5. Electric Fields in Material Space

5.2 매질의 성질

	재료	전도율 σ (S/m)
도체	Ag Cu Al Au	6.1×10^{7} 5.8×10^{7} 3.5×10^{7} 4.1×10^{7}
반도체	Ge Si	2.2 4.4×10^{-4}
절연체	Glass	10^{-12}

5.3 Convection and Conduction Currents

σ: 전기전도율, 도전율, Conductivity
단위: Ω^{-1}/m, Siemen/m, Mho/m
R=L/(Aσ)

Current : 단위 시간에 흐르는 전하량 (단위: C/sec).
- **convection current**
- **conduction current**
- **displacement current**

$$\nabla \times \frac{\vec{B}}{\mu} = \boxed{\varepsilon \frac{\partial \vec{E}}{\partial t}} + \vec{J} : \text{Ampere's Eq}$$

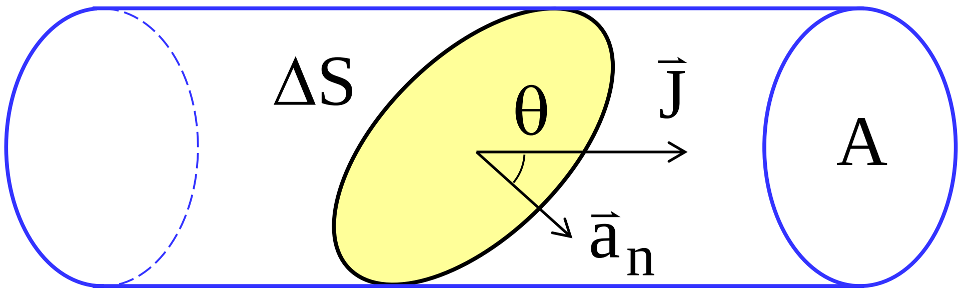
$$I = \frac{dQ}{dt} \; [C/sec] \qquad (5.1)$$

$$J_n = \frac{\Delta I}{\Delta S} \; [C/sec/m^2]$$

$$\Delta I = J_n \Delta S \qquad (5.2)$$

$$\Delta I = \vec{J} \cdot \Delta\vec{S} \qquad (5.3)$$

$$I = \int_S \vec{J} \cdot d\vec{S} \qquad (5.4)$$



$$A = \cos\theta \Delta S$$

$$I = JA = \vec{J} \cdot \vec{a}_n \Delta S$$

Convection current : 공간을 자유롭게 움직이는 전자의 운동에 의한 전류.

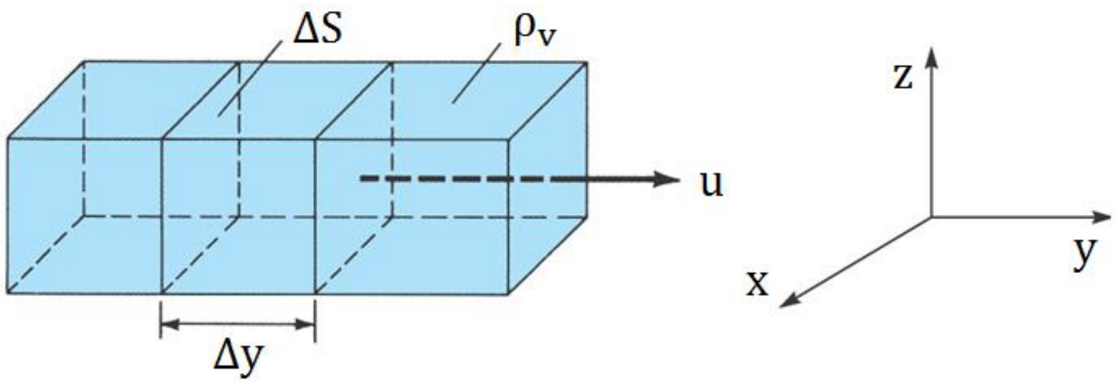


Figure 5.1 Current in a filament.

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta S \Delta y}{\Delta t} = \rho_v \Delta S \frac{\Delta y}{\Delta t} = \rho_v \Delta S u_y \qquad (5.5)$$

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y \qquad (5.6)$$

$$\vec{J} = \rho_v \vec{u} \qquad (5.7)$$

: Convection Current Density [A/sec/m^2]

Conduction current: 금속과 같은 물질에 구속된 전자들에 의해 전달되는 전류.

Electron Momentum Eq :

$$\vec{F} = \cancel{mn_e \frac{d\vec{u}}{dt}} = \boxed{-en_e(\vec{E}} + \cancel{\vec{u}\times\vec{B}}) \boxed{- \frac{mn_e\vec{u}}{\tau}}$$

$$\frac{mn_e\vec{u}}{\tau} = -en_e\vec{E}$$

$$\frac{m\boxed{(-en_e\vec{u})}}{\tau} = e^2 n_e \vec{E}$$

$$\boxed{\vec{J}_e} = \frac{e^2\tau n_e}{m}\vec{E}$$

$$\vec{J}_e = \sigma\vec{E} : \text{Ohm's Law} \qquad (5.11)$$

$$\text{where } \sigma \equiv \frac{e^2\tau n_e}{m}$$

$\sigma = \dfrac{e^2\tau n_e}{m}$: Conductivity

$\tau =$ Collision Time

$\nu = 1/\tau$: Collision Frequency

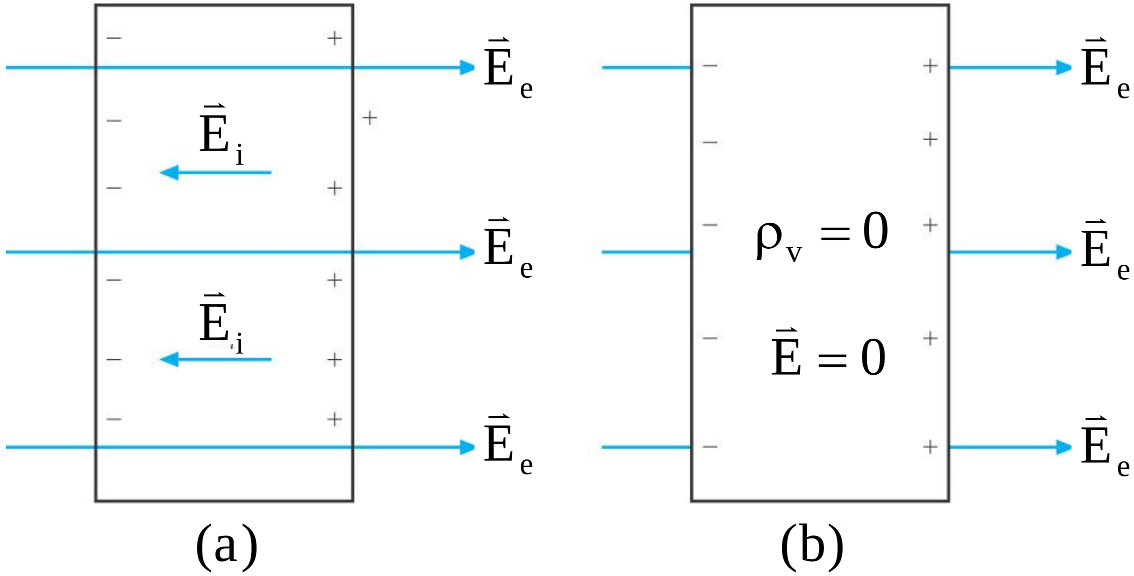
$n_e =$ Electron Density

$m =$ Electron Mass

5.4 Conductors

A perfect conductor cannot contain an electrostatic field within it.

$\vec{E} = -\nabla V = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0 \qquad$ inside a conductor



Floating Conductor

Conductor 내부

Resistivity 정의

$$E = \frac{V}{L} \qquad (5.13)$$

$$J = \frac{I}{S} \qquad (5.14)$$

$$= \sigma E = \frac{\sigma V}{L} \qquad (5.15)$$

$$R = \frac{V}{I} = \frac{L}{\sigma S}$$

$$= \frac{\rho_c L}{S} \qquad (5.16)$$

$$\begin{cases} \rho_c = 1/\sigma : \text{ Resistivity} \\ \text{unit} : \ \Omega \cdot \text{m} \end{cases}$$

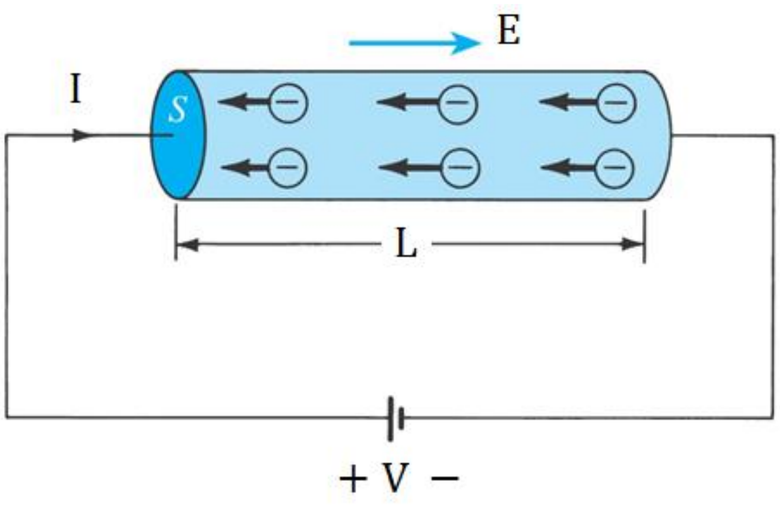
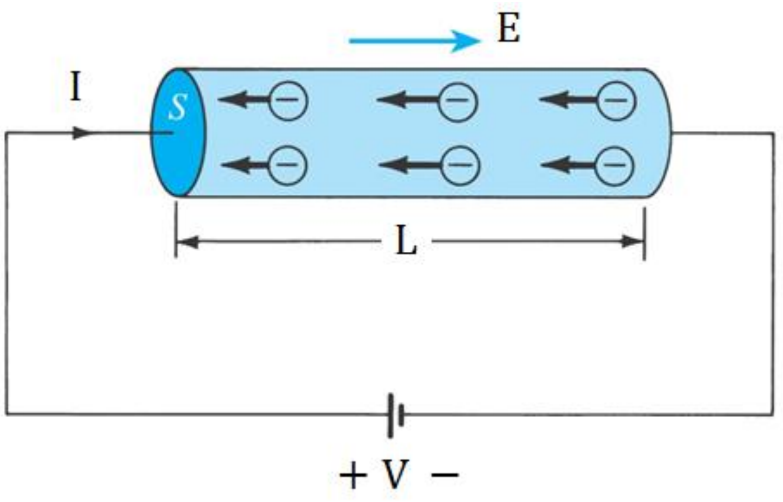


Figure 5.3 A conductor of uniform cross section under an applied **E** field. Conductor has a uniform cross section S and is of length L.

불균일한 단면적을 가진 전선의 Resistance 정의

$$R = \frac{V}{I} = \frac{\int \vec{E}\cdot d\vec{L}}{\int \sigma\vec{E}\cdot d\vec{S}} = \frac{\int \vec{E}\cdot d\vec{L}}{\int \vec{J}\cdot d\vec{S}} \qquad (5.17)$$



Power의 정의

$$dW = -\vec{F} \cdot d\vec{L}$$

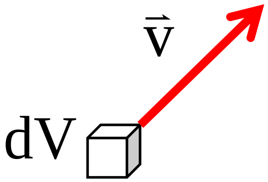
$$P \equiv \frac{dW}{dt} = -\vec{F} \cdot \frac{d\vec{L}}{dt} = -\vec{F} \cdot \vec{v}$$

$$\begin{cases} \text{unit} : [F] = kg \cdot m/sec^2 & = \text{Newton} : \text{Force} \\ \quad [W] = kg \cdot m^2/sec^2 & = \text{Joule} : \text{Work} \\ \quad [P] = kg \cdot m^2/sec^2/sec & \\ \qquad = J/sec & = \text{Watt} : \text{Power} \end{cases}$$

Joule's Law : Ohmic Power Dissipation

$$
\begin{aligned}
dP &= (dQ\vec{E})\cdot\vec{v} \\
&= (\rho_v\,dxdydz)\vec{E}\cdot\vec{v} \\
&= \vec{E}\cdot(\rho_v\vec{v})dxdydz \\
&= \vec{E}\cdot\vec{J}dxdydz
\end{aligned}
$$

$$P = \iiint(\vec{E}\cdot\vec{J})dV \qquad (5.18)$$



$$dQ = \rho_v dV$$

$$w_P \equiv \frac{dP}{dV} = \vec{E}\cdot\vec{J} = \sigma\left|\vec{E}\right|^2 \qquad (5.19)$$

: Power Density

균일한 단면적을 가진 전기선

$$
\begin{aligned}
P &= \int_L E\,dL\int_S J\,dS = VI \\
&= I^2R \qquad (5.20)
\end{aligned}
$$

Ex 5.1 $\vec{J} = r^{-3}(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)$ 일 때 다음 단면을 흐르는 전류는?

(a) 반경 20 cm 의 반구 껍데기

$$\iint \vec{J}\cdot\vec{a}_r dS$$

$$= \iint \vec{J}\cdot\vec{a}_r (rd\theta)(r\sin\theta d\phi)$$

$$= \int_0^{\pi/2}\int_0^{2\pi}\frac{1}{r^3}(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)\cdot\vec{a}_r(r^2\sin\theta d\phi d\theta)$$

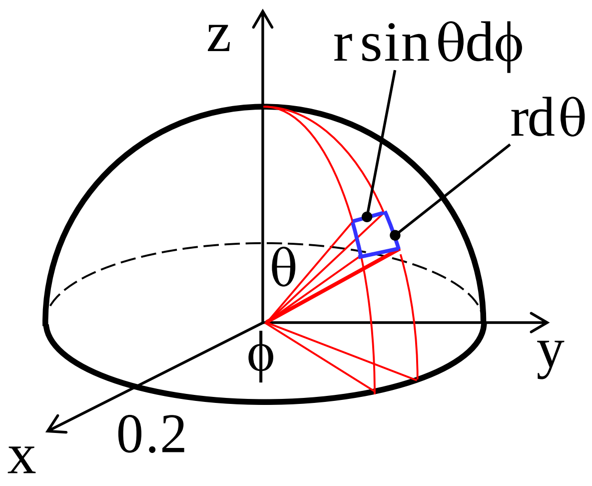
$$= \int_0^{\pi/2}\int_0^{2\pi}\frac{1}{r^3}(2\cos\theta)(r^2\sin\theta d\phi d\theta)$$

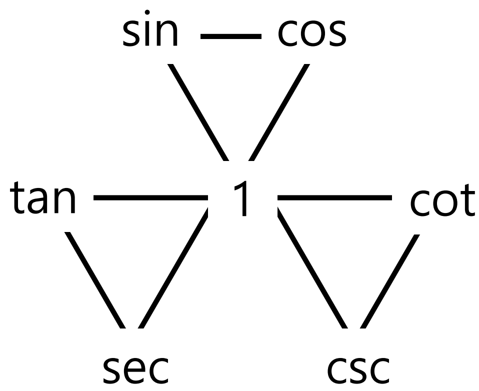
$$= 2\pi\int_0^{\pi/2}\frac{1}{r^3}(2\cos\theta)(r^2\sin\theta)d\theta \qquad (r = 0.2\ \text{m})$$

$$= 10\pi\int_0^{\pi/2}\sin 2\theta d\theta$$

$$= 10\pi\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= 10\pi\ \text{A}$$





$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\boxed{\sin(2\alpha)=2\sin\alpha\cos\alpha}$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\frac{d\sin\theta}{d\theta}=\cos\theta$$

$$\frac{d\cos\theta}{d\theta}=-\sin\theta$$

$$\frac{d\tan\theta}{d\theta}=\sec^2\theta$$

(b) 반경 10 cm 의 구각

$$\iint \vec{J}\cdot\vec{a}_r (rd\theta)(r\sin\theta d\phi)$$

$$= \int_0^{\pi}\int_0^{2\pi}\frac{1}{r^3}(2\cos\theta\vec{a}_r + \sin\theta\vec{a}_\theta)\cdot\vec{a}_r (r^2\sin\theta d\phi d\theta)$$

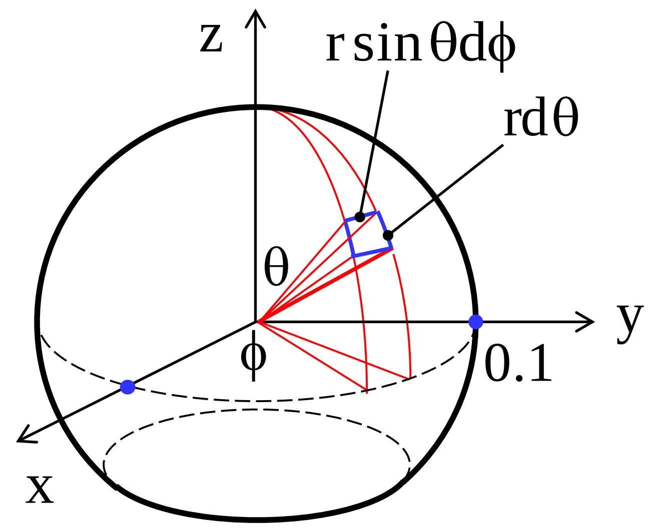
$$= \int_0^{\pi}\int_0^{2\pi}\frac{1}{r^3}(2\cos\theta)(r^2\sin\theta d\phi d\theta)$$

$$= 2\pi\int_0^{\pi}\frac{1}{r^3}(2\cos\theta)(r^2\sin\theta)d\theta \qquad (r = 0.1\ \text{m})$$

$$= 20\pi\int_0^{\pi}\sin 2\theta d\theta$$

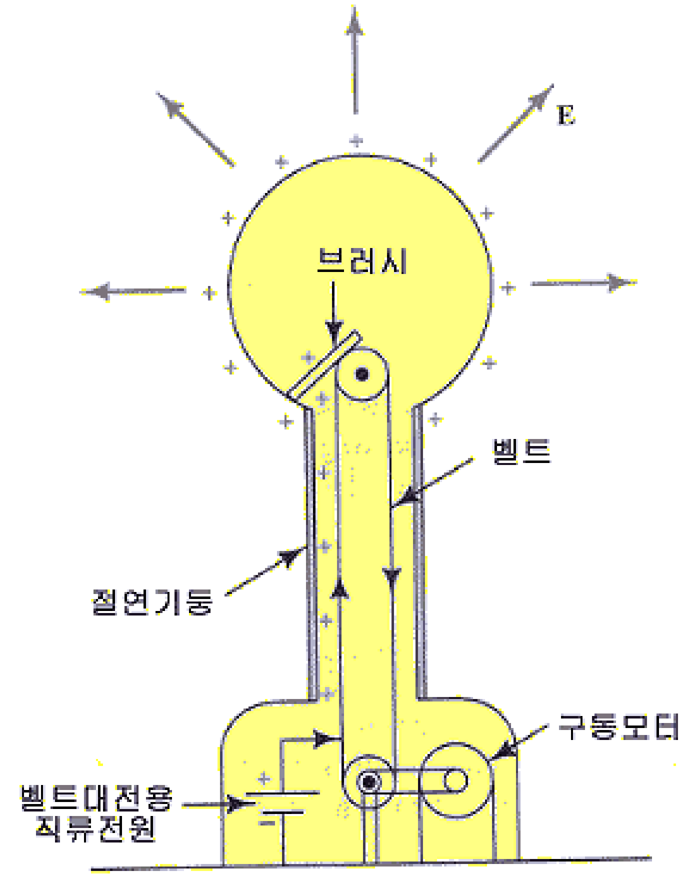
$$= 20\pi\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi}$$

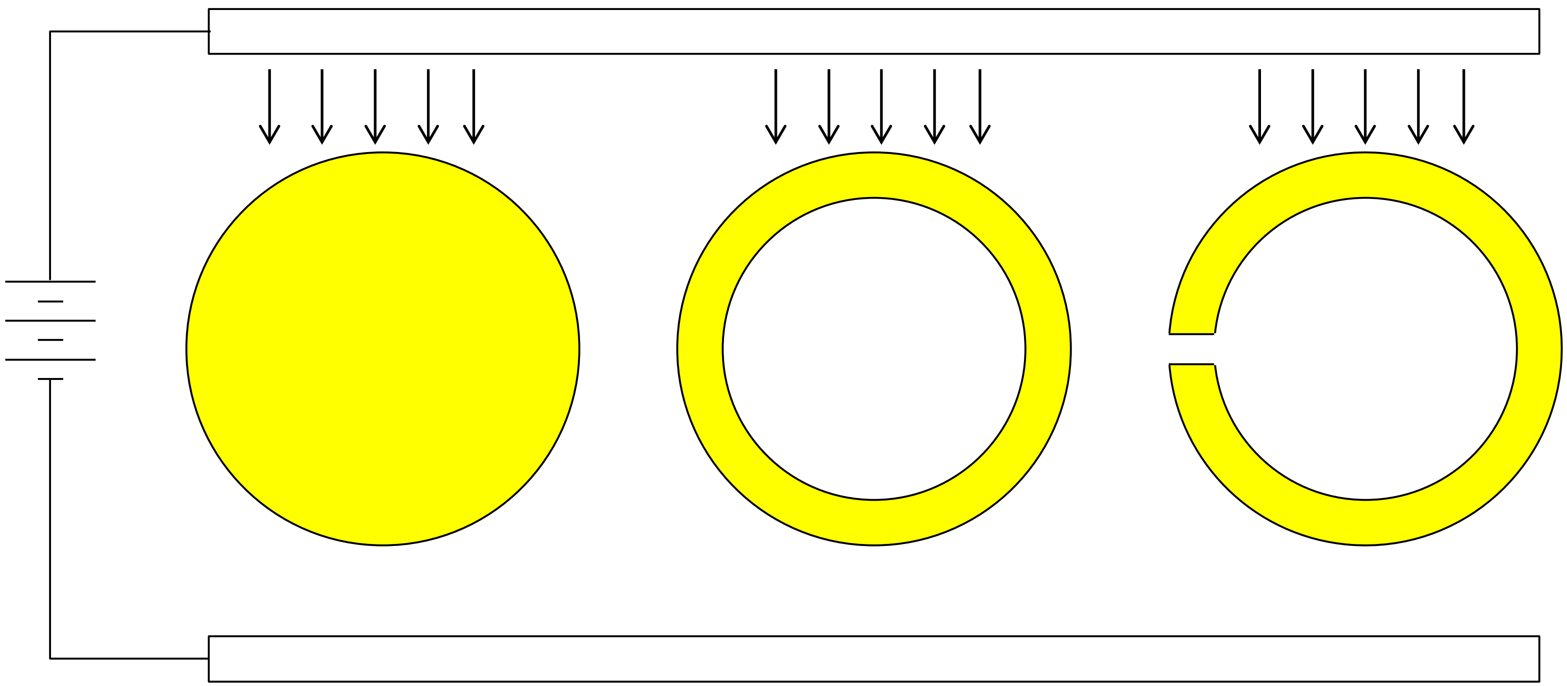
$$= 0$$



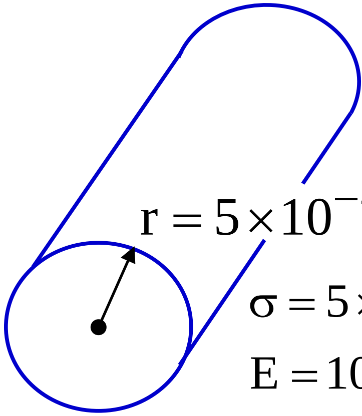
Ex 5.2 Van de Graaff 발전기에서 $\rho_S = 10^{-7} C/m^2$ 인 전하가 $u = 2m/s$의 속도로 폭 $w = 10cm$인 Belt가 이동할 때 5초 동안 축적된 전하는?

$$Q = It$$
$$= \rho_S uwt$$
$$= 10^{-7}[C/m^2] \times 2[m/sec] \times 0.1[m] \times 5[sec]$$
$$= 100\ nC$$





Ex 5.3 **직경 1mm** 인 다음 그림의 전선에 **10 mV/m**의 전장을 가했을 때 자유전자의 수 $\mathbf{n=10^{29}\ /m^3}$ 이었다. $\boldsymbol{\sigma = 5 \times 10^7/\Omega m}$.



$r = 5\times10^{-4}$ m

$\sigma = 5\times10^{-7}[S/m = 1/\Omega m]$

$E = 10\ [mV/m]$

(a) 자유전자에 의한 전하 밀도 ρ_v?

$$\begin{aligned}\rho_v &= nQ_e \\ &= (10^{29}\ \#/m^3)\times(-1.6\times10^{-19}\ C) \\ &= -1.6\times10^{10}\ C/m^3\end{aligned}$$

(b) 전류밀도 J?

$$\begin{aligned}J &= \sigma E \\ &= (5\times10^{7}\ [1/\Omega m])\times(10^{-2}\ [V/m]) \\ &= 500\ kA/m^2\end{aligned}$$

(c) 전류 I?

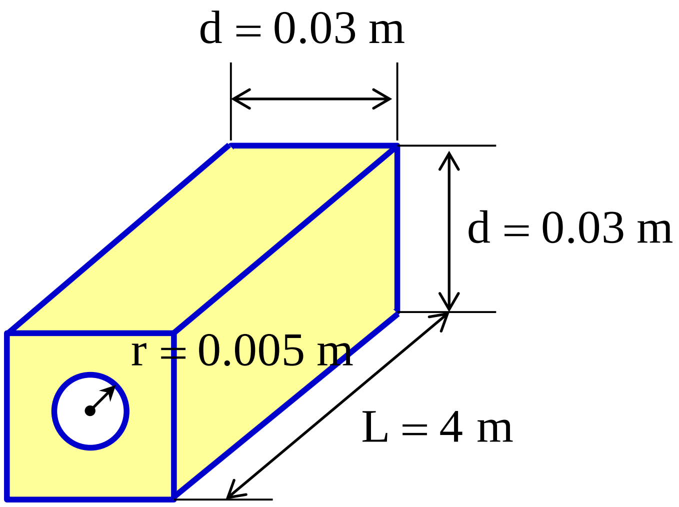
$$\begin{aligned}I &= JS = J\pi r^2 \\ &= (5\times10^{5}\ [A/m^2])\times\pi 0.0005^2\ [m^2] \\ &= 0.393\ \ A\end{aligned}$$

(d) 전자 속도 u?

$$J = \rho_v[C/m^2]\times u[m/sec]$$

$$\begin{aligned}u &= J/\rho_v \\ &= \frac{5\times10^5}{1.6\times10^{10}} \\ &= 3.125\times10^{10} \\ &= 0.393\ \ [m/sec]\end{aligned}$$

Ex 5.4 전도율 **σ=5x10⁶ S/m**, 길이 **L=4 m** 일 때 가운데 둥근 구멍이 있는 다음 그림의 양단의 저항은?



$$R = \frac{L}{\sigma S}$$

$$\left(\begin{aligned} S &= d^2 - \pi r^2 \\ &= 0.03^2 - \pi(0.5\times10^{-2})^2 \ \ \mathrm{m}^2 \\ &= (9-\pi/4)\times10^{-4} \ \ \mathrm{m}^2 \end{aligned} \right)$$

$$= \frac{4}{(5\times10^6)\times(9-\pi/4)\times10^{-4}}$$

$$= 974\ \mu\Omega$$

5.5 Polarization in Dielectrics

$$\vec{p} = Q\vec{d} \quad \text{(dipole)} \quad [C \cdot m] \qquad (5.21)$$

$$Q_1\vec{d}_1 + Q_2\vec{d}_2 + \cdot \ \cdot \ \cdot + Q_n\vec{d}_n = \sum_{k=1}^{N} Q_k\vec{d}_k \qquad (5.22)$$

$$\vec{P} = \frac{\lim\limits_{\Delta v \to 0} \sum\limits_{k=1}^{N} Q_k\vec{d}_k}{\Delta v} \quad [C/m^2] \qquad (5.23)$$

(**Polarization** $\vec{P}$ is defined as the dipole moment per unit volume)

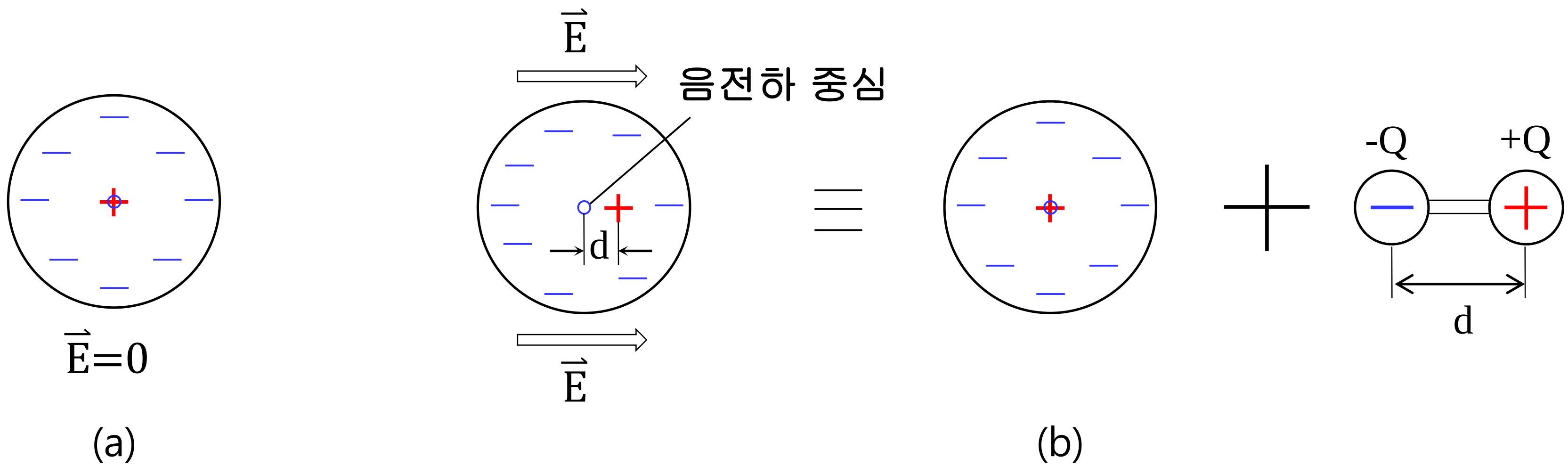


Figure 5.6 Polarization of a nonpolar atom or molecule.

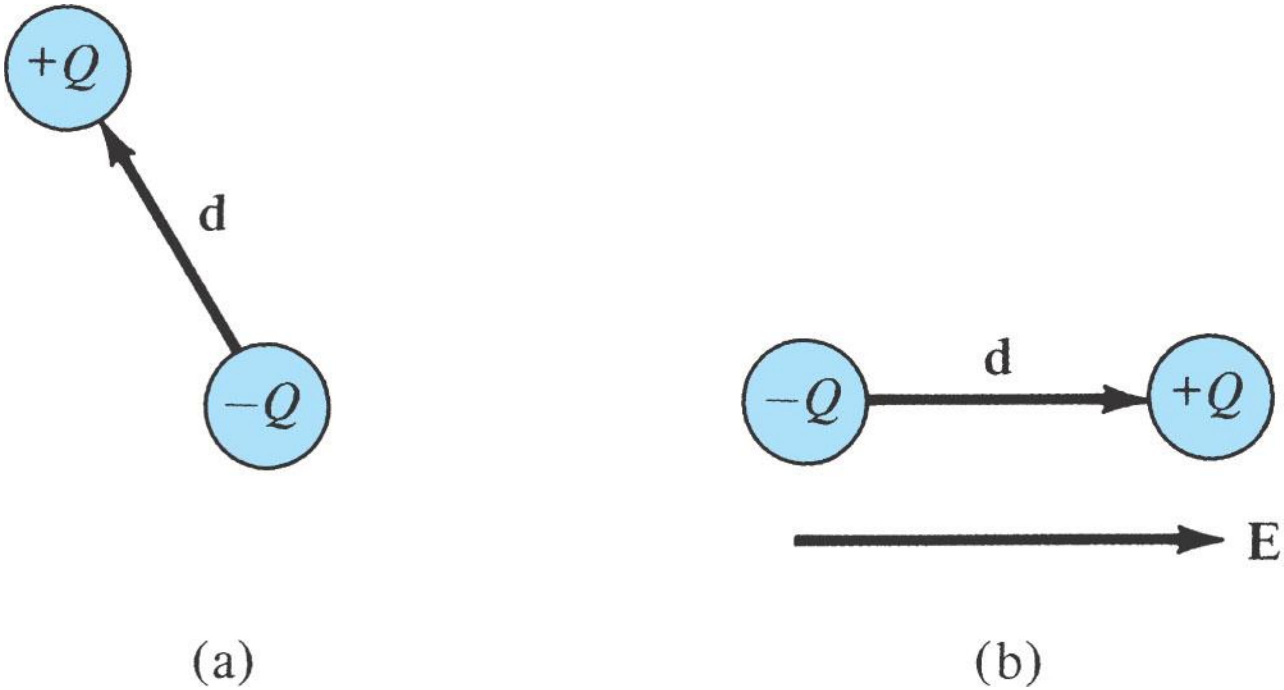
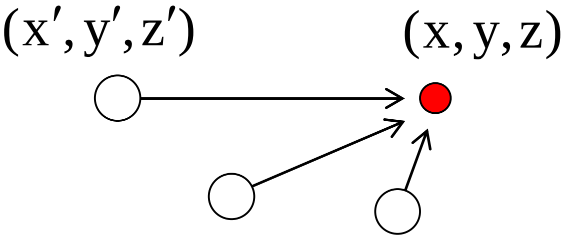


Figure 5.7 Polarization of a polar molecule:
(a) permanent dipole ($\vec{E}$=0), **(b)** induced dipole ($\vec{E} \neq 0$).



$$V = \frac{\vec{p}\cdot\vec{a}_r}{4\pi\varepsilon_0 r^2} \qquad (4.80)$$

$$dV = \frac{\vec{P}\cdot\vec{a}_r dv'}{4\pi\varepsilon_0 R^2} \qquad (5.24)$$

$$\text{where } R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\vec{a}_r dv'}{R^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\vec{P}\cdot\nabla'\frac{1}{R}dv'$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\nabla'\cdot\frac{\vec{P}}{R} - \frac{\nabla'\cdot\vec{P}}{R}\right)dv' \qquad \leftarrow \left(\nabla'\cdot\frac{\vec{P}}{R} = \frac{\nabla'\cdot\vec{P}}{R} + \vec{P}\cdot\nabla'\frac{1}{R}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{v'}\left(\nabla'\cdot\frac{\vec{P}}{R} - \frac{\nabla'\cdot\vec{P}}{R}\right)dv'$$

참고

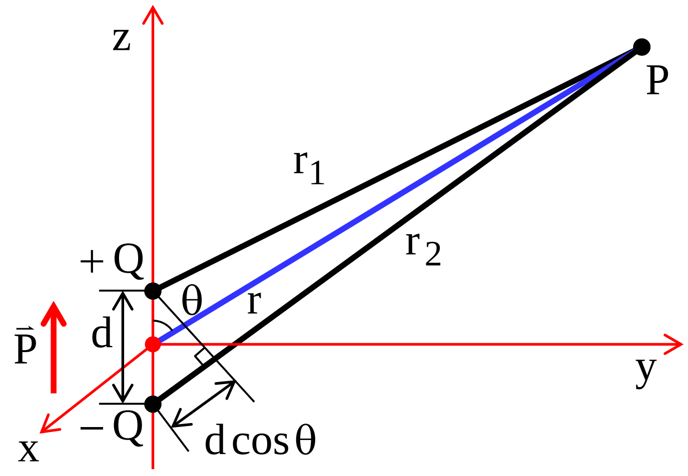
$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right] \quad (4.77)$$

$$V = \frac{\boxed{Q \quad d\cos\theta}}{4\pi\varepsilon_0 \quad r^2} \quad (4.78)$$

$$\vec{p} \equiv Q\vec{d}(\mathrm{Cm}) \quad \text{(Dipole moment)} \quad (4.79)$$

$$V = \frac{\vec{p}\cdot\vec{a}_r}{4\pi\varepsilon_0 r^2} \quad (4.80)$$

$$V(r) = \frac{\vec{p}\cdot(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^3} \quad (4.81)$$



$$\nabla' \frac{1}{R} = \frac{\vec{a}_r}{R^2}$$ 유도

$$R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$$

$$\left(\nabla' \frac{1}{R}\right)_{x'} = \frac{\partial}{\partial x'} \frac{1}{R} = \frac{\partial}{\partial x'} \left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{-1/2}$$

$$= -\frac{1}{2}\left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{-3/2}\left[-2(x - x')\right]$$

$$= \left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{-3/2}(x - x')$$

$$\nabla' \frac{1}{R} = \frac{(x - x')\vec{a}_x + (y - y')\vec{a}_y + (z - z')\vec{a}_z}{R^3}$$

$$= \frac{\vec{R}}{R^3}$$

$$= \frac{\vec{a}_r}{R^2}$$

$$(4.68)\quad V(r) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S(r')dS'}{|r-r'|}$$

$$(4.69)\quad V(r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_v(r')dv'}{|r-r'|}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{v'}\left(\nabla'\cdot\frac{\vec{P}}{R} - \frac{\nabla'\cdot\vec{P}}{R}\right)dv'$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_{S'}\frac{\boxed{\vec{P}\cdot\vec{a}_n}}{R}dS' + \frac{1}{4\pi\varepsilon_0}\int_{v'}\left(\frac{\boxed{-\nabla'\cdot\vec{P}}}{R}\right)dv' \qquad (5.26)$$

$$\boxed{\rho_{ps} = \vec{P}\cdot\vec{a}_n} \qquad (5.27a)$$

$$\boxed{\rho_{pv} = -\nabla\cdot\vec{P}} \qquad (5.27b)$$

Bound (or polarization) surface and volume charge density

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n \qquad (5.27a)$$

$$\rho_{pv} = -\nabla \cdot \vec{P} \qquad (5.27b)$$

Free surface and volume charge density

$$\rho_s = \vec{D} \cdot \vec{a}_n$$

$$\rho_v = \nabla \cdot \vec{D}$$

$$\left(\begin{aligned} &\rho = \nabla \cdot \vec{D} \\ &Q = \int \nabla \cdot \vec{D} dv \\ &\quad = \oint_S \vec{D} \cdot d\vec{S} \\ &\quad = \oint_S \vec{D} \cdot \vec{a}_n dS \\ &\rightarrow \rho_s = \vec{D} \cdot \vec{a}_n \end{aligned} \right.$$

$$Q_b \equiv \oint \vec{P} \cdot d\vec{S} \equiv \int \rho_{ps} dS \qquad \text{(bound charge on surface)} \quad (5.28a)$$

$$-Q_b \equiv \int_v \rho_{pv} dv \equiv -\int_v \nabla \cdot \vec{P}\, dv \quad \text{(charge inside the surface)} \quad (5.28b)$$

$$(5.27) \begin{cases} \rho_{ps} = \vec{P} \cdot a_n \\ \rho_{pv} = -\nabla \cdot \vec{P} \end{cases}$$

$$\text{Total charge} = \oint_S \rho_{ps} dS + \int_v \rho_{pv} dv = Q_b - Q_b = 0$$

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_0 \vec{E} \quad (5.29)$$

$$\begin{aligned} \rho_v &= \nabla \cdot \varepsilon_0 \vec{E} - \rho_{pv} \\ &= \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) \\ &= \nabla \cdot (\varepsilon \vec{E}) \\ &= \nabla \cdot \vec{D} \qquad (5.30) \end{aligned}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad (5.31)$$

$$\vec{P} = \chi_e \varepsilon_0 \vec{E} \qquad (5.32)$$

5.6 Dielectric Constant and Strength

$$\vec{D} = \varepsilon_0(1+\chi_e)\vec{E} = \varepsilon_0\varepsilon_r\vec{E} \qquad (5.33)$$

$$= \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon\vec{E} \qquad (5.34)$$

$$\varepsilon = \varepsilon_0\varepsilon_r \qquad (5.35)$$

dielectric constant

susceptibility

permittivity

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0} \qquad (5.36)$$

진공의 permittivity

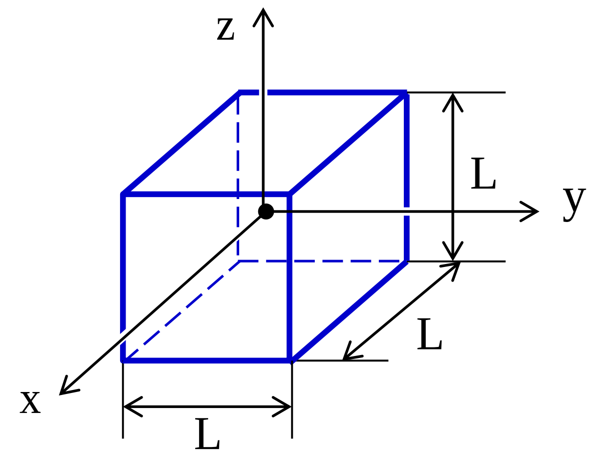
5.7 Linear, Isotropic, and Homogeneous Dielectrics

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (5.37)$$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r R^2}\vec{a}_R \quad (5.38)$$

$$W = \frac{1}{2}\int \varepsilon_0\varepsilon_r E^2 dv \quad (5.39)$$

Ex 5.5 중심이 원점에 있고, 한 변의 길이가 L인 정육면체에서 $\vec{P} = a\vec{r}$로 분극 되었다. ρ_{ps}, ρ_{pv}, Q_S, Q_v ?



(a) Total Bound Surface Charge Q_s?

$\vec{a}_x$ Direction

(5.27a) $\rho_{ps} = \vec{P} \cdot \vec{a}_x$

$$= a(x\vec{a}_x + y\vec{a}_y + z\vec{a}_z) \cdot \vec{a}_x \Big|_{x=L/2}$$

$$= aL/2$$

$$Q_s = \int_s \rho_{ps} dS$$

$$= 6\left[\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \rho_{ps} dydz\right]$$

$$= 3aL^3$$

(b) Total Bound Volume Charge Q_v?

(5.27b) $\rho_{pv} = -\nabla \cdot \vec{P}$

$$= -a\nabla \cdot (x\vec{a}_x + y\vec{a}_y + z\vec{a}_z)$$

$$= -3a$$

$$Q_v = \int_v \rho_{pv} dv$$

$$= -3aL^3$$

Ex 5.6 평행평판 Capacitor에 ε_r=2.55 의 유전체로 채워져 있다.
극판 사이의 간격은 1.5mm이고 10 kV/m의 전장이 가해졌다.

(a) $D = \varepsilon_0 \varepsilon_r E = \dfrac{10^{-9}}{36\pi} \times 2.55 \times 10000 = 225.4\ \mathrm{nC/m^2}$

(b) $P = \chi_e \varepsilon_0 E = 1.55 \times \dfrac{10^{-9}}{36\pi} \times 10000 = 137\ \mathrm{nC/m^2}$

(c) 극판에서의 자유전하의 면전하밀도

$$\rho_s = \vec{D} \cdot \vec{a}_n = \pm D_n = \pm 225.4\ \mathrm{nC/m^2}$$

(d) 분극전하의 면전하밀도

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n = \pm P_n = \pm 137\ \mathrm{nC/m^2}$$

(e) 두 극판 사이의 전위차

$$V = Ed = 10^4 \times 0.0015 = 15\ \mathrm{volt}$$

Ex 5.7 r=0.1m, ε_r=5.7 인 구 중심에 2pC의 전하가 있다. 다음을 구하라.

(a) 구 표면에서의 분극 전하의 면밀도.

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\vec{a}_r$$

$$\vec{P} = \chi_e\varepsilon_0\vec{E} = \frac{\chi_e Q}{4\pi\varepsilon_r r^2}\vec{a}_r$$

$$\rho_{ps} = \vec{P}\cdot\vec{a}_r = \frac{(\varepsilon_r - 1)Q}{4\pi\varepsilon_r r^2} = \frac{4.7\times 2\times 10^{-12}}{4\pi\times 5.7\times 0.1^2} = 13.12 \ \ \mathrm{pC/m^2}$$

(b) 구 표면의 -4 pC 전하에 작용하는 힘?

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2}\vec{a}_r = \frac{-4\times 2\times 10^{-24}}{4\pi\times\frac{10^{-9}}{36\pi}5.7\times 0.1^2}\vec{a}_r = -1.263\vec{a}_r \ \ \mathrm{pN}$$

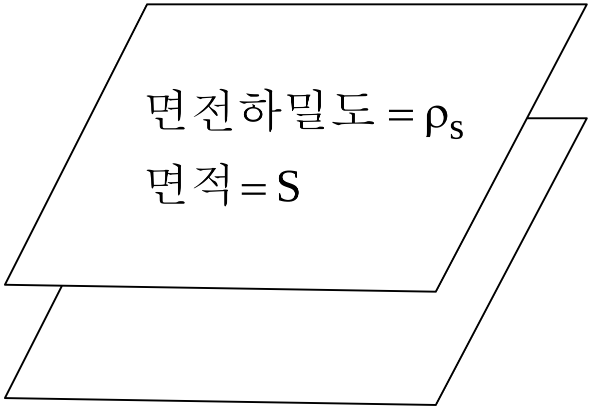
Ex 5.8 평행평판 Capacitor의 양 극판에 작용하는 힘을 구하라.
극판에 작용하는 압력을 구하라.

극판이 무한 평판이라고 가정.

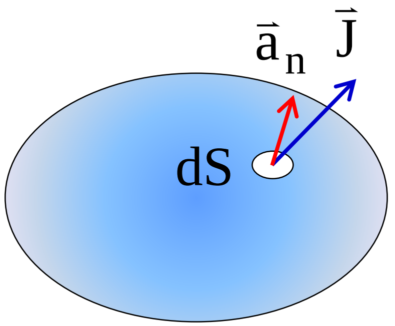
$$\vec{E} = \frac{\rho_s}{2\varepsilon}\vec{a}_n$$

$$F = QE = \rho_s S \frac{\rho_s}{2\varepsilon} = \frac{\rho_s^2 S}{2\varepsilon} = \frac{Q^2}{2\varepsilon S}$$

$$p = F/S = \frac{\rho_s^2}{2\varepsilon}$$



5.8 Continuity Equation and Relaxation Time



$$I_{out} = \oint \vec{J} \cdot d\vec{S} = \frac{-\partial Q_{in}}{\partial t} \qquad (5.40)$$

$$\oint_S \vec{J} \cdot d\vec{S} = \int_v \boxed{\nabla \cdot \vec{J}} dv \qquad (5.41)$$

$$\frac{-\partial Q_{in}}{\partial t} = -\frac{\partial}{\partial t}\int_v \rho_v dv = -\int_v \boxed{\frac{\partial \rho_v}{\partial t}} dv \qquad (5.42)$$

$$\int_v \nabla \cdot \vec{J}\, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}} \qquad (5.43)$$

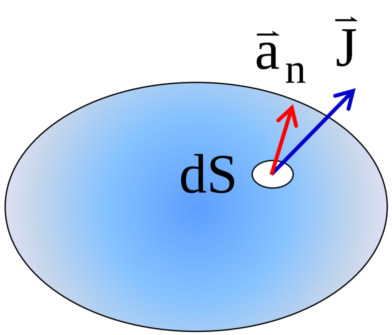
$$\vec{J} = \sigma\vec{E} \qquad (5.44)$$

$$\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon} \qquad (5.45)$$

$$\nabla \cdot \sigma\vec{E} = \frac{\sigma\rho_v}{\varepsilon}$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0 \qquad (5.46)$$



$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0 \qquad (5.46)$$

$$\frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon}\partial t \qquad (5.47)$$

$$\ln \rho_v = -\frac{\sigma t}{\varepsilon} + \ln \rho_{v0}$$

$$\rho_v = \exp\left(-\frac{\sigma t}{\varepsilon} + \ln \rho_{v0}\right)$$

$$= \exp(\ln \rho_{v0})\exp\left(-\frac{\sigma t}{\varepsilon}\right)$$

$$= \rho_{v0}\exp\left(-\frac{\sigma t}{\varepsilon}\right)$$

$$= \rho_{v0} e^{-t/T_r} \qquad (5.48)$$

$$\text{where } T_r = \varepsilon / \sigma \qquad (5.49)$$

$$= \text{Relaxation time}$$

$$\rho_v = \rho_{v0} e^{-t/T_r} \qquad (5.48)$$

$$\text{where } T_r = \varepsilon / \sigma \qquad (5.49)$$

$$= \text{Relaxation time}$$

(Example)

$$\text{Cu} \quad \sigma = 5.8 \times 10^7 \text{ mhos/m}, \ \varepsilon_r = 1$$

$$T_r = \frac{\varepsilon_r \varepsilon_0}{\sigma} = 1 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{5.8 \times 10^7}$$

$$= 1.53 \times 10^{-19} \text{s}$$

$$\text{유전층} \quad \sigma = 10^{-17} \text{ mhos/m}, \varepsilon_r = 5.0$$

$$T_r = 5 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{1}{10^{-17}}$$

$$= 51.2 \text{ days}$$

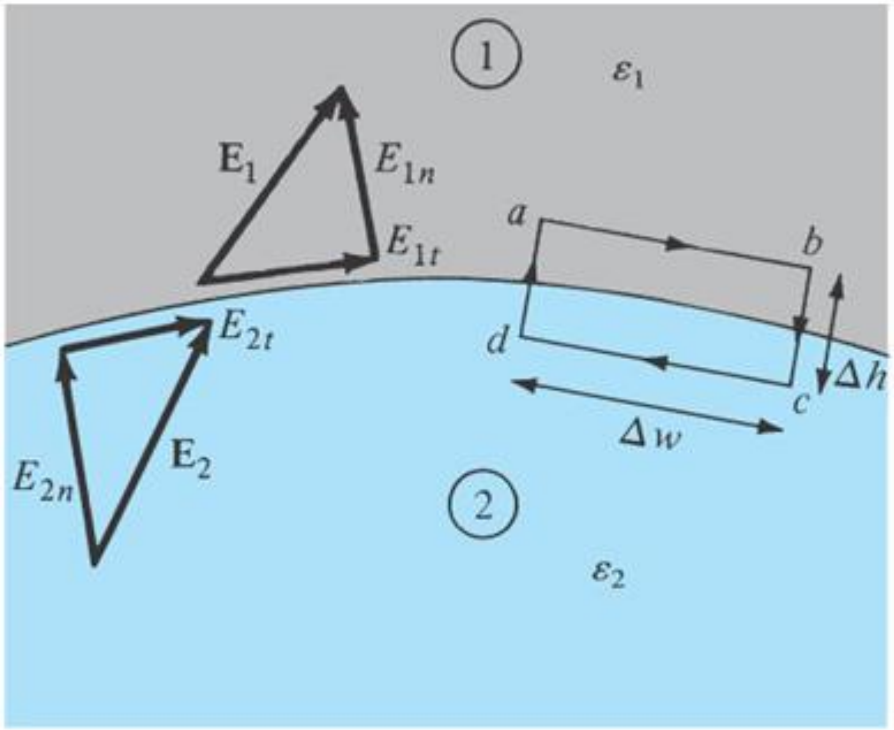
5.9 Boundary Conditions

$$\oint \vec{E} \cdot d\vec{L}$$
$$= \oint (-\nabla V) \cdot d\vec{L}$$
$$= -\oint \left(\frac{\partial V}{\partial x} \vec{a}_x + \frac{\partial V}{\partial y} \vec{a}_y + \frac{\partial V}{\partial z} \vec{a}_z \right) \cdot \left(dx\vec{a}_x + dy\vec{a}_y + dz\vec{a}_z \right)$$
$$= -\oint \left(\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \right)$$
$$= \oint dV$$
$$= 0$$

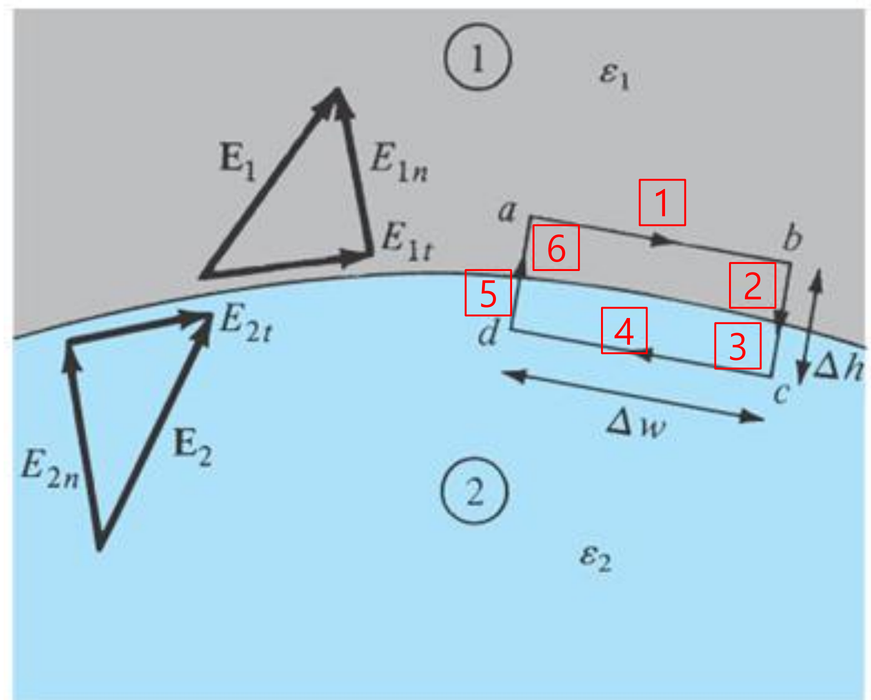
$$\oint \vec{E} \cdot d\vec{L} = 0 \qquad (5.52)$$

$$\oint \vec{D} \cdot d\vec{S} = Q_{enc} \qquad (5.53)$$

$$\vec{E} = \vec{E}_t + \vec{E}_n \qquad (5.54)$$



A. Dielectric-Dielectric Boundary Conditions



$$\vec{E} = \vec{E}_t + \vec{E}_n \qquad (5.54)$$

$$\begin{cases} \vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n} & (5.55a) \\ \vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} & (5.55b) \end{cases}$$

$$\oint \vec{E} \cdot d\vec{L} = 0 \qquad (5.52)$$

$$0 = \underset{1}{\boxed{E_{1t}\Delta w}} - \underset{2}{\cancel{E_{1n}\frac{\Delta h}{2}}} - \underset{3}{\cancel{E_{2n}\frac{\Delta h}{2}}} \underset{4}{\boxed{-E_{2t}\Delta w}} + \underset{5}{\cancel{E_{2n}\frac{\Delta h}{2}}} + \underset{6}{\cancel{E_{1n}\frac{\Delta h}{2}}} \qquad (5.56)$$

$$\boxed{E_{1t} = E_{2t}} \qquad (5.57)$$

$$E_{1t} = E_{2t} \qquad (5.57)$$

Since $\vec{D} = \varepsilon\vec{E} = \vec{D}_t + \vec{D}_n$

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2} \qquad (5.58)$$

$$Q = \oiint \vec{D} \cdot d\vec{S}$$

$$\Delta Q = \rho_S \Delta S = D_{1n}\Delta S - D_{2n}\Delta S \quad \leftarrow \quad (\Delta h \to 0)$$

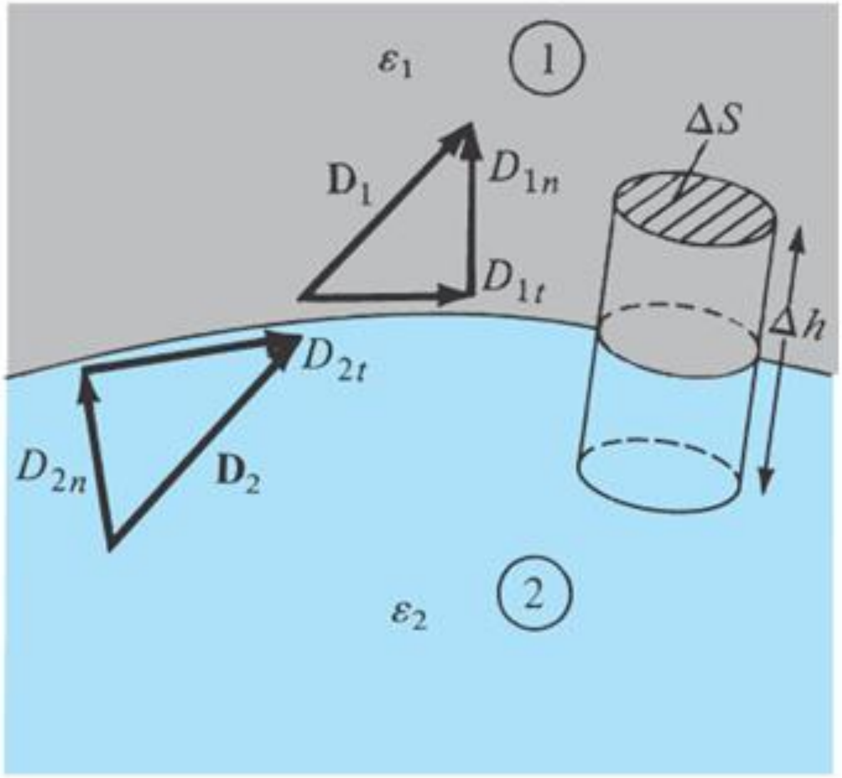
$$\boxed{D_{1n} - D_{2n} = \rho_S} \qquad (5.59)$$

$$\rho_S = 0$$

$$\boxed{D_{1n} = D_{2n}} \qquad (5.60)$$

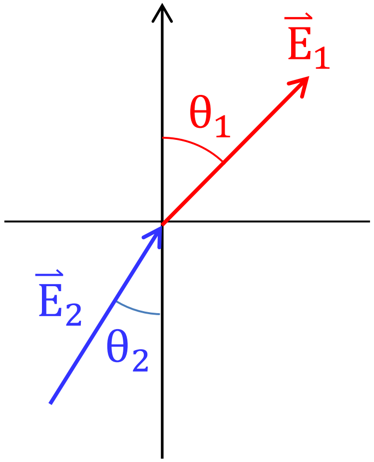
$$\vec{D} = \varepsilon \vec{E}$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \qquad (5.60)$$



$$E_1 \sin\theta_1 = \boxed{E_{1t} = E_{2t}} = E_2 \sin\theta_2$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2 \qquad (5.62)$$



$$\varepsilon_1 E_1 \cos\theta_1 = \boxed{D_{1n} = D_{2n}} = \varepsilon_2 E_2 \cos\theta_2$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2 \qquad (5.63)$$

$$(5.62)/(5.63) \rightarrow$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}} \qquad (5.65)$$

B. Conductor– Dielectric Boundary Conditions

$$(5.52)\ \oint \vec{E}\cdot d\vec{L} = 0$$

$$0 = \underset{\boxed{4}}{\cancel{0\cdot\Delta w}} + \underset{\boxed{5}}{\cancel{0\cdot\frac{\Delta h}{2}}} + \underset{\boxed{6}}{\cancel{E_n\cdot\frac{\Delta h}{2}}} - \underset{\boxed{1}}{E_t\cdot\Delta w} - \underset{\boxed{2}}{\cancel{E_n\cdot\frac{\Delta h}{2}}} - \underset{\boxed{3}}{\cancel{0\cdot\frac{\Delta h}{2}}} \quad (5.66)$$

$$\Delta h \to 0$$

$$E_t = 0 \qquad (5.67)$$

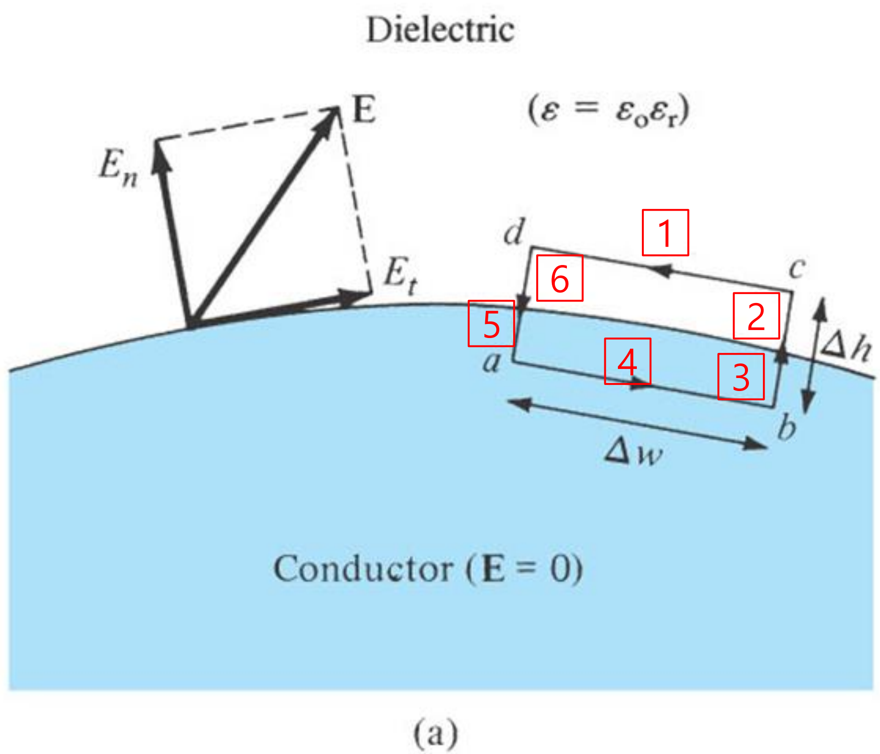


Figure 5.12 Conductor–dielectric boundary.

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S \qquad (5.68)$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S \qquad (5.69)$$

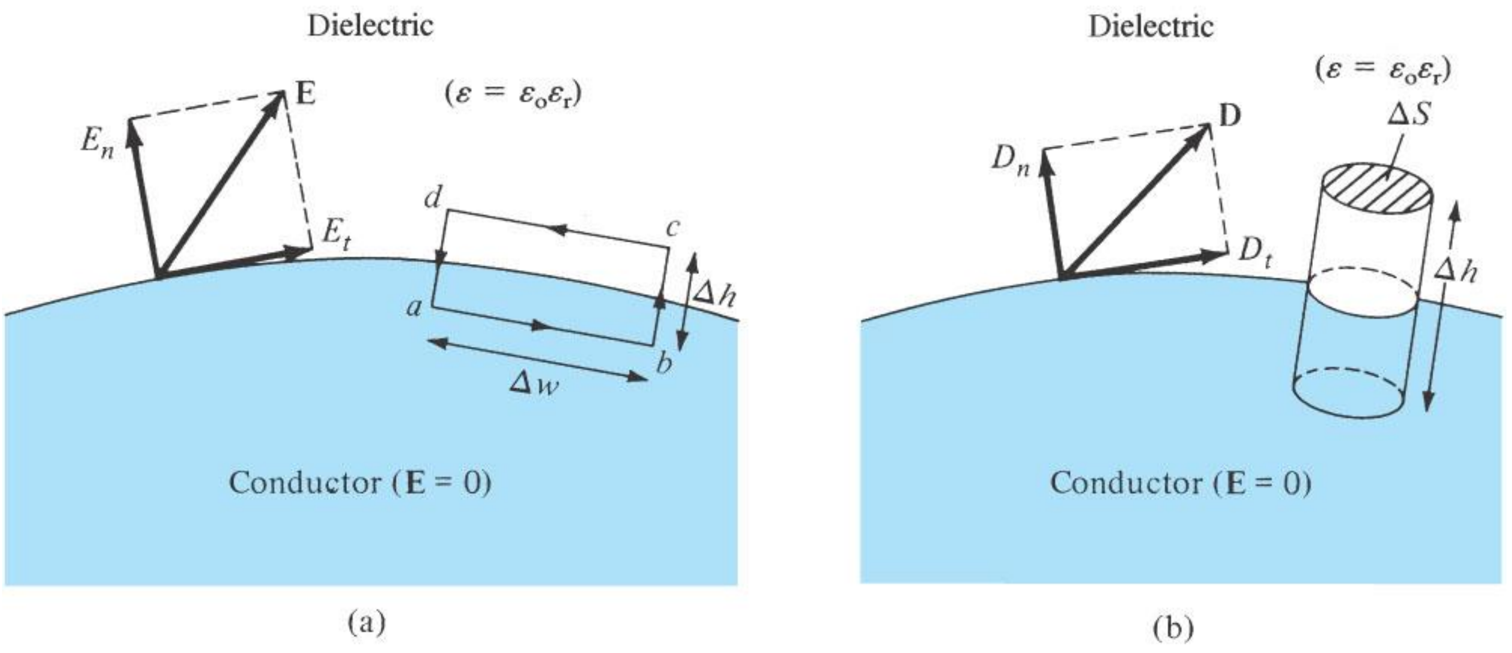
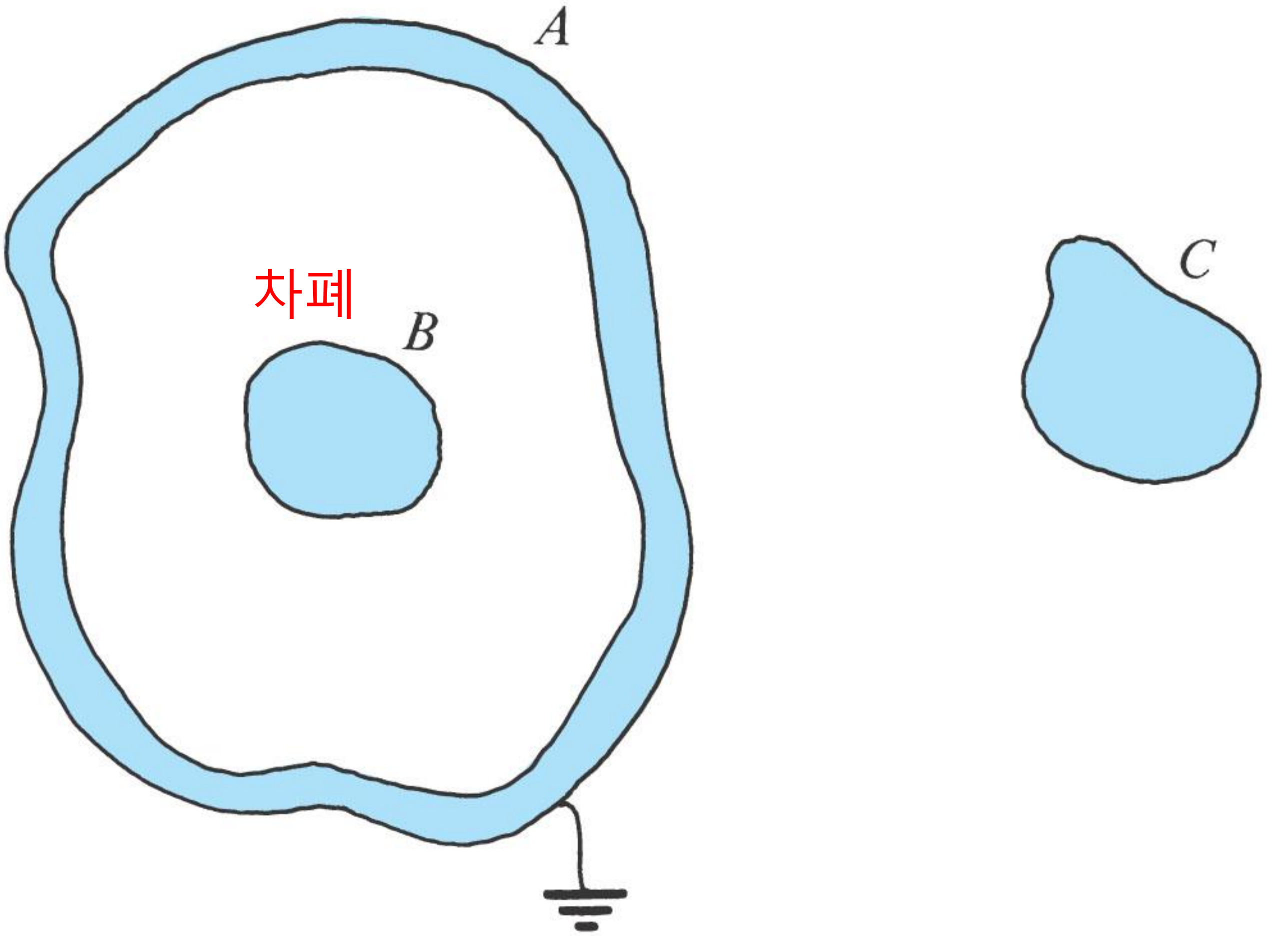


Figure 5.12 Conductor–dielectric boundary.

Electrostatic screening.
정전 차폐



1. No electric field may exist within a conductor; that is,

$$\boxed{\rho_v = 0, \qquad E = 0} \tag{5.70}$$

2. Since $E = -\nabla V = 0$, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body

3. The electric field E can be external to the conductor and nomal to its surface; that is

$$\boxed{D_t = \varepsilon_0 \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_0 \varepsilon_r E_n = \rho_S} \tag{5.71}$$

C. Conductor-Free Space Boundary Conditions

$$\boxed{D_t = \varepsilon_0 E_t = 0, \qquad D_n = \varepsilon_0 E_n = \rho_S} \tag{5.72}$$

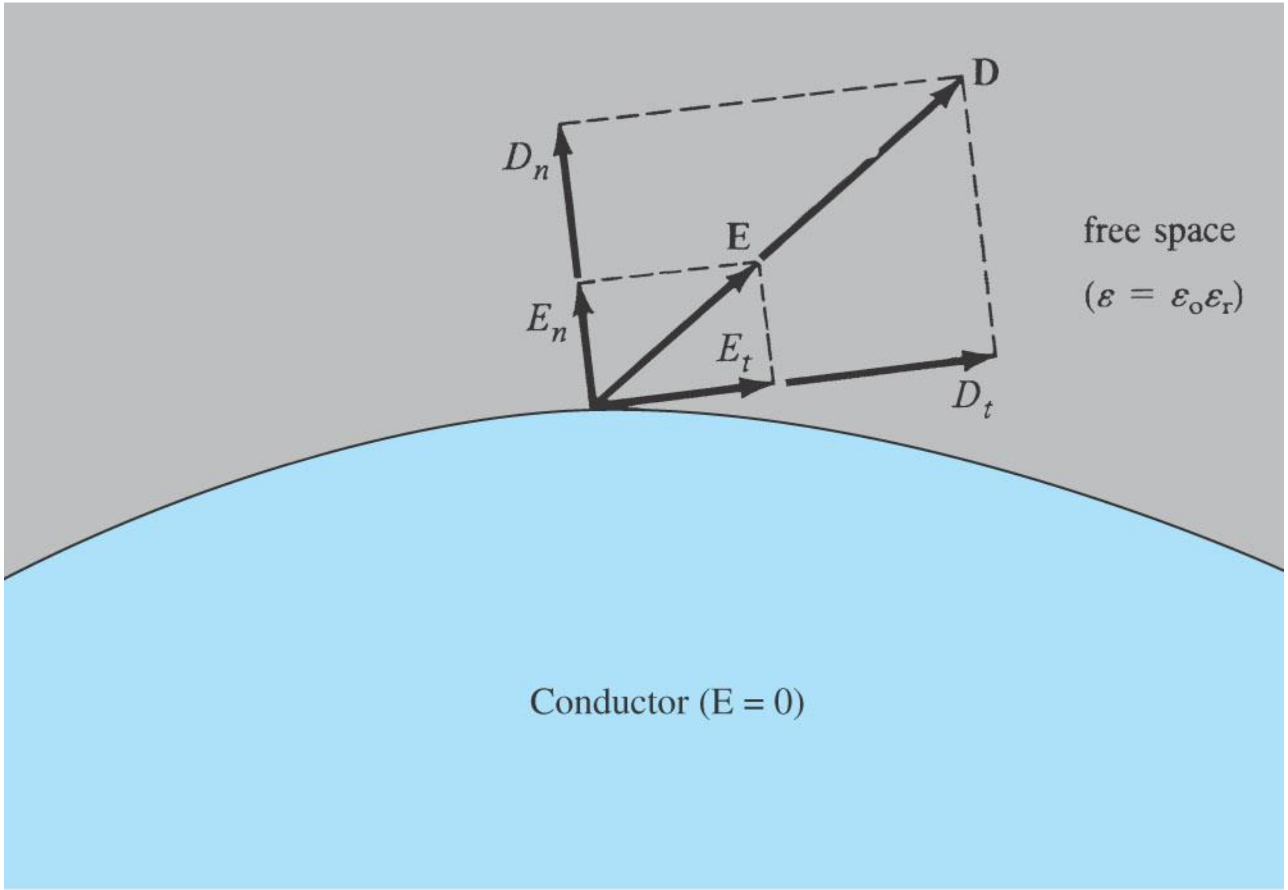
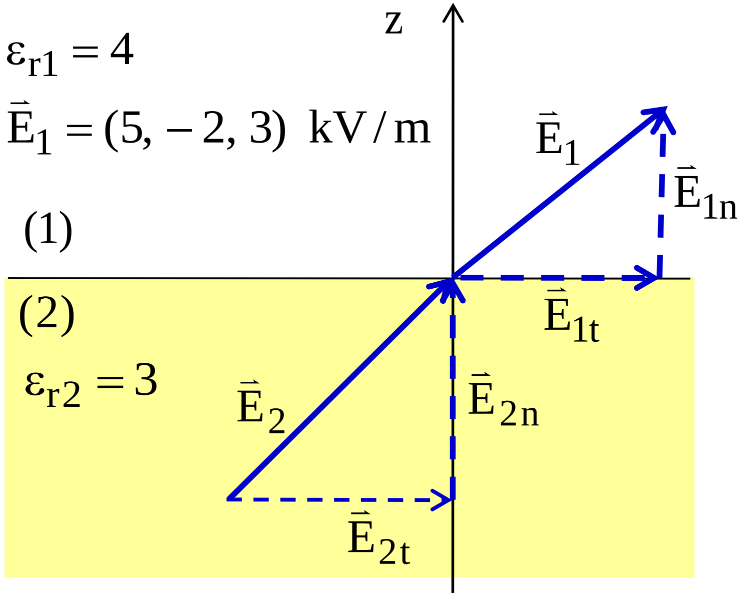


Figure 5.14 Conductor–free space boundary.

Ex 5.9 다음 그림과 같이 z=0 평면을 기준으로 두 물질이 분포하고 있다.
(1) 영역의 전장 $\vec{\mathbf{E}}_1$=(5,-2,3) kV/m 이다.



(a) $\vec{\mathbf{E}}_2$를 구하라.

$$\vec{E}_{1n} = (0, 0, 3)$$
$$\vec{E}_{1t} = (5, -2, 0)$$
$$\vec{E}_{2t} = \vec{E}_{1t} = (5, -2, 0)$$

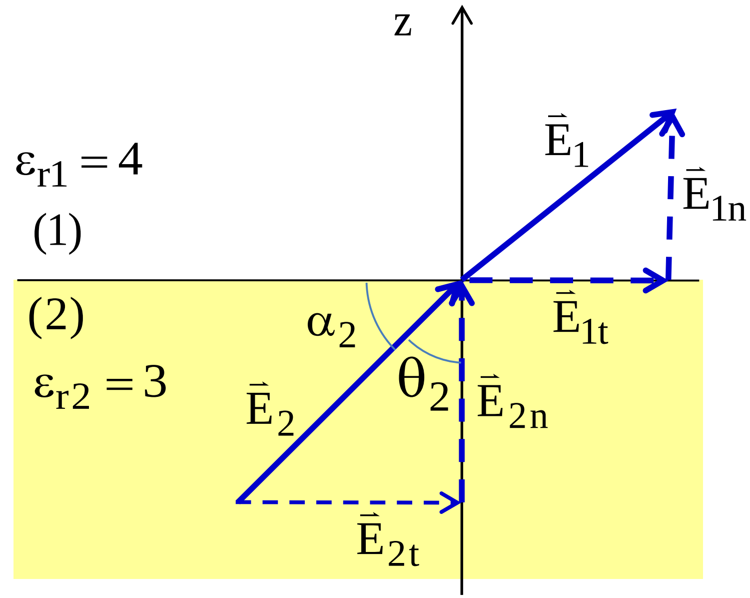
$$\vec{D}_{2n} = \vec{D}_{1n}$$
$$\vec{E}_{2n} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\vec{E}_{1n} = \frac{4}{3}(0, 0, 3) = (0, 0, 4)$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n} = (5, -2, 0) + (0, 0, 4) = (5, -2, 4)$$

$$\vec{E}_{1n} = (0,\ 0,\ 3)$$
$$\vec{E}_{1t} = (5,\ -2,\ 0)$$
$$\vec{E}_{2t} = (5,\ -2,\ 0)$$
$$\vec{E}_{2n} = (0,\ 0,\ 4)$$
$$\vec{E}_{1} = (5,\ -2,\ 3)$$
$$\vec{E}_{2} = (5,\ -2,\ 4)$$

(b) α_2를 구하라.

$$\sin\alpha_2 = \frac{|\vec{E}_{2n}|}{|\vec{E}_2|}$$
$$= \frac{|(0,\ 0,\ 4)|}{|(5,\ -2,\ 4)|}$$
$$= \frac{4}{\sqrt{25+4+16}} = 0.5963$$
$$\alpha_2 = 36.6^{o}$$



(c) 두 유전체의 에너지 밀도.

$$w_{E_1} = \frac{1}{2}\varepsilon_1|\vec{E}_1|^2 = \frac{1}{2}\times 4\times\frac{10^{-9}}{36\pi}\times(25+4+9)\times 10^{6} = 672\ \ \mu J/m^3$$

$$w_{E_2} = \frac{1}{2}\varepsilon_2|\vec{E}_2|^2 = \frac{1}{2}\times 3\times\frac{10^{-9}}{36\pi}\times(25+4+16)\times 10^{6} = 597\ \ \mu J/m^3$$

(d) 중심을 (3,4,-5)에 두고 한 변이 2m인 정육면체의 에너지?

$$W_E = \int_v w_{E_2} dv$$

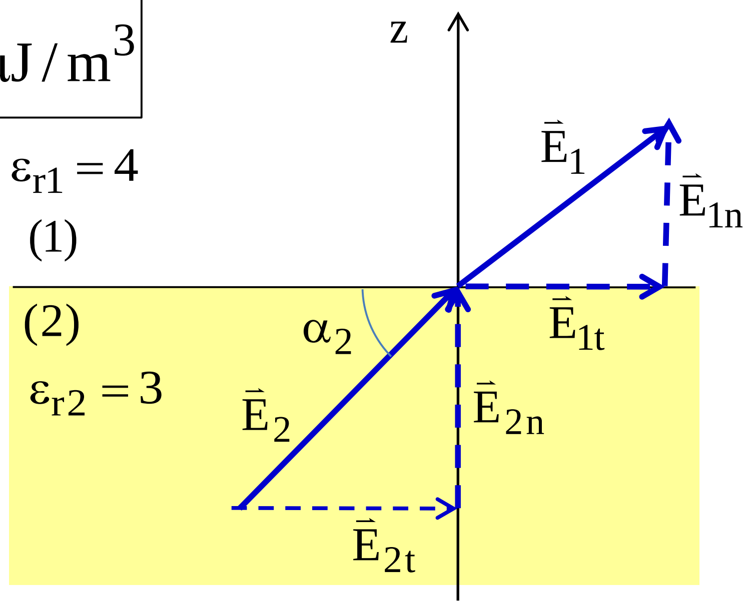
$$= \int_{-6}^{-4}\int_{3}^{5}\int_{2}^{4} w_{E_2} dxdydz$$

$$= w_{E_2} \int_{-6}^{-4}\int_{3}^{5}\int_{2}^{4} dxdydz$$

$$= 597 \times 2 \times 2 \times 2 \ \mu J$$

$$= 4.776 \ mJ$$

$$\boxed{\begin{aligned} w_{E_1} &= 672 \ \mu J / m^3 \\ w_{E_2} &= 597 \ \mu J / m^3 \end{aligned}}$$

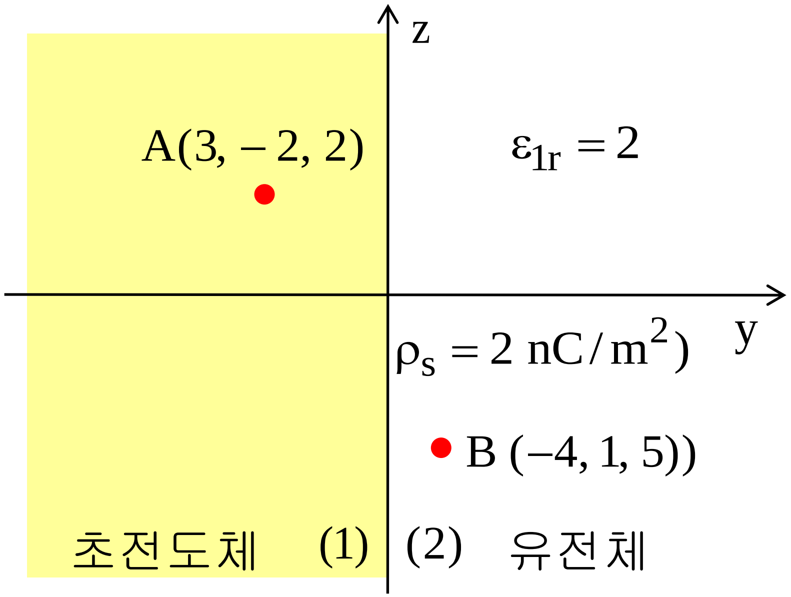


Ex 5.10 다음 그림과 같이 y=0 평면을 기준으로 초전도체와 유전체가 있다. 도체 표면에 . ρ_s=2 nC/m²의 전하가 있다.

(a) A(3,-2,2)에서 $\vec{E}$와 $\vec{D}$를 구하라.

도체 내 이므로 $\vec{\mathbf{E}} = \vec{\mathbf{D}} = \mathbf{0}$

(b) A(-4,1,5)에서 $\vec{E}$와 $\vec{D}$를 구하라.



$$D_n = \rho_s = 2\ \mathrm{nC/m^2}$$

$$\vec{D} = 2\vec{a}_y\ \mathrm{nC/m^2}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$$

$$= 2\times 10^{-9} / \left(2\times \frac{10^{-9}}{36\pi} \right) \vec{a}_y$$

$$= 113.1\, \vec{a}_y\ \ \mathrm{V/m}$$