrt{66^2 + 27^2 + 20^2} \\ &= 74.06 \end{aligned}$$

**Ex 2.3**  $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z$ ,  $\vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z$  일 때

(b) Point P(5,  $\pi/2$ , 3) 에서 Line (x=2, z=3)에 평행한  $\vec{E}$ 의 성분.

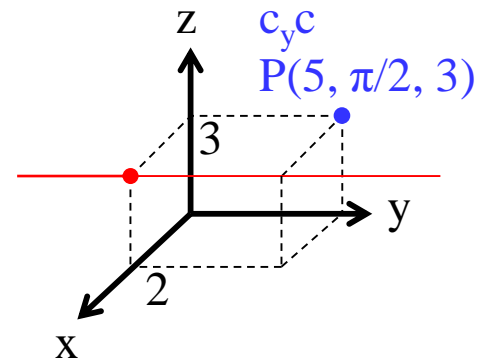
Line (x=2, z=3)는 y축에 평행.

y축에 평행한  $\vec{E}$ 의 성분:  $(\vec{E} \cdot \vec{a}_y)\vec{a}_y = (\vec{E} \cdot \vec{a}_\rho)\vec{a}_\rho = -5\vec{a}_\rho$

$$\vec{a}_y = \sin \phi \vec{a}_\rho + \cos \phi \vec{a}_\phi \quad (2.9)$$

$$= \sin \frac{\pi}{2} \vec{a}_\rho + \cos \frac{\pi}{2} \vec{a}_\phi$$

$$= \vec{a}_\rho$$



**Ex 2.3**  $\vec{E} = -5\vec{a}_\rho + 10\vec{a}_\phi + 3\vec{a}_z$ ,  $\vec{F} = \vec{a}_\rho + 2\vec{a}_\phi - 6\vec{a}_z$  일 때  
 (c) Point P(5,  $\pi/2$ , 3) 에서 Surface ( $z=3$ ) 와 이루는 각.

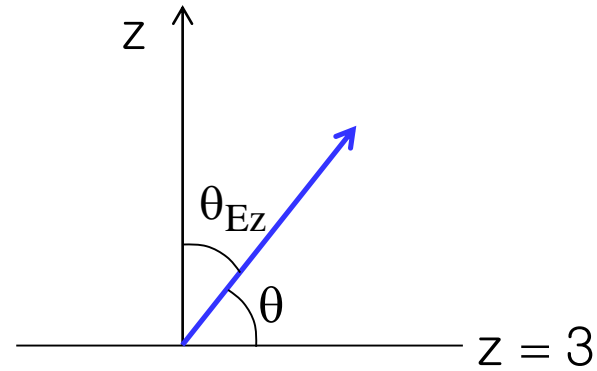
$\vec{a}_z$  는 Surface  $z=3$  과 수직.

$$\vec{E} \cdot \vec{a}_z = E \cos \theta_{Ez}$$

$$(-5, 10, 3) \cdot (0, 0, 1) = \sqrt{134} \cos \theta_{Ez}$$

$$\theta_{Ez} = 74.98^\circ$$

$$\theta = 90^\circ - \theta_{Ez} = 15.2^\circ$$





**Ex 2.4**  $\vec{D} = r\sin\varphi\vec{a}_r - \sin\theta\cos\varphi\vec{a}_\theta/r + r^2\vec{a}_\varphi$  일 때

(a) Point P(10,150°,330°) 에서  $\vec{D}$

(b) Point P에서 Surface  $r=10$ 에 접하는  $\vec{D}$ 의 Vector 성분

(a)

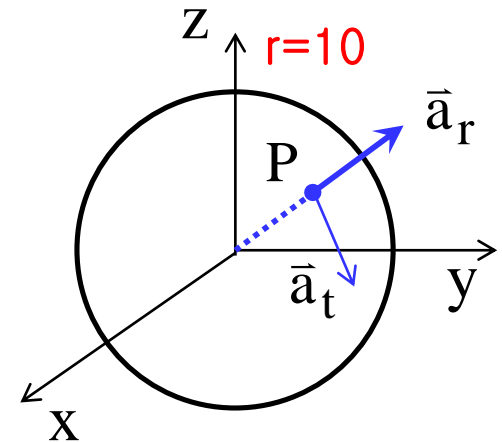
$$\begin{aligned}\vec{D} &= 10 \sin(330^\circ) \vec{a}_r - \frac{1}{10} \sin(150^\circ) \cos(330^\circ) \vec{a}_\theta + 10^2 \vec{a}_\varphi \\ &= (-5, 0.043, 100)\end{aligned}$$

(b)

$\vec{a}_r$  은 Surface  $r=10$  에 수직.

$\vec{D}$  의 수직인 성분:  $\vec{D}_n = (\vec{D} \cdot \vec{a}_r) \vec{a}_r = (-5, 0, 0)$

$\vec{D}$  의 평행인 성분:  $\vec{D}_t = \vec{D} - \vec{D}_n = (0, 0.043, 100)$



**Ex 2.4**  $\vec{D} = r\sin\phi\vec{a}_r - \sin\theta\cos\phi\vec{a}_\theta/r + r^2\vec{a}_\phi$  일 때

(c) Point P(10,150°,330°) 에서  $\vec{D}$ 에 수직이고 Cone  $\theta=150^\circ$ 에 접하는 Unit Vector

$$\vec{D} \times \vec{a}_\theta = \begin{vmatrix} \vec{a}_r & \vec{a}_\theta & \vec{a}_\phi \\ -5 & 0.043 & 100 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= -100\vec{a}_r - 5\vec{a}_\phi$$

$$\vec{a} = \frac{-100\vec{a}_r - 5\vec{a}_\phi}{\sqrt{100^2 + 5^2}}$$

$$= -0.9988\vec{a}_r - 0.0499\vec{a}_\phi$$

$\vec{a}_\theta$  성분을 제외해야 하는데  
 $\vec{a}_\theta$  성분 없음  $\rightarrow$  그대로 답

