

# **5. Electric Fields in Material Space**

## 5.2 매질의 성질

	재료	전도율 $\sigma$ (S/m)
도체	Ag	$6.1 \times 10^7$
	Cu	$5.8 \times 10^7$
	Al	$3.5 \times 10^7$
	Au	$4.1 \times 10^7$
반도체	Ge	2.2
	Si	$4.4 \times 10^{-4}$
절연체	Glass	$10^{-12}$

## 5.3 Convection and Conduction Currents

$\sigma$ : 전기전도율, 도전율, Conductivity  
단위:  $\Omega^{-1}/\text{m}$ , Siemen/m, Mho/m

$$R=L/(A\sigma)$$

**Current** : 단의 시간에 흐르는 전하량 (단위: C/sec).

- convection current
- conduction current
- displacement current

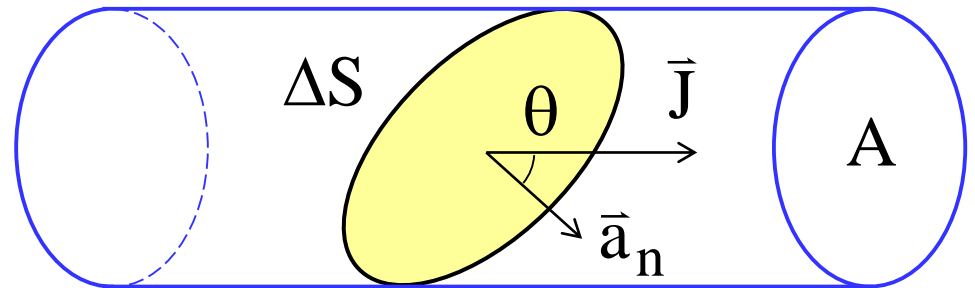
$$\nabla \times \frac{\vec{B}}{\mu} = \epsilon \frac{\partial \vec{E}}{\partial t} + \vec{J} : \text{Ampere's Eq}$$

$$I = \frac{dQ}{dt} \text{ [C/sec]} \quad (5.1)$$

$$J_n = \frac{\Delta I}{\Delta S} \text{ [C/sec/m}^2\text{]}$$
$$\Delta I = J_n \Delta S \quad (5.2)$$

$$\Delta I = \vec{J} \cdot \Delta \vec{S} \quad (5.3)$$

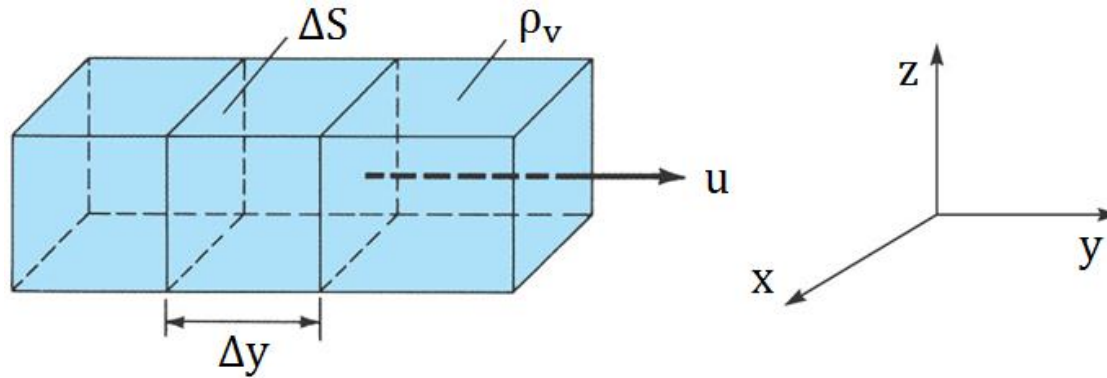
$$I = \int_S \vec{J} \cdot d\vec{S} \quad (5.4)$$



$$A = \cos \theta \Delta S$$

$$I = JA = \vec{J} \cdot \vec{a}_n \Delta S$$

**Convection current** : 공간을 자유롭게 움직이는 전자의 운동에 의한 전류.



**Figure 5.1** Current in a filament.

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta S \Delta y}{\Delta t} = \rho_v \Delta S \frac{\Delta y}{\Delta t} = \rho_v \Delta S u_y \quad (5.5)$$

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y \quad (5.6)$$

$$\vec{J} = \rho_v \vec{u} \quad (5.7)$$

: Convection Current Density [A/sec/m<sup>2</sup>]

**Conduction current:** 금속과 같은 물질에 구속된 전자들에 의해 전달되는 전류.

Electron Momentum Eq :

$$\vec{F} = mn_e \frac{d\vec{u}}{dt} = -en_e(\vec{E} + \vec{u} \times \vec{B}) - \frac{mn_e \vec{u}}{\tau}$$

$$\frac{mn_e \vec{u}}{\tau} = -en_e \vec{E}$$

$$\frac{m(-en_e \vec{u})}{\tau} = e^2 n_e \vec{E}$$

$$\vec{J}_e = \frac{e^2 \tau n_e}{m} \vec{E}$$

$$\vec{J}_e = \sigma \vec{E} : \text{Ohm's Law} \quad (5.11)$$

$$\text{where } \sigma \equiv \frac{e^2 \tau n_e}{m}$$

$$\sigma = \frac{e^2 \tau n_e}{m} : \text{Conductivity}$$

$\tau = \text{Collision Time}$

$\nu = 1/\tau : \text{Collision Frequency}$

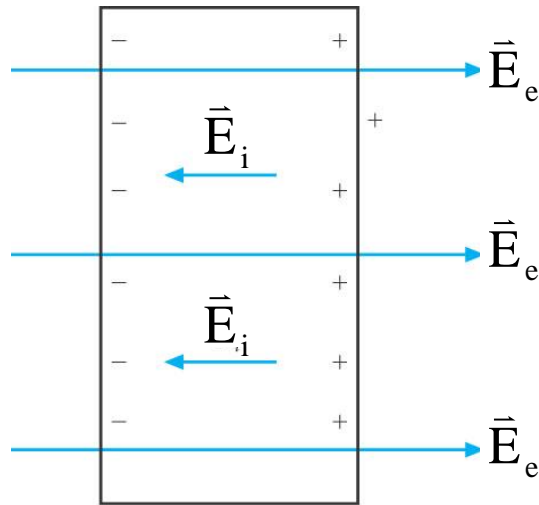
$n_e = \text{Electron Density}$

$m = \text{Electron Mass}$

## 5.4 Conductors

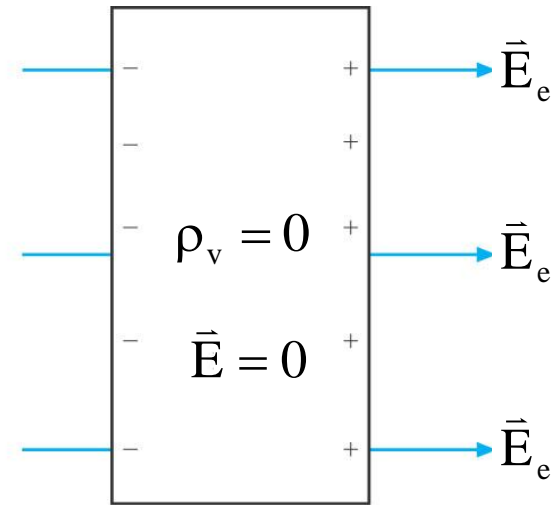
A perfect conductor cannot contain an electrostatic field within it.

$$\vec{E} = -\nabla V = 0, \quad \rho_v = 0, \quad V_{ab} = 0 \quad \text{inside a conductor}$$



(a)

Floating Conductor



(b)

Conductor 내부

## Resistivity 정의

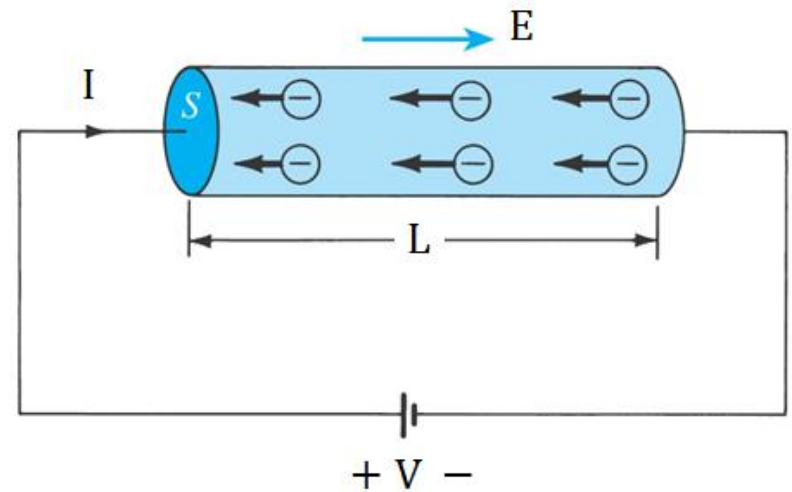
$$E = \frac{V}{L} \quad (5.13)$$

$$J = \frac{I}{S} \quad (5.14)$$

$$= \sigma E = \frac{\sigma V}{L} \quad (5.15)$$

$$R = \frac{V}{I} = \frac{L}{\sigma S} \\ = \frac{\rho_c L}{S} \quad (5.16)$$

$\left( \rho_c = 1/\sigma : \text{Resistivity} \right.$   
 $\left. \text{unit : } \Omega \cdot \text{m} \right)$

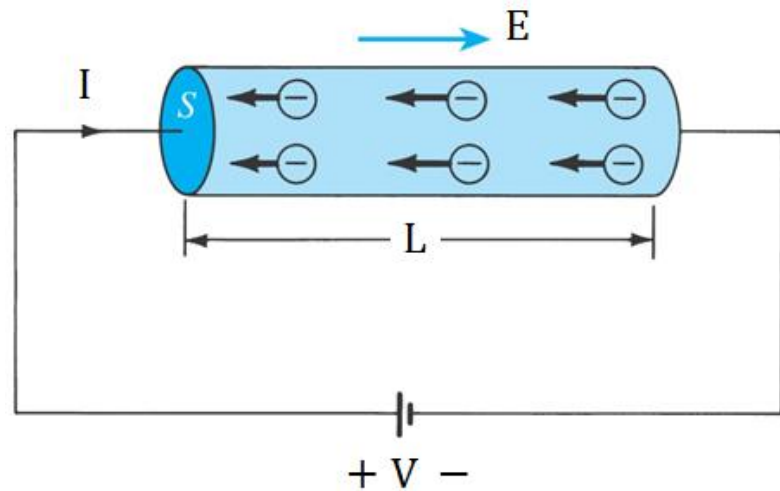


**Figure 5.3** A conductor of uniform cross section under an applied  $\mathbf{E}$  field. Conductor has a uniform cross section  $S$  and is of length  $L$ .



## 불균일한 단면적을 가진 전선의 Resistance 정의

$$\begin{aligned} R &= \frac{V}{I} \\ &= \frac{\int \vec{E} \cdot d\vec{L}}{\int \sigma \vec{E} \cdot d\vec{S}} \\ &= \frac{\int \vec{E} \cdot d\vec{L}}{\int \vec{J} \cdot d\vec{S}} \end{aligned} \quad (5.17)$$



## Power의 정의

$$dW = -\vec{F} \cdot d\vec{L}$$

$$P \equiv \frac{dW}{dt} = -\vec{F} \cdot \frac{d\vec{L}}{dt} = -\vec{F} \cdot \vec{v}$$

$$\left( \begin{array}{ll} \text{unit : } [F] = \text{kg} \cdot \text{m} / \text{sec}^2 & = \text{Newton : Force} \\ [W] = \text{kg} \cdot \text{m}^2 / \text{sec}^2 & = \text{Joule : Work} \\ [P] = \text{kg} \cdot \text{m}^2 / \text{sec}^2 / \text{sec} & \\ = \text{J} / \text{sec} & = \text{Watt : Power} \end{array} \right.$$

## Joule's Law : Ohmic Power Dissipation

$$\begin{aligned}dP &= (dQ\vec{E}) \cdot \vec{v} \\ &= (\rho_v dx dy dz) \vec{E} \cdot \vec{v} \\ &= \vec{E} \cdot (\rho_v \vec{v}) dx dy dz \\ &= \vec{E} \cdot \vec{J} dx dy dz\end{aligned}$$

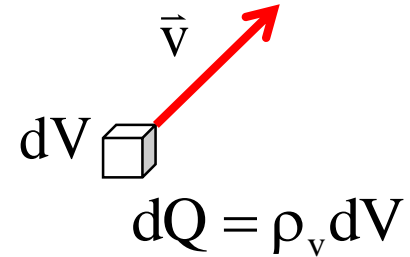
$$P = \iiint (\vec{E} \cdot \vec{J}) dV \quad (5.18)$$

$$w_p \equiv \frac{dP}{dV} = \vec{E} \cdot \vec{J} = \sigma |\vec{E}|^2 \quad (5.19)$$

: Power Density

균일한 단면적을 가진 전기선

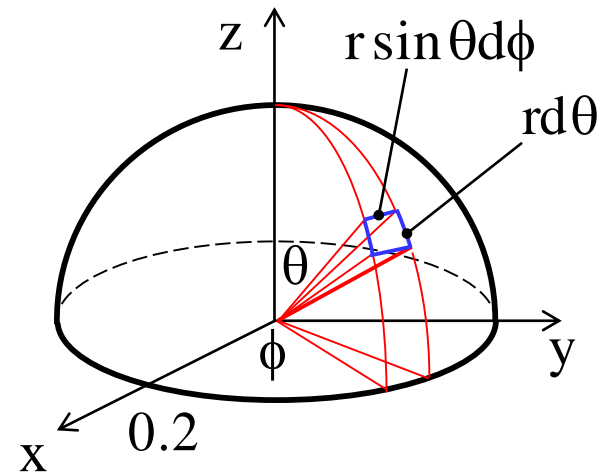
$$\begin{aligned}P &= \int_L E dL \int_S J dS = VI \\ &= I^2 R \quad (5.20)\end{aligned}$$

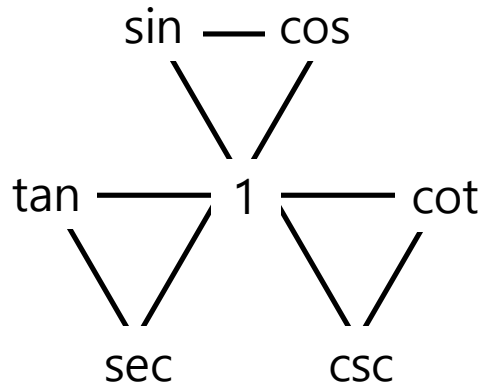


**Ex 5.1**  $\vec{J} = r^{-3}(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$  일 때 다음 단면을 흐르는 전류는?

(a) 반경 20 cm 의 반구 꺾이기

$$\begin{aligned}
 & \iint \vec{J} \cdot \vec{a}_r dS \\
 &= \iint \vec{J} \cdot \vec{a}_r (r d\theta)(r \sin \theta d\phi) \\
 &= \int_0^{\pi/2} \int_0^{2\pi} \frac{1}{r^3} (2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta) \cdot \vec{a}_r (r^2 \sin \theta d\phi d\theta) \\
 &= \int_0^{\pi/2} \int_0^{2\pi} \frac{1}{r^3} (2\cos\theta)(r^2 \sin \theta d\phi d\theta) \\
 &= 2\pi \int_0^{\pi/2} \frac{1}{r^3} (2\cos\theta)(r^2 \sin \theta) d\theta \quad (r = 0.2 \text{ m}) \\
 &= 10\pi \int_0^{\pi/2} \sin 2\theta d\theta \\
 &= 10\pi \left[ -\frac{\cos 2\theta}{2} \right]_0^{\pi/2} \\
 &= 10\pi \text{ A}
 \end{aligned}$$





$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

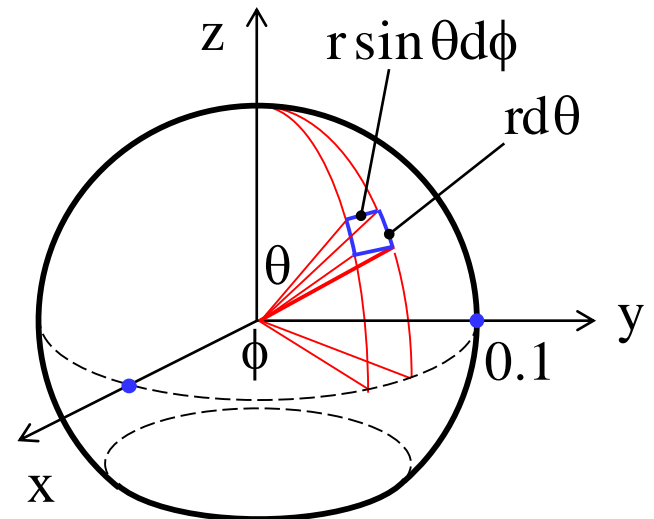
$$\frac{d \sin \theta}{d\theta} = \cos \theta$$

$$\frac{d \cos \theta}{d\theta} = -\sin \theta$$

$$\frac{d \tan \theta}{d\theta} = \sec^2 \theta$$

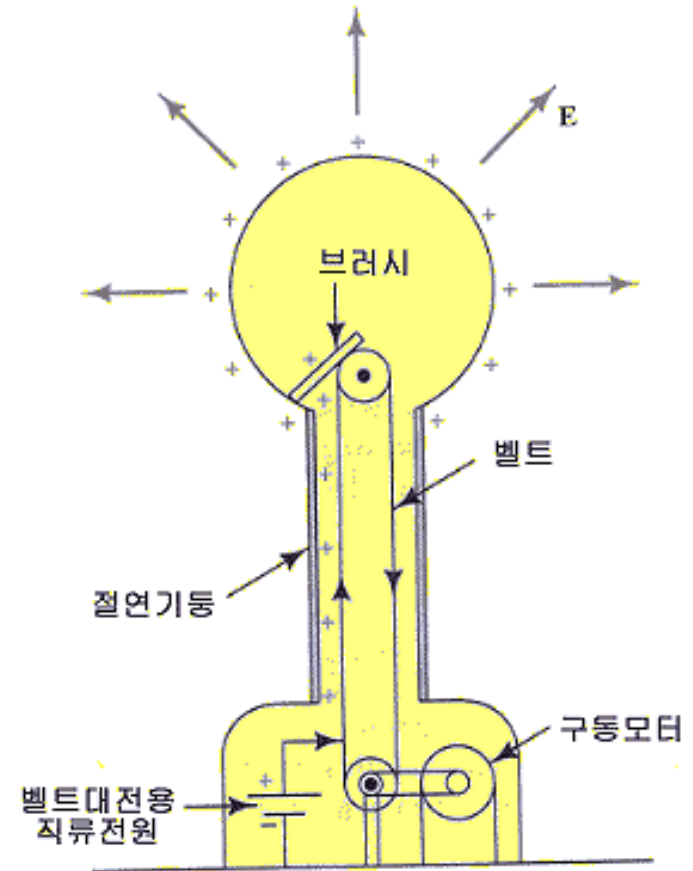
(b) 반경 10 cm 의 구각

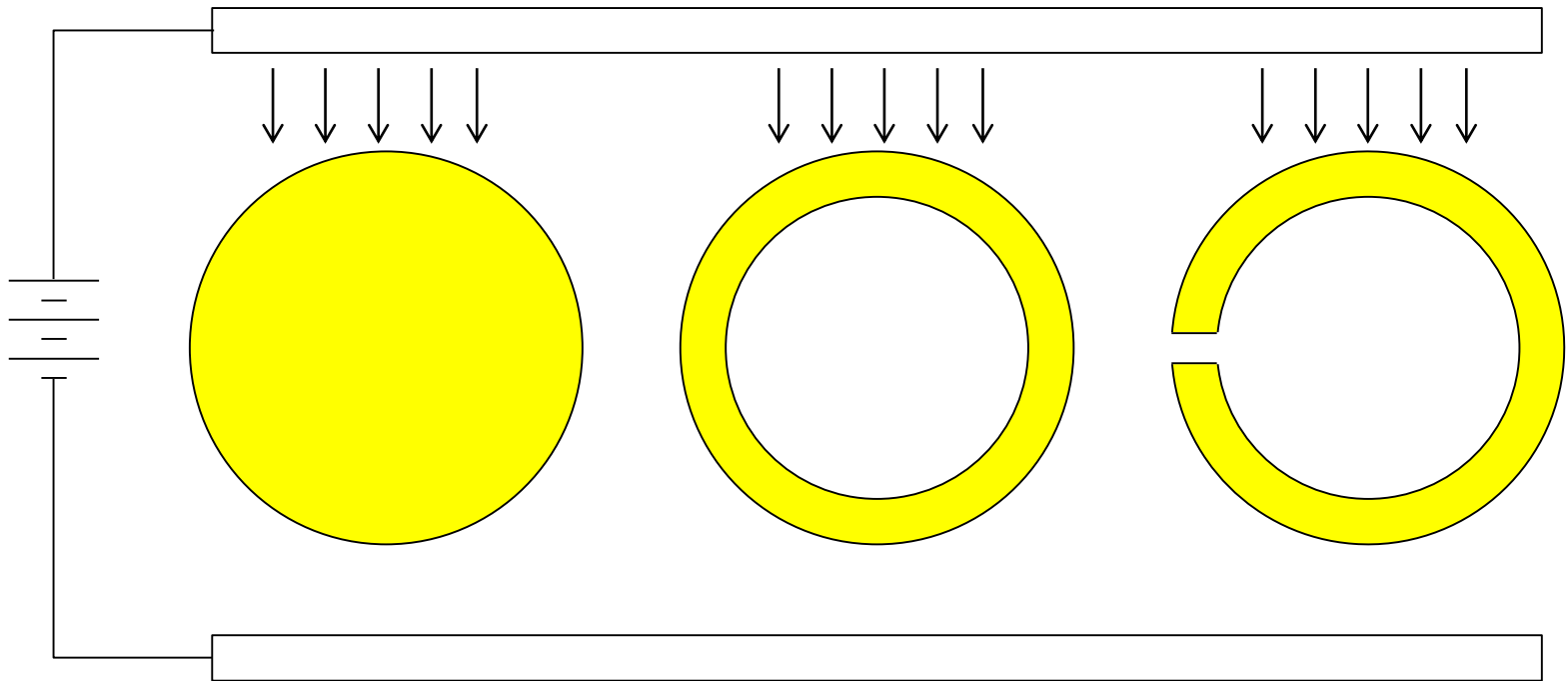
$$\begin{aligned}
 & \iint \vec{J} \cdot \vec{a}_r (r d\theta)(r \sin \theta d\phi) \\
 &= \int_0^\pi \int_0^{2\pi} \frac{1}{r^3} (2 \cos \theta \vec{a}_r + \sin \theta \vec{a}_\theta) \cdot \vec{a}_r (r^2 \sin \theta d\phi d\theta) \\
 &= \int_0^\pi \int_0^{2\pi} \frac{1}{r^3} (2 \cos \theta) (r^2 \sin \theta d\phi d\theta) \\
 &= 2\pi \int_0^\pi \frac{1}{r^3} (2 \cos \theta) (r^2 \sin \theta) d\theta \quad (r = 0.1 \text{ m}) \\
 &= 20\pi \int_0^\pi \sin 2\theta d\theta \\
 &= 20\pi \left[ -\frac{\cos 2\theta}{2} \right]_0^\pi \\
 &= 0
 \end{aligned}$$



**Ex 5.2** Van de Graaff 발전기에서  $\rho_s=10^{-7}\text{C/m}^2$  인 전하가  $u=2\text{m/s}$ 의 속도로 폭  $w=10\text{cm}$ 인 Belt가 이동할 때 5초 동안 축적된 전하는?

$$\begin{aligned}
 Q &= It \\
 &= \rho_s u w t \\
 &= 10^{-7} [\text{C/m}^2] \times 2 [\text{m/sec}] \times 0.1 [\text{m}] \times 5 [\text{sec}] \\
 &= 100 \text{ nC}
 \end{aligned}$$







**Ex 5.3** 직경 1mm 인 다음 그림의 전선에 10 mV/m의 전장을 가했을 때 자유전자의 수  $n=10^{29}/\text{m}^3$  이었다.  $\sigma = 5 \times 10^7/\Omega\text{m}$ .

(a) 자유전자에 의한 전하 밀도  $\rho_v$ ?

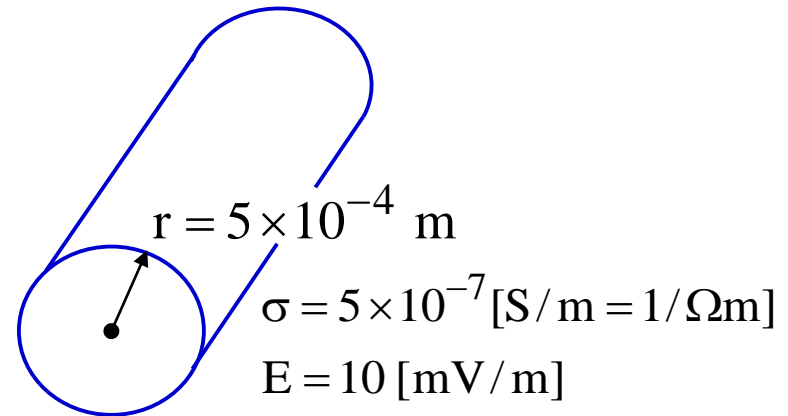
$$\begin{aligned}\rho_v &= nQ_e \\ &= (10^{29} \text{ \#/m}^3) \times (-1.6 \times 10^{-19} \text{ C}) \\ &= -1.6 \times 10^{10} \text{ C/m}^3\end{aligned}$$

(b) 전류밀도  $J$ ?

$$\begin{aligned}J &= \sigma E \\ &= (5 \times 10^7 \text{ [1/\Omega m]}) \times (10^{-2} \text{ [V/m]}) \\ &= 500 \text{ kA/m}^2\end{aligned}$$

(c) 전류  $I$ ?

$$\begin{aligned}I &= JS = J\pi r^2 \\ &= (5 \times 10^5 \text{ [A/m}^2]) \times \pi (0.0005)^2 \text{ [m}^2] \\ &= 0.393 \text{ A}\end{aligned}$$



(d) 전자 속도  $u$ ?

$$\begin{aligned}J &= \rho_v [\text{C/m}^2] \times u [\text{m/sec}] \\ u &= J / \rho_v \\ &= \frac{5 \times 10^5}{1.6 \times 10^{10}} \\ &= 3.125 \times 10^{10} \\ &= 0.393 \text{ [m/sec]}\end{aligned}$$

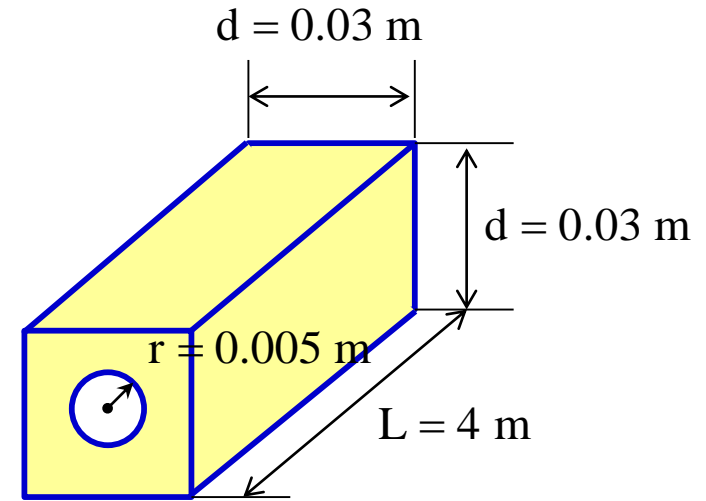
**Ex 5.4** 전도율  $\sigma = 5 \times 10^6 \text{ S/m}$ , 길이  $L = 4 \text{ m}$  일 때  
가운데 둥근 구멍이 있는 다음 그림의 양단의 저항은?

$$R = \frac{L}{\sigma S}$$

$$\left( \begin{array}{l} S = d^2 - \pi r^2 \\ = 0.03^2 - \pi(0.5 \times 10^{-2})^2 \text{ m}^2 \\ = (9 - \pi/4) \times 10^{-4} \text{ m}^2 \end{array} \right)$$

$$= \frac{4}{(5 \times 10^6) \times (9 - \pi/4) \times 10^{-4}}$$

$$= 974 \mu\Omega$$



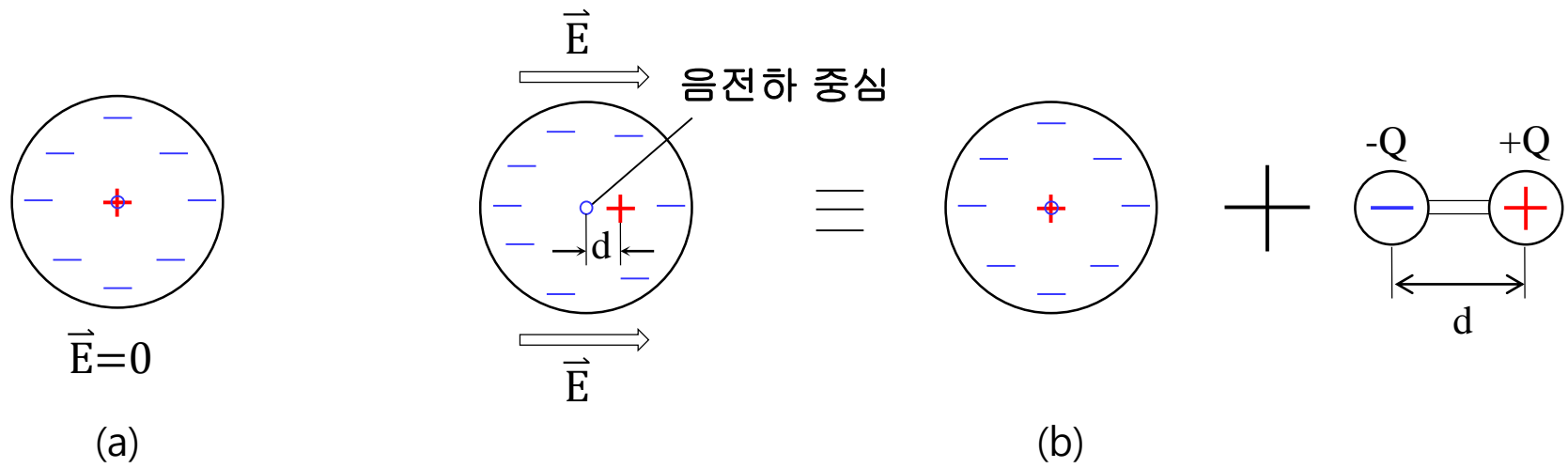
## 5.5 Polarization in Dielectrics

$$\vec{p} = Q\vec{d} \quad (\text{dipole}) \quad [\text{C} \cdot \text{m}] \quad (5.21)$$

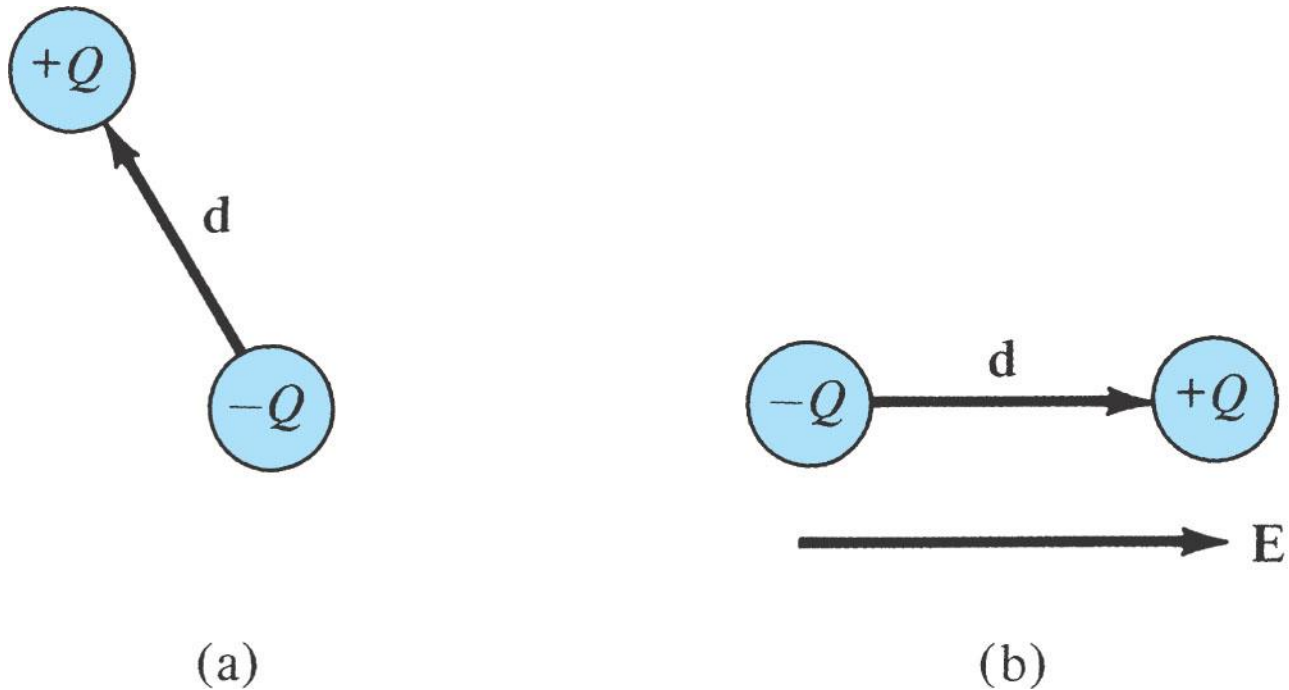
$$Q_1\vec{d}_1 + Q_2\vec{d}_2 + \cdots + Q_n\vec{d}_n = \sum_{k=1}^N Q_k\vec{d}_k \quad (5.22)$$

$$\vec{P} = \frac{\lim_{\Delta v \rightarrow 0} \sum_{k=1}^N Q_k\vec{d}_k}{\Delta v} \quad [\text{C}/\text{m}^2] \quad (5.23)$$

(**Polarization**  $\vec{P}$  is defined as the dipole moment per unit volume)



**Figure 5.6** Polarization of a nonpolar atom or molecule.



**Figure 5.7** Polarization of a polar molecule:  
(a) permanent dipole ( $\vec{E}=0$ ), (b) induced dipole ( $\vec{E} \neq 0$ ).

$$V = \frac{\bar{\mathbf{p}} \cdot \bar{\mathbf{a}}_r}{4\pi\epsilon_0 r^2} \quad (4.80)$$

$$dV = \frac{\bar{\mathbf{P}} \cdot \bar{\mathbf{a}}_r dv'}{4\pi\epsilon_0 R^2} \quad (5.24)$$

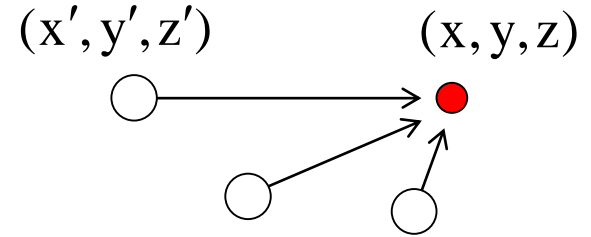
$$\text{where } R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\bar{\mathbf{P}} \cdot \bar{\mathbf{a}}_r dv'}{R^2}$$

$$= \frac{1}{4\pi\epsilon_0} \bar{\mathbf{P}} \cdot \nabla' \frac{1}{R} dv'$$

$$= \frac{1}{4\pi\epsilon_0} \left( \nabla' \cdot \frac{\bar{\mathbf{P}}}{R} - \frac{\nabla' \cdot \bar{\mathbf{P}}}{R} \right) dv' \quad \leftarrow \left( \nabla' \cdot \frac{\bar{\mathbf{P}}}{R} = \frac{\nabla' \cdot \bar{\mathbf{P}}}{R} + \bar{\mathbf{P}} \cdot \nabla' \frac{1}{R} \right)$$

$$V = \frac{1}{4\pi\epsilon_0} \int_{v'} \left( \nabla' \cdot \frac{\bar{\mathbf{P}}}{R} - \frac{\nabla' \cdot \bar{\mathbf{P}}}{R} \right) dv'$$

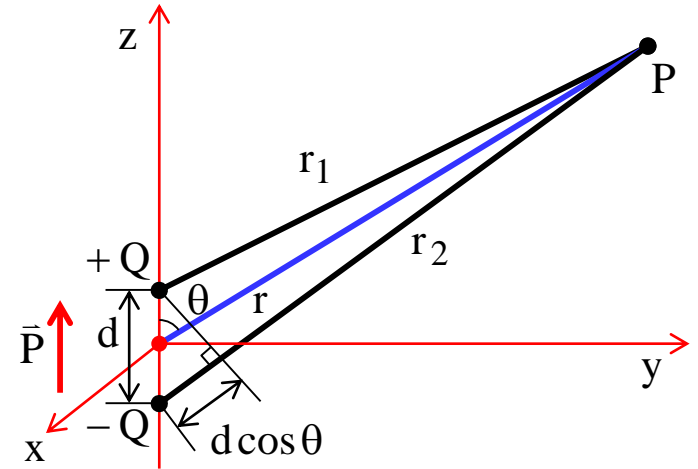


## 참고

$$V = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{Q}{4\pi\epsilon_0} \left[ \frac{r_2 - r_1}{r_1 r_2} \right] \quad (4.77)$$

$$V = \frac{Q d \cos \theta}{4\pi\epsilon_0 r^2} \quad (4.78)$$

$$\vec{p} \equiv Q\vec{d} \text{ (Cm) (Dipole moment)} \quad (4.79)$$



$$V = \frac{\vec{p} \cdot \vec{a}_r}{4\pi\epsilon_0 r^2} \quad (4.80)$$

$$V(\mathbf{r}) = \frac{\vec{p} \cdot (\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3} \quad (4.81)$$

$$\nabla' \frac{1}{R} = \frac{\bar{\mathbf{a}}_r}{R^2} \quad \text{ㅇㅇ}$$

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$$R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$$

$$\begin{aligned} \left( \nabla' \frac{1}{R} \right)_{x'} &= \frac{\partial}{\partial x'} \frac{1}{R} = \frac{\partial}{\partial x'} \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{-1/2} \\ &= -\frac{1}{2} \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{-3/2} [-2(x - x')] \\ &= \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{-3/2} (x - x') \end{aligned}$$

$$\begin{aligned} \nabla' \frac{1}{R} &= \frac{(x - x')\bar{\mathbf{a}}_x + (y - y')\bar{\mathbf{a}}_y + (z - z')\bar{\mathbf{a}}_z}{R^3} \\ &= \frac{\bar{\mathbf{R}}}{R^3} \\ &= \frac{\bar{\mathbf{a}}_r}{R^2} \end{aligned}$$



$$(4.68) \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_S(\mathbf{r}') dS'}{|\mathbf{r} - \mathbf{r}'|}$$

$$(4.69) \quad V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_V(\mathbf{r}') dv'}{|\mathbf{r} - \mathbf{r}'|}$$

$$\begin{aligned} V &= \frac{1}{4\pi\epsilon_0} \int_{V'} \left( \nabla' \cdot \frac{\vec{P}}{R} - \frac{\nabla' \cdot \vec{P}}{R} \right) dv' \\ &= \frac{1}{4\pi\epsilon_0} \oint_{S'} \frac{\vec{P} \cdot \vec{a}_n}{R} dS' + \frac{1}{4\pi\epsilon_0} \int_{V'} \left( \frac{-\nabla' \cdot \vec{P}}{R} \right) dv' \end{aligned} \quad (5.26)$$

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n \quad (5.27a)$$

$$\rho_{pv} = -\nabla \cdot \vec{P} \quad (5.27b)$$

### Bound (or polarization) surface and volume charge density

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n \quad (5.27a)$$

$$\rho_{pv} = -\nabla \cdot \vec{P} \quad (5.27b)$$

### Free surface and volume charge density

$$\rho_s = \vec{D} \cdot \vec{a}_n$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$\left( \begin{array}{l} \rho = \nabla \cdot \vec{D} \\ Q = \int \nabla \cdot \vec{D} dv \\ \quad = \oint_S \vec{D} \cdot d\vec{S} \\ \quad = \oint_S \vec{D} \cdot \vec{a}_n dS \\ \rightarrow \rho_s = \vec{D} \cdot \vec{a}_n \end{array} \right.$$

$$Q_b \equiv \oint \vec{P} \cdot d\vec{S} \equiv \int \rho_{ps} dS \quad (\text{bound charge on surface}) \quad (5.28a)$$

$$-Q_b \equiv \int_V \rho_{pv} dv \equiv -\int_V \nabla \cdot \vec{P} dv \quad (\text{charge inside the surface}) \quad (5.28b)$$

$$(5.27) \quad \begin{cases} \rho_{ps} = \vec{P} \cdot \mathbf{a}_n \\ \rho_{pv} = -\nabla \cdot \vec{P} \end{cases}$$

$$\text{Total charge} = \oint_S \rho_{ps} dS + \int_V \rho_{pv} dv = Q_b - Q_b = 0$$

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \epsilon_0 \vec{E} \quad (5.29)$$

$$\begin{aligned} \rho_v &= \nabla \cdot \epsilon_0 \vec{E} - \rho_{pv} \\ &= \nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) \\ &= \nabla \cdot (\epsilon \vec{E}) \\ &= \nabla \cdot \vec{D} \end{aligned} \quad (5.30)$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (5.31)$$

$$\vec{P} = \chi_e \epsilon_0 \vec{E} \quad (5.32)$$

## 5.6 Dielectric Constant and Strength

$$\begin{aligned}\vec{D} &= \varepsilon_0(1 + \chi_e)\vec{E} = \varepsilon_0\varepsilon_r\vec{E} & (5.33) \\ &= \varepsilon_0\vec{E} + \vec{P}\end{aligned}$$

$$\vec{D} = \varepsilon\vec{E} \quad (5.34)$$

$$\varepsilon = \varepsilon_0\varepsilon_r \quad (5.35)$$

dielectric constant      susceptibility      permittivity

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0} \quad (5.36)$$

진공의 permittivity

## 5.7 Linear, Isotropic, and Homogeneous Dielectrics

$$\begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \\ \mathbf{D}_z \end{bmatrix} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{bmatrix} \quad (5.37)$$

$$\vec{\mathbf{F}} = \frac{Q_1 Q_2}{4\pi\epsilon_0\epsilon_r R^2} \vec{\mathbf{a}}_R \quad (5.38)$$

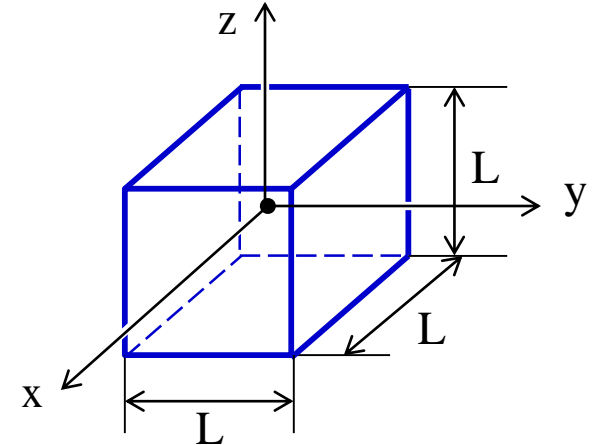
$$W = \frac{1}{2} \int \epsilon_0 \epsilon_r \mathbf{E}^2 dv \quad (5.39)$$

**Ex 5.5** 중심이 원점에 있고, 한 변의 길이가 L인 정육면체에서  $\vec{P} = a\vec{r}$ 로 분극 되었다.  $\rho_{ps}, \rho_{pv}, Q_s, Q_v$  ?

(a) Total Bound Surface Charge  $Q_s$ ?

$\vec{a}_x$  Direction

$$\begin{aligned}
 (5.27a) \quad \rho_{ps} &= \vec{P} \cdot \vec{a}_x \\
 &= a(x\vec{a}_x + y\vec{a}_y + z\vec{a}_z) \cdot \vec{a}_x \Big|_{x=L/2} \\
 &= aL/2
 \end{aligned}$$



$$\begin{aligned}
 Q_s &= \int_s \rho_{ps} dS \\
 &= 6 \left[ \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \rho_{ps} dy dz \right] \\
 &= 3aL^3
 \end{aligned}$$

(b) Total Bound Volume Charge  $Q_v$ ?

$$\begin{aligned}
 (5.27b) \quad \rho_{pv} &= -\nabla \cdot \vec{P} \\
 &= -a \nabla \cdot (x\vec{a}_x + y\vec{a}_y + z\vec{a}_z) \\
 &= -3a
 \end{aligned}$$

$$\begin{aligned}
 Q_v &= \int_v \rho_{pv} dv \\
 &= -3aL^3
 \end{aligned}$$

**Ex 5.6** 평행평판 Capacitor에  $\epsilon_r=2.55$ 의 유전체로 채워져 있다.  
극판 사이의 간격은 1.5mm이고 10 kV/m의 전장이 가해졌다.

(a)  $D = \epsilon_0 \epsilon_r E = \frac{10^{-9}}{36\pi} \times 2.55 \times 10000 = 225.4 \text{ nC/m}^2$

(b)  $P = \chi_e \epsilon_0 E = 1.55 \times \frac{10^{-9}}{36\pi} \times 10000 = 137 \text{ nC/m}^2$

(c) 극판에서의 자유전하의 면전하밀도

$$\rho_s = \vec{D} \cdot \vec{a}_n = \pm D_n = \pm 225.4 \text{ nC/m}^2$$

(d) 분극전하의 면전하밀도

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n = \pm P_n = \pm 137 \text{ nC/m}^2$$

(e) 두 극판 사이의 전위차

$$V = Ed = 10^4 \times 0.0015 = 15 \text{ volt}$$

**Ex 5.7**  $r=0.1\text{m}$ ,  $\epsilon_r=5.7$  인 구 중심에  $2\text{pC}$ 의 전하가 있다. 다음을 구하라.

(a) 구 표면에서의 분극 전하의 면밀도.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2} \vec{a}_r$$

$$\vec{P} = \chi_e \epsilon_0 \vec{E} = \frac{\chi_e Q}{4\pi\epsilon_r r^2} \vec{a}_r$$

$$\rho_{ps} = \vec{P} \cdot \vec{a}_r = \frac{(\epsilon_r - 1)Q}{4\pi\epsilon_r r^2} = \frac{4.7 \times 2 \times 10^{-12}}{4\pi \times 5.7 \times 0.1^2} = 13.12 \text{ pC/m}^2$$

(b) 구 표면의  $-4 \text{ pC}$  전하에 작용하는 힘?

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0\epsilon_r r^2} \vec{a}_r = \frac{-4 \times 2 \times 10^{-24}}{4\pi \times \frac{10^{-9}}{36\pi} \times 5.7 \times 0.1^2} \vec{a}_r = -1.263 \vec{a}_r \text{ pN}$$



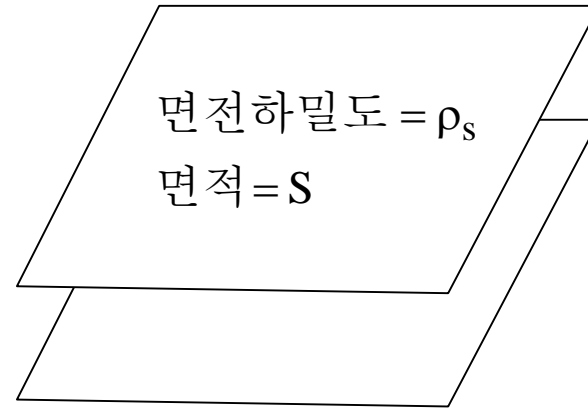
**Ex 5.8** 평행평판 Capacitor의 양 극판에 작용하는 힘을 구하라.  
극판에 작용하는 압력을 구하라.

극판이 무한 평판이라고 가정.

$$\vec{E} = \frac{\rho_s}{2\epsilon} \vec{a}_n$$

$$F = QE = \rho_s S \frac{\rho_s}{2\epsilon} = \frac{\rho_s^2 S}{2\epsilon} = \frac{Q^2}{2\epsilon S}$$

$$p = F/S = \frac{\rho_s^2}{2\epsilon}$$



## 5.8 Continuity Equation and Relaxation Time

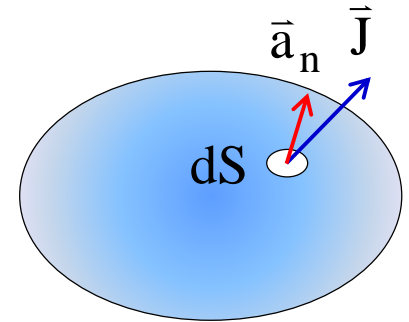
$$I_{\text{out}} = \oint \vec{J} \cdot d\vec{S} = \frac{-\partial Q_{\text{in}}}{\partial t} \quad (5.40)$$

$$\oint_S \vec{J} \cdot d\vec{S} = \int_V \nabla \cdot \vec{J} dv \quad (5.41)$$

$$\frac{-\partial Q_{\text{in}}}{\partial t} = -\frac{\partial}{\partial t} \int_V \rho_v dv = -\int_V \frac{\partial \rho_v}{\partial t} dv \quad (5.42)$$

$$\int_V \nabla \cdot \vec{J} dv = -\int_V \frac{\partial \rho_v}{\partial t} dv$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad (5.43)$$



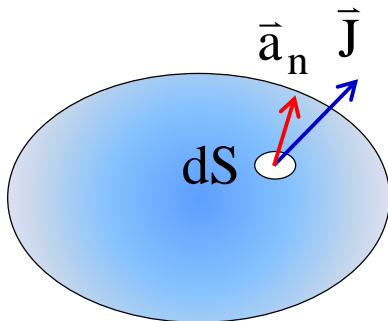
$$\vec{J} = \sigma \vec{E} \quad (5.44)$$

$$\nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon} \quad (5.45)$$

$$\nabla \cdot \sigma \vec{E} = \frac{\sigma \rho_v}{\epsilon}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0 \quad (5.46)$$



$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon} \rho_v = 0 \quad (5.46)$$

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} \partial t \quad (5.47)$$

$$\ln \rho_v = -\frac{\sigma t}{\epsilon} + \ln \rho_{v0}$$

$$\rho_v = \exp\left(-\frac{\sigma t}{\epsilon} + \ln \rho_{v0}\right)$$

$$= \exp(\ln \rho_{v0}) \exp\left(-\frac{\sigma t}{\epsilon}\right)$$

$$= \rho_{v0} \exp\left(-\frac{\sigma t}{\epsilon}\right)$$

$$= \rho_{v0} e^{-t/T_r} \quad (5.48)$$

$$\text{where } T_r = \epsilon / \sigma \quad (5.49)$$

= Relaxation time

$$\rho_v = \rho_{v0} e^{-t/T_r} \quad (5.48)$$

$$\text{where } T_r = \epsilon / \sigma \quad (5.49)$$

= Relaxation time

(Example)

$$\text{Cu } \sigma = 5.8 \times 10^7 \text{ mhos/m, } \epsilon_r = 1$$

$$\begin{aligned} T_r &= \frac{\epsilon_r \epsilon_0}{\sigma} = 1 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{5.8 \times 10^7} \\ &= 1.53 \times 10^{-19} \text{ s} \end{aligned}$$

$$\text{유전층 } \sigma = 10^{-17} \text{ mhos/m, } \epsilon_r = 5.0$$

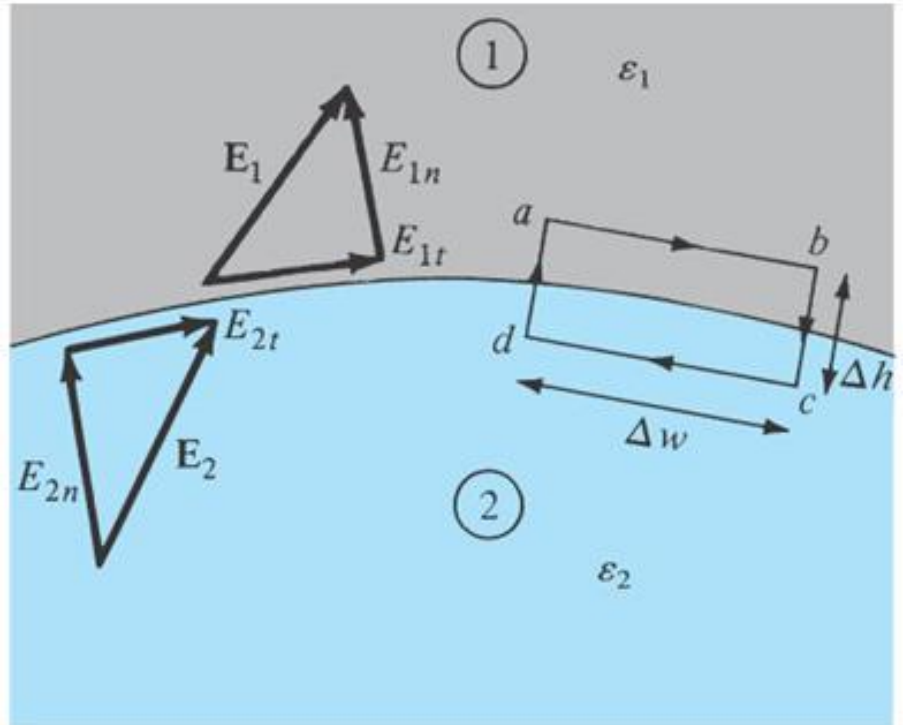
$$\begin{aligned} T_r &= 5 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{10^{-17}} \\ &= 51.2 \text{ days} \end{aligned}$$

## 5.9 Boundary Conditions

$$\begin{aligned}
 & \oint \vec{E} \cdot d\vec{L} \\
 &= \oint (-\nabla V) \cdot d\vec{L} \\
 &= -\oint \left( \frac{\partial V}{\partial x} \vec{a}_x + \frac{\partial V}{\partial y} \vec{a}_y + \frac{\partial V}{\partial z} \vec{a}_z \right) \cdot (dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z) \\
 &= -\oint \left( \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \right) \\
 &= \oint dV \\
 &= 0 \\
 & \oint \vec{E} \cdot d\vec{L} = 0 \tag{5.52}
 \end{aligned}$$

$$\oint \vec{D} \cdot d\vec{S} = Q_{\text{enc}} \tag{5.53}$$

$$\vec{E} = \vec{E}_t + \vec{E}_n \tag{5.54}$$



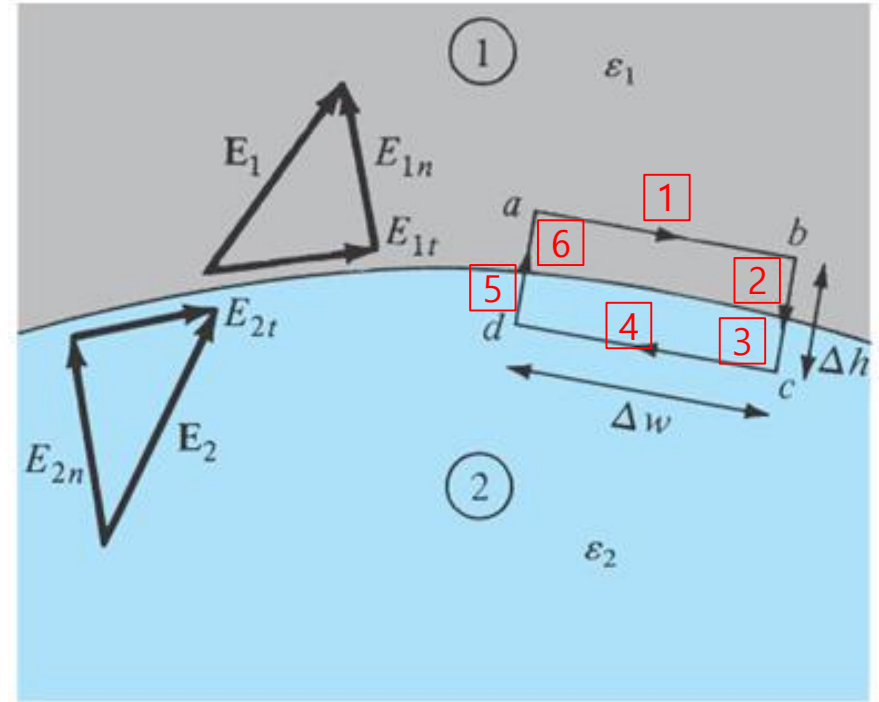
# A. Dielectric-Dielectric Boundary Conditions

$$\vec{E} = \vec{E}_t + \vec{E}_n \quad (5.54)$$

$$\left( \vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} \quad (5.55a) \right.$$

$$\left. \vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} \quad (5.55b) \right.$$

$$\oint \vec{E} \cdot d\vec{L} = 0 \quad (5.52)$$



$$0 = \underbrace{E_{1t}\Delta w}_1 - \underbrace{E_{1n}\frac{\Delta h}{2}}_2 - \underbrace{E_{2n}\frac{\Delta h}{2}}_3 - \underbrace{E_{2t}\Delta w}_4 + \underbrace{E_{2n}\frac{\Delta h}{2}}_5 + \underbrace{E_{1n}\frac{\Delta h}{2}}_6 \quad (5.56)$$

$$\boxed{E_{1t} = E_{2t}} \quad (5.57)$$

$$E_{1t} = E_{2t} \quad (5.57)$$

Since  $\bar{D} = \epsilon \bar{E} = \bar{D}_t + \bar{D}_n$

$$\frac{D_{1t}}{\epsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\epsilon_2}$$

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2} \quad (5.58)$$

$$Q = \oiint \vec{D} \cdot d\vec{S}$$

$$\Delta Q = \rho_S \Delta S = D_{1n} \Delta S - D_{2n} \Delta S \quad \leftarrow \quad (\Delta h \rightarrow 0)$$

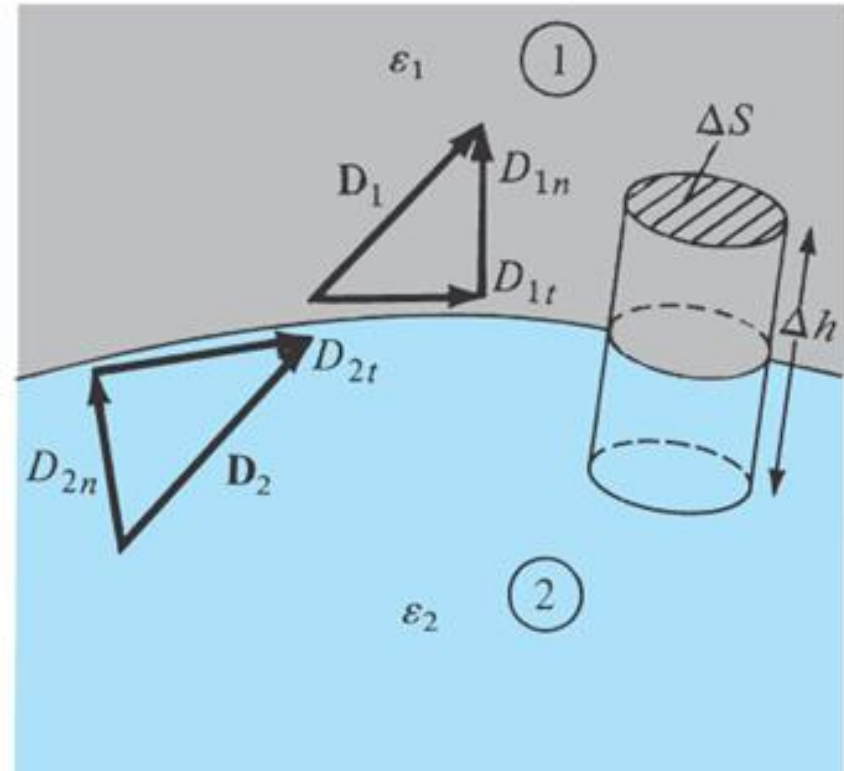
$$D_{1n} - D_{2n} = \rho_S \quad (5.59)$$

$$\rho_S = 0$$

$$D_{1n} = D_{2n} \quad (5.60)$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n} \quad (5.60)$$





$$E_1 \sin \theta_1 = \boxed{E_{1t} = E_{2t}} = E_2 \sin \theta_2$$

$$E_1 \sin \theta_1 = E_2 \sin \theta_2 \quad (5.62)$$

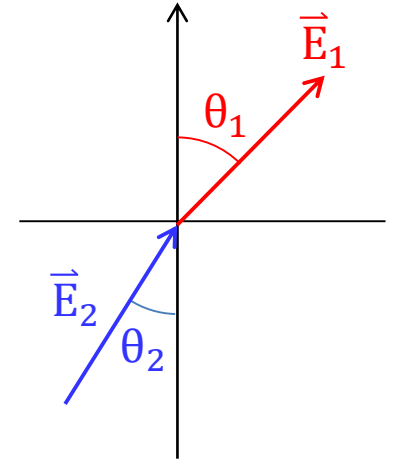
$$\epsilon_1 E_1 \cos \theta_1 = \boxed{D_{1n} = D_{2n}} = \epsilon_2 E_2 \cos \theta_2$$

$$\epsilon_1 E_1 \cos \theta_1 = \epsilon_2 E_2 \cos \theta_2 \quad (5.63)$$

(5.62)/(5.63)  $\rightarrow$

$$\frac{\tan \theta_1}{\epsilon_1} = \frac{\tan \theta_2}{\epsilon_2}$$

$$\boxed{\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}} \quad (5.65)$$



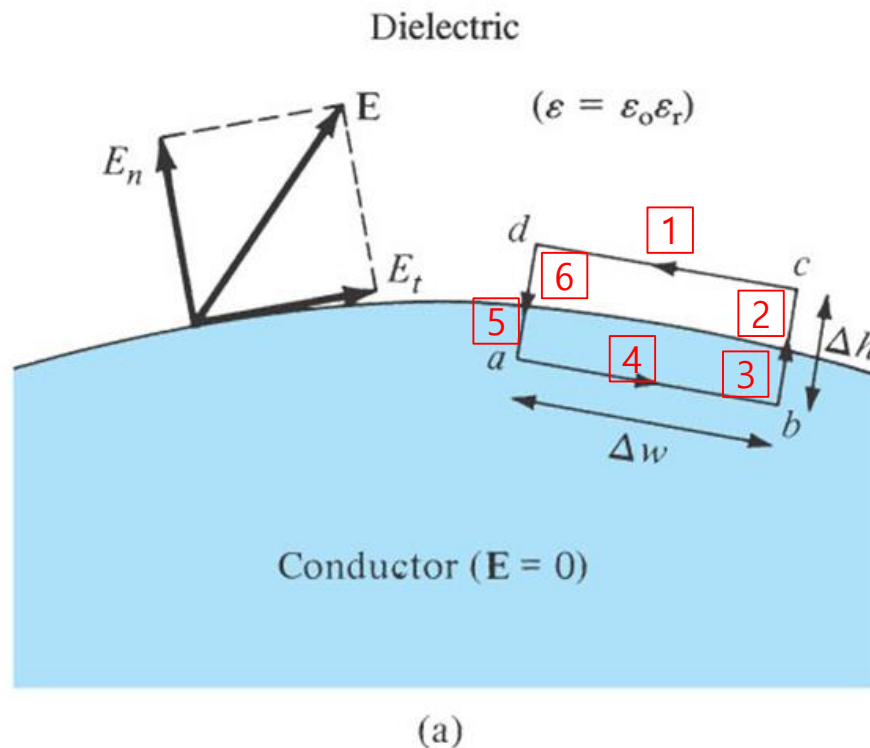
## B. Conductor– Dielectric Boundary Conditions

$$(5.52) \oint \vec{E} \cdot d\vec{L} = 0$$

$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2} \quad (5.66)$$

$$\Delta h \rightarrow 0$$

$$E_t = 0 \quad (5.67)$$

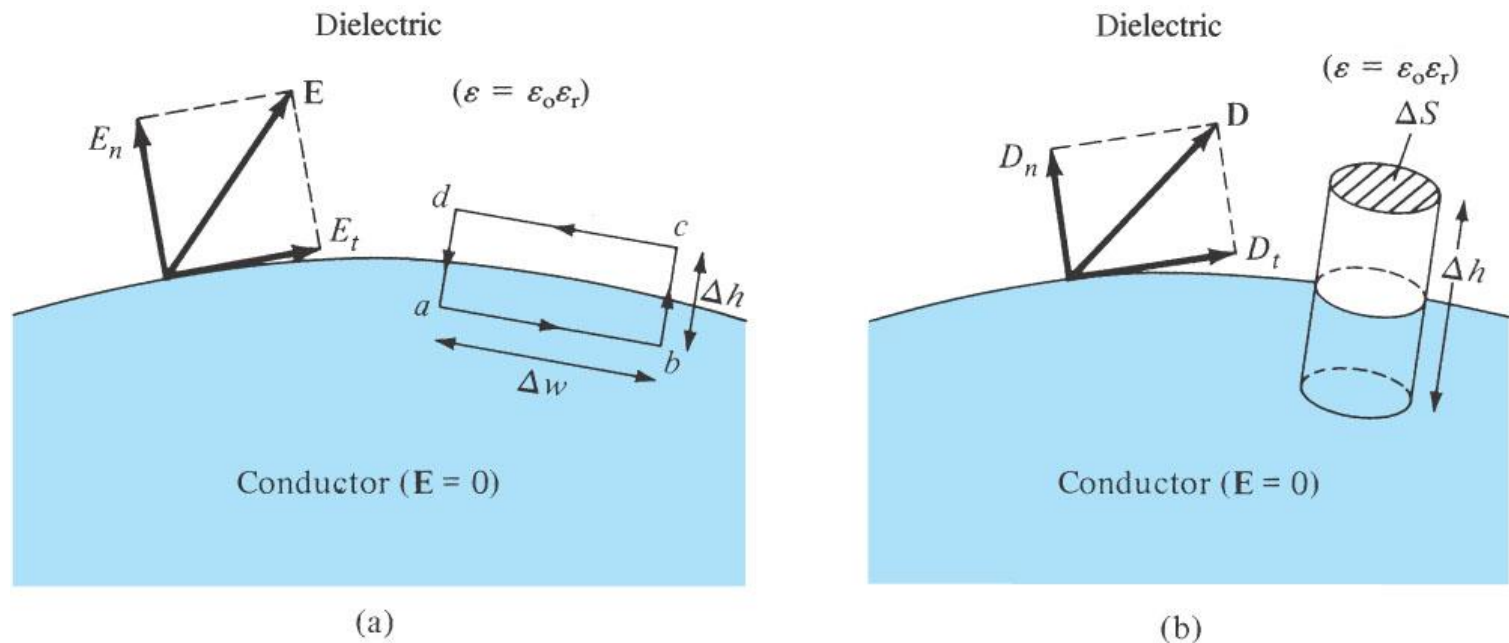


**Figure 5.12** Conductor–dielectric boundary.

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S \quad (5.68)$$

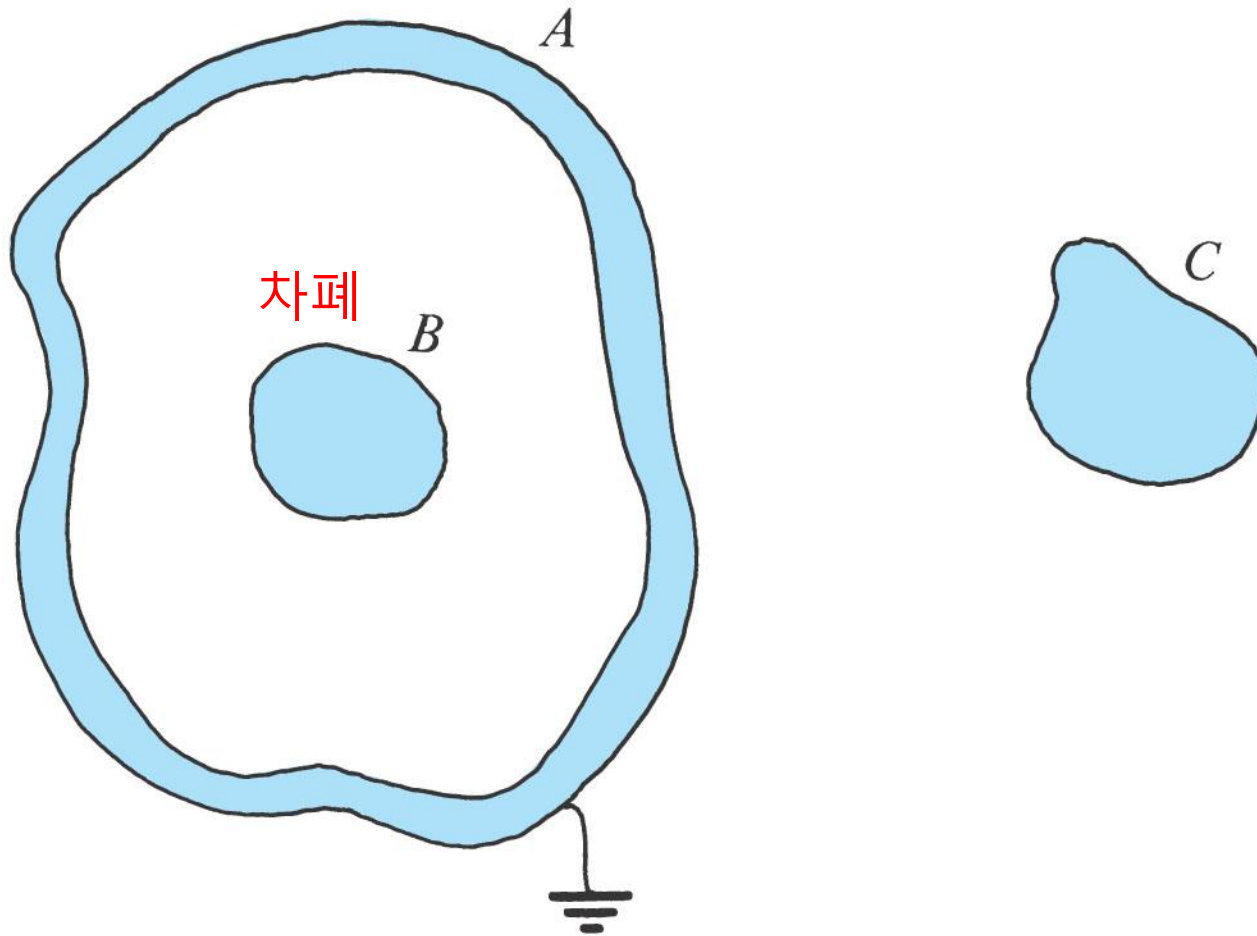
$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S \quad (5.69)$$



**Figure 5.12** Conductor–dielectric boundary.

Electrostatic screening.  
정전 차폐



1. No electric field may exist within a conductor; that is,

$$\rho_v = 0, \quad E = 0 \quad (5.70)$$

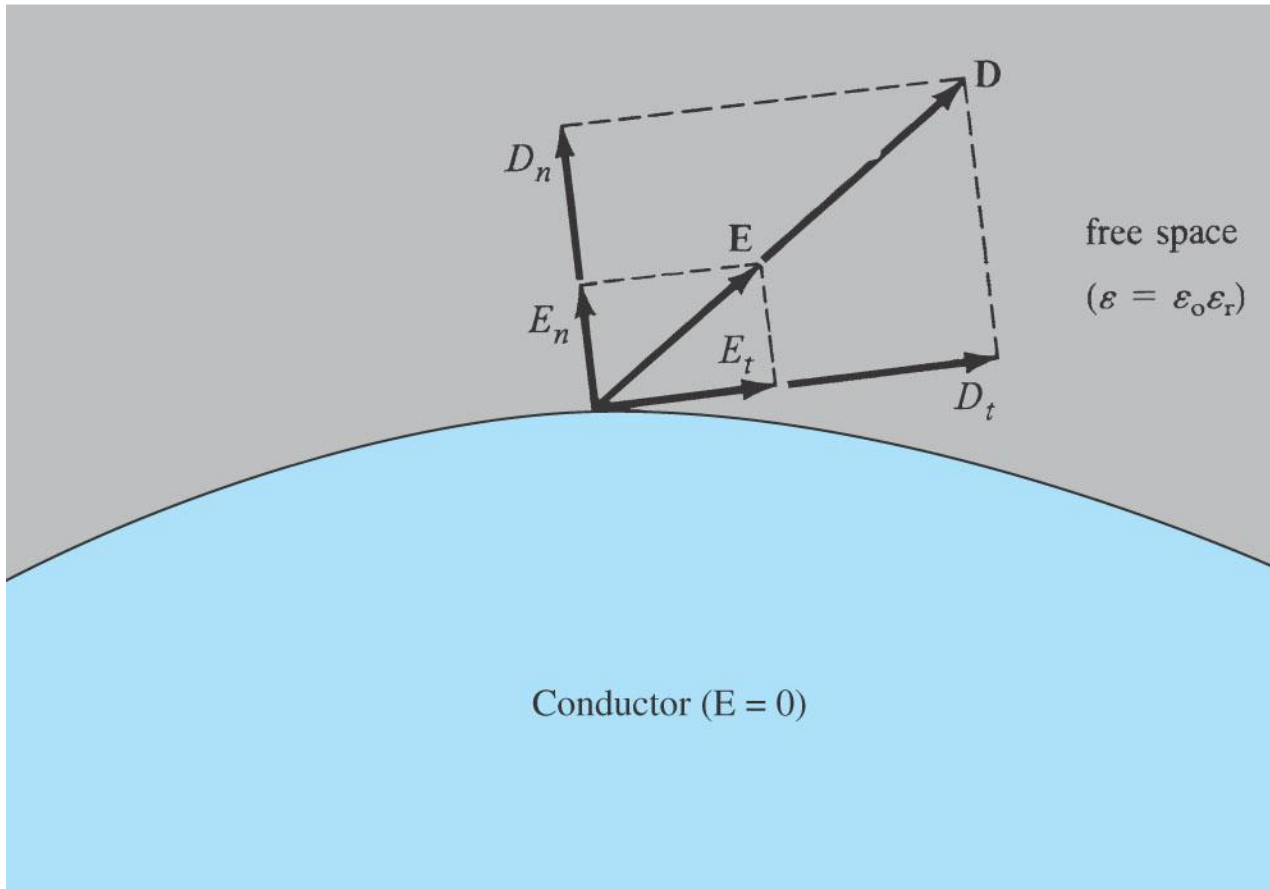
2. Since  $E = -\nabla V = 0$ , there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body

3. The electric field  $E$  can be external to the conductor and normal to its surface; that is

$$D_t = \epsilon_0 \epsilon_r E_t = 0, \quad D_n = \epsilon_0 \epsilon_r E_n = \rho_S \quad (5.71)$$

### C. Conductor-Free Space Boundary Conditions

$$D_t = \epsilon_0 E_t = 0, \quad D_n = \epsilon_0 E_n = \rho_S \quad (5.72)$$



**Figure 5.14** Conductor–free space boundary.

**Ex 5.9** 다음 그림과 같이  $z=0$  평면을 기준으로 두 물질이 분포하고 있다.  
 (1) 영역의 전장  $\vec{E}_1 = (5, -2, 3)$  kV/m 이다.

(a)  $\vec{E}_2$ 를 구하라.

$$\vec{E}_{1n} = (0, 0, 3)$$

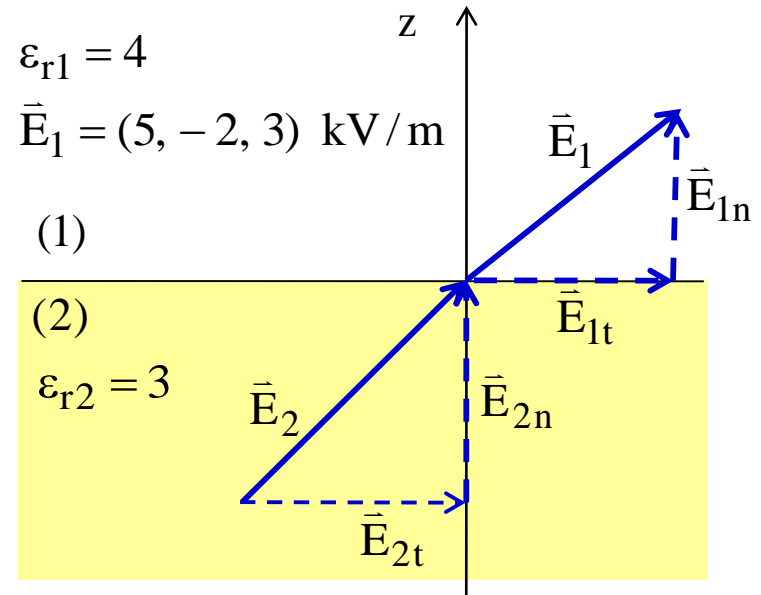
$$\vec{E}_{1t} = (5, -2, 0)$$

$$\vec{E}_{2t} = \vec{E}_{1t} = (5, -2, 0)$$

$$\vec{D}_{2n} = \vec{D}_{1n}$$

$$\vec{E}_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \vec{E}_{1n} = \frac{4}{3} (0, 0, 3) = (0, 0, 4)$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} = (5, -2, 0) + (0, 0, 4) = (5, -2, 4)$$

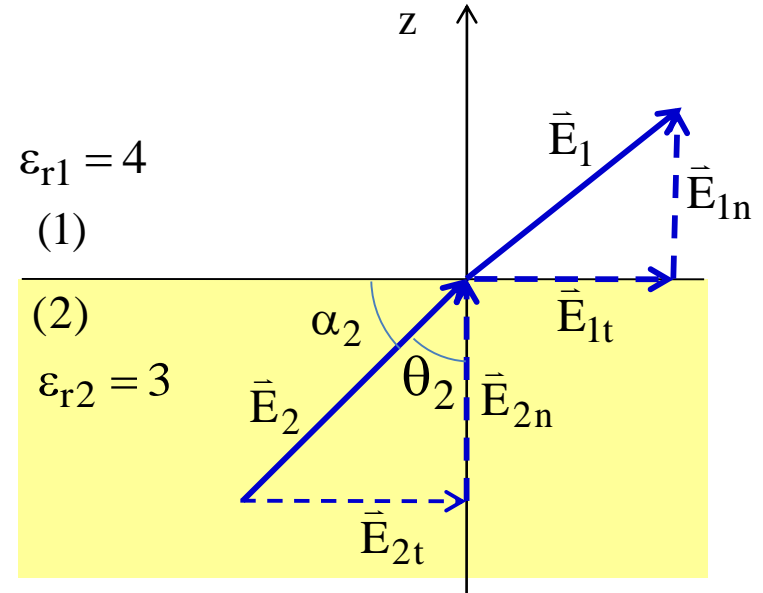


(b)  $\alpha_2$ 를 구하라.

$$\begin{aligned} \sin \alpha_2 &= \frac{|\vec{E}_{2n}|}{|\vec{E}_2|} \\ &= \frac{|(0, 0, 4)|}{|(5, -2, 4)|} \\ &= \frac{4}{\sqrt{25 + 4 + 16}} = 0.5963 \end{aligned}$$

$$\alpha_2 = 36.6^\circ$$

$$\begin{aligned} \vec{E}_{1n} &= (0, 0, 3) \\ \vec{E}_{1t} &= (5, -2, 0) \\ \vec{E}_{2t} &= (5, -2, 0) \\ \vec{E}_{2n} &= (0, 0, 4) \\ \vec{E}_1 &= (5, -2, 3) \\ \vec{E}_2 &= (5, -2, 4) \end{aligned}$$



(c) 두 유전체의 에너지 밀도.

$$w_{E_1} = \frac{1}{2} \epsilon_1 |\vec{E}_1|^2 = \frac{1}{2} \times 4 \times \frac{10^{-9}}{36\pi} \times (25 + 4 + 9) \times 10^6 = 672 \mu\text{J}/\text{m}^3$$

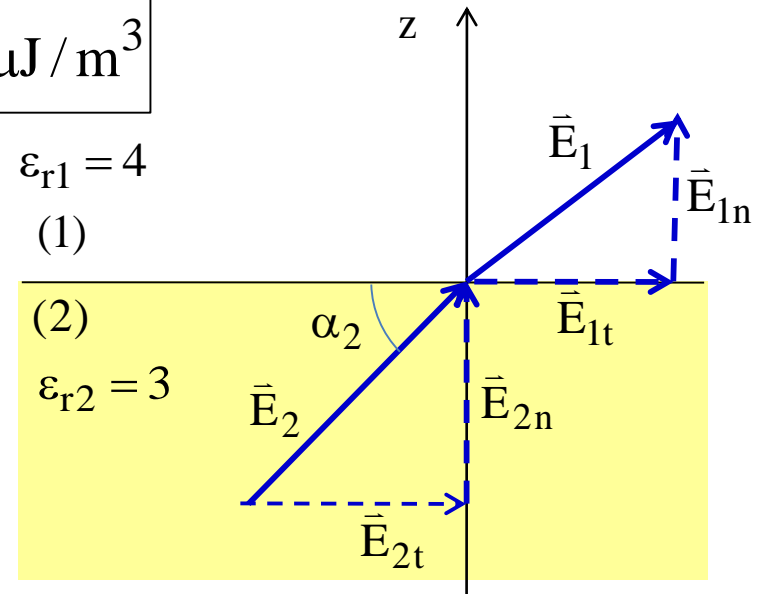
$$w_{E_2} = \frac{1}{2} \epsilon_2 |\vec{E}_2|^2 = \frac{1}{2} \times 3 \times \frac{10^{-9}}{36\pi} \times (25 + 4 + 16) \times 10^6 = 597 \mu\text{J}/\text{m}^3$$



(d) 중심을 (3,4,-5)에 두고 한 변이 2m인 정육면체의 에너지?

$$\begin{aligned}
 W_E &= \int_V w_{E_2} dv \\
 &= \int_{-6}^{-4} \int_3^5 \int_2^4 w_{E_2} dx dy dz \\
 &= w_{E_2} \int_{-6}^{-4} \int_3^5 \int_2^4 dx dy dz \\
 &= 597 \times 2 \times 2 \times 2 \mu\text{J} \\
 &= 4.776 \text{ mJ}
 \end{aligned}$$

$$\begin{aligned}
 w_{E_1} &= 672 \mu\text{J}/\text{m}^3 \\
 w_{E_2} &= 597 \mu\text{J}/\text{m}^3
 \end{aligned}$$



**Ex 5.10** 다음 그림과 같이  $y=0$  평면을 기준으로 초전도체와 유전체가 있다. 도체 표면에  $\rho_s = 2 \text{ nC/m}^2$ 의 전하가 있다.

(a)  $A(3, -2, 2)$ 에서  $\vec{E}$ 와  $\vec{D}$ 를 구하라.  
도체 내 이므로  $\vec{E} = \vec{D} = \mathbf{0}$

(b)  $A(-4, 1, 5)$ 에서  $\vec{E}$ 와  $\vec{D}$ 를 구하라.

$$D_n = \rho_s = 2 \text{ nC/m}^2$$

$$\vec{D} = 2\vec{a}_y \text{ nC/m}^2$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0 \epsilon_r}$$

$$= 2 \times 10^{-9} / \left( 2 \times \frac{10^{-9}}{36\pi} \right) \vec{a}_y$$

$$= 113.1 \vec{a}_y \text{ V/m}$$

