9. Maxwell’s Equations

* Stationary Charge → Electrostatic Fields

$$\nabla \cdot \vec{E} = \rho$$

$$\nabla \times \vec{E} = -\partial\vec{B}/\partial t$$

* Steady Current → Magnetostatic Fields

$$\nabla \times \vec{H} = \partial\vec{D}/\partial t + \vec{J}$$

$$= \vec{J}$$

$$\vec{H}_1 = 0$$

* Time-Varying Current → Electromagnetic Fields (or Waves)

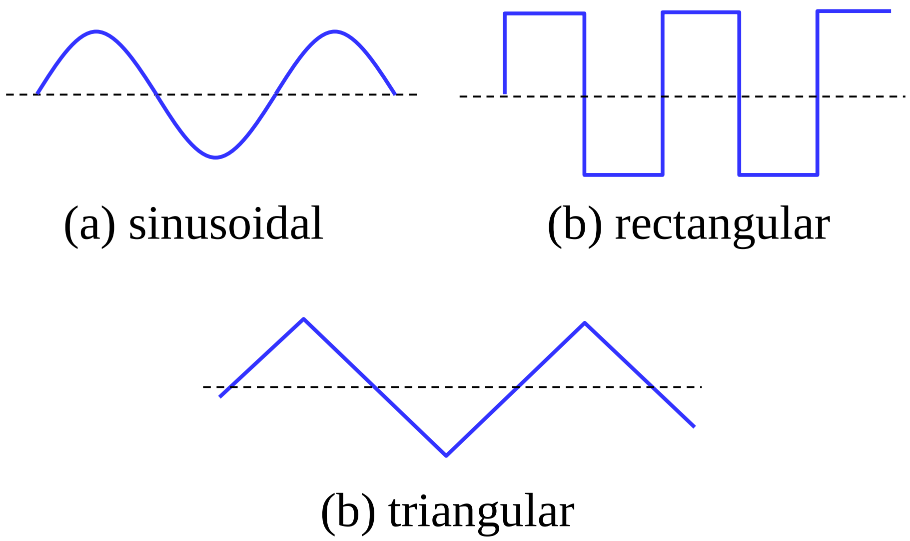
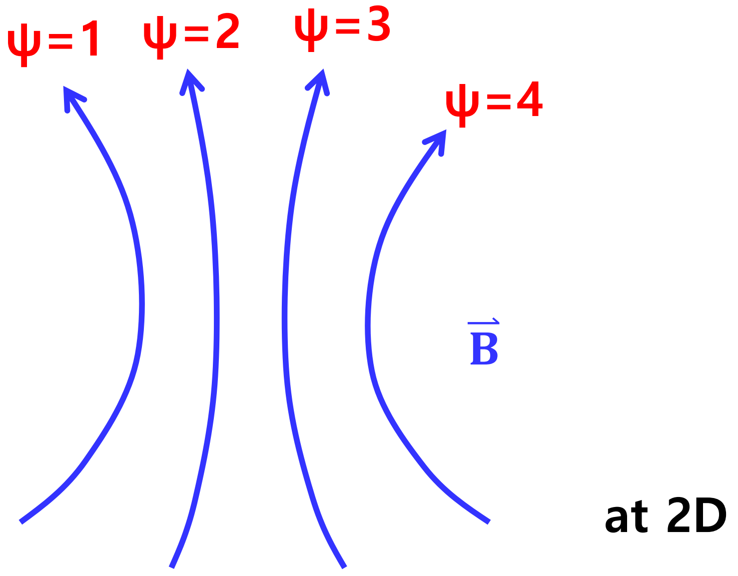


Fig 9.1 Various type of time-varying current

Magnetic Flux: $\psi = \iint \vec{B} \cdot d\vec{S}$

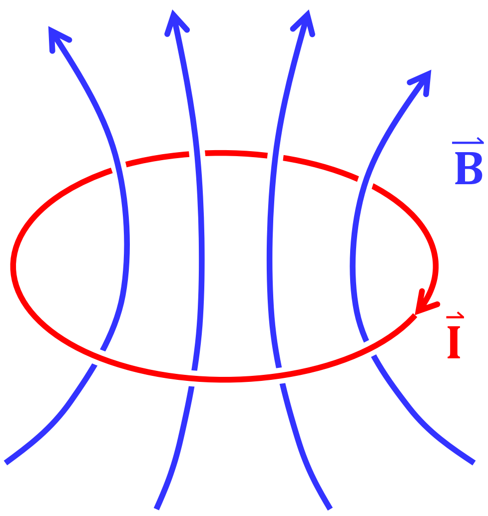


9.2 Faraday's Law

Faraday's law states that the induced emf (electromotive force), V_{emf} [volt], in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit.

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt} \qquad (9.1)$$

"**- sign**" is from Lenz's Law & diamagnetic.



From Fig. 9.2 (E_f : emf-produced field, E_e : electrostaic field)

$$\vec{E} = \vec{E}_f + \vec{E}_e \qquad (9.2)$$

$$\oint_L \vec{E} \cdot d\vec{L} = \oint_L \vec{E}_f \cdot d\vec{L} + 0 = \int_N^P \vec{E}_f \cdot d\vec{L} \qquad (9.3a) \quad \text{(through battery)}$$

$$V_{emf} = \int_N^P \vec{E}_f \cdot d\vec{L} = -\int_N^P \vec{E}_e \cdot d\vec{L} = IR \qquad (9.3b)$$

- E_e cannot maintain a steady current in a closed circuit
- E_f is nonconservative

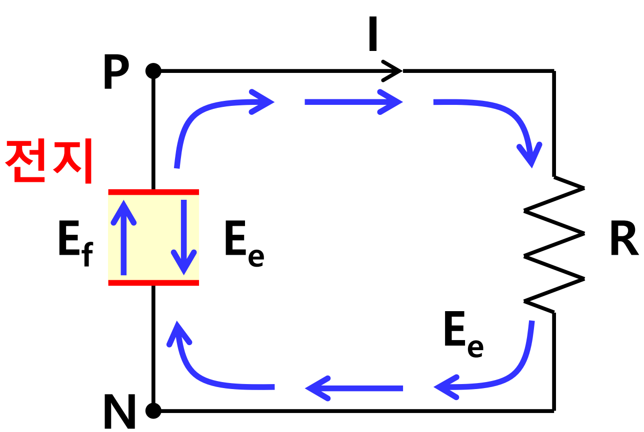
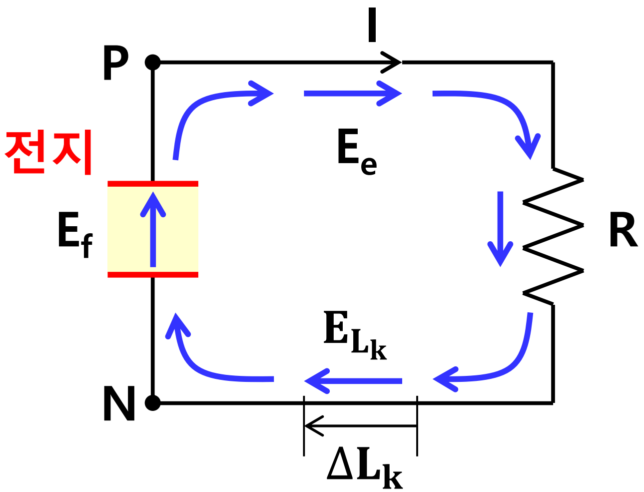


Fig 9.2 Circuit showing emf-producing field E_f and electrostatic field E_e.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$$ Electrostatic 경우



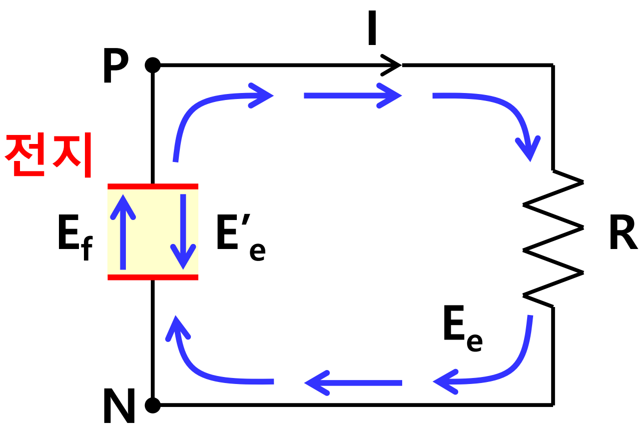
$$\iint \nabla \times \vec{E} \cdot d\vec{S} = 0$$

$$\oint_L \vec{E} \cdot d\vec{L} = \sum E_{L_k} \Delta L_k \neq 0$$ 실제의 경우

$$\oint_L \vec{E} \cdot d\vec{L} = \oint_L (-\nabla V) \cdot d\vec{L}$$

$$= -\oint_L dV$$

$$= [V]_P^P = 0$$



$$\Rightarrow \vec{E} = 0 \; ?$$

$$\Rightarrow \vec{E} = \vec{E}_f + \vec{E}_e$$

(*)

$$\nabla V \cdot d\vec{L} = \left(\vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z} \right) \left(\vec{a}_x dx + \vec{a}_y dy + \vec{a}_z dz \right)$$

$$= \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= dV$$

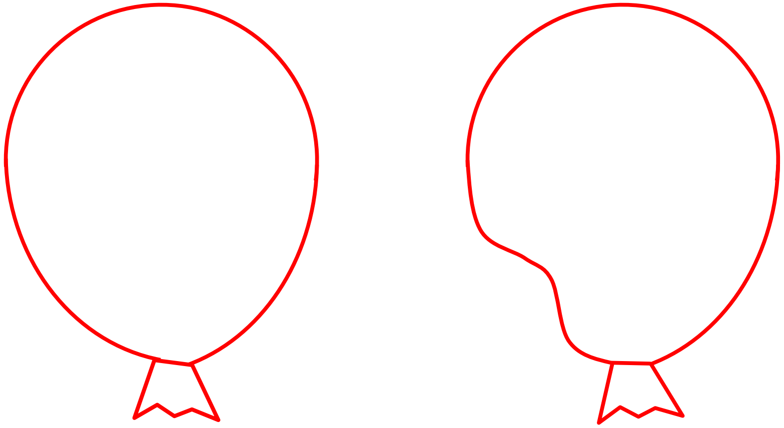
(*)

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \frac{\partial x}{\partial t} \frac{\partial}{\partial x} + \frac{\partial y}{\partial t} \frac{\partial}{\partial y} + \frac{\partial z}{\partial t} \frac{\partial}{\partial z}$$

$$(*)\ \ \nabla \cdot \vec{E} = \frac{dE_L}{dL} = 0 \Rightarrow (\vec{E}\ \text{ is conservative})$$

$$(\text{ex})\quad \nabla \cdot \vec{v} = 0$$

** 물 풍선: 부피가 보존



9.3 Transformer and Motional EMFs (EMF=electromotive force)

$$V_{emf} = -\frac{d\psi}{dt} \qquad (9.4)$$

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{L} = \iint_S \nabla \times \vec{E} \cdot d\vec{S} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S}$$

$$= -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S} \qquad (9.5)$$

A. Stationary Loop in Time-Varying B Field (Transformer emf)

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{L} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \qquad (9.6)$$

$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \qquad (9.7)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (9.8)$$

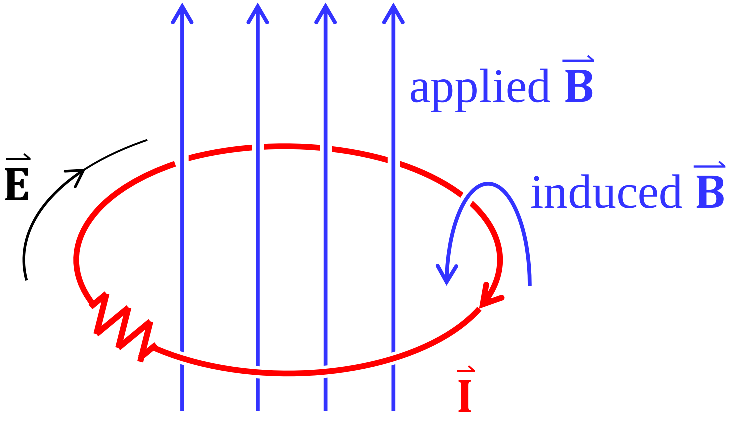


Fig 9.3 Induced emf due to a stationary loop in a time varying B field.

B. Moving Loop in Static B Field (Motional emf)

$$\vec{F}_m = Q\vec{u} \times \vec{B} = It\vec{u} \times \vec{B} = I\vec{\ell} \times \vec{B} \qquad (8.2)$$

$$\vec{E}_m = \frac{\vec{F}_m}{Q} = \vec{u} \times \vec{B} \qquad (9.9)$$

$$V_{emf} = \oint_L \vec{E}_m \cdot d\vec{L} = \oint_L (\vec{u} \times \vec{B}) \cdot d\vec{L} \qquad (9.10)$$

$$\begin{cases} \vec{F}_m = I\vec{\ell} \times \vec{B} & (9.11) \\ F_m = I\ell B & (9.12) \\ V_{emf} = uB\ell & (9.13) \end{cases}$$

$$\nabla \times \vec{E}_m = \nabla \times (\vec{u} \times \vec{B}) \qquad (9.14)$$

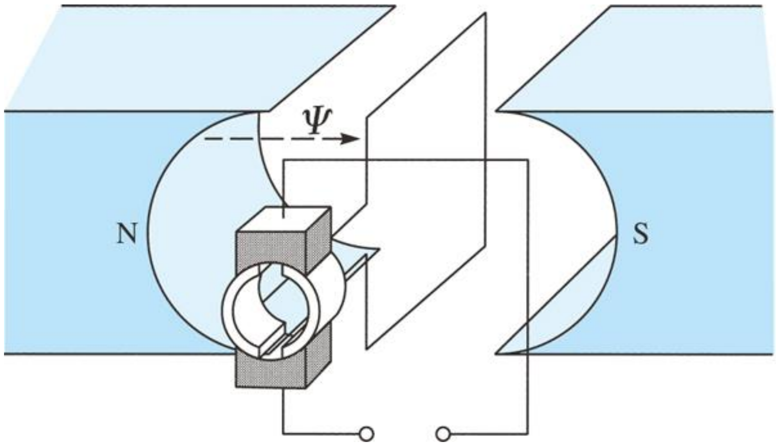
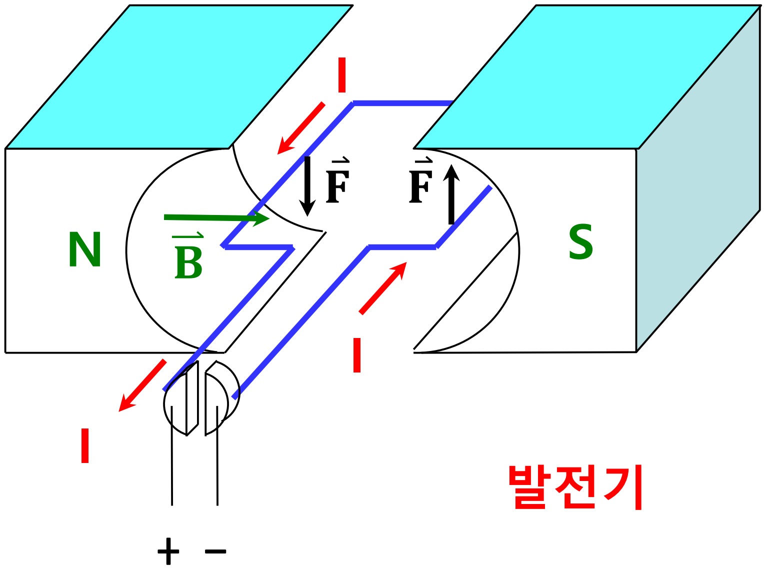


Fig 9.4 A direct-current machine.



발전기

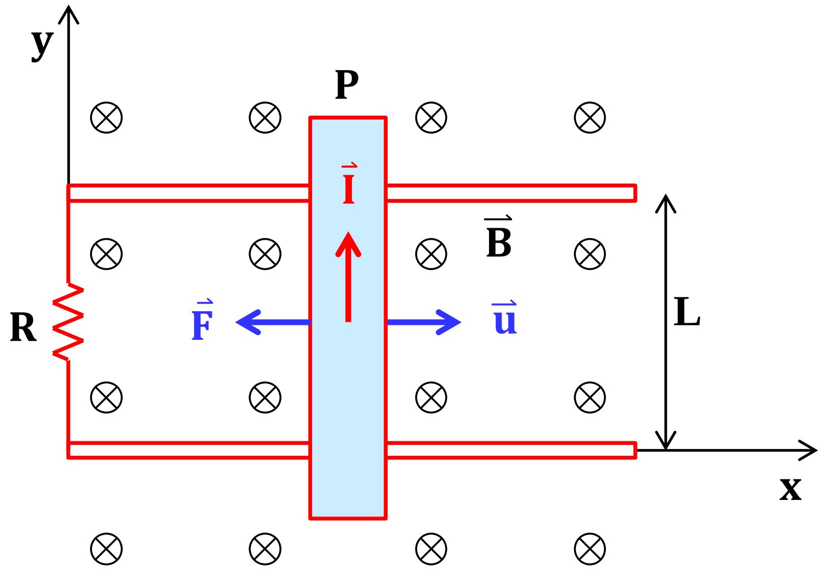


Fig 9.5 Induced emf due to a moving loop in a static B field.

A. Stationary Loop in Time-Varying B Field (Transformer emf)

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \qquad (9.6)$$

$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \qquad (9.7)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (9.8)$$

C. Moving Loop in Time-Varying Field

$$V_{emf} = \oint_L \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint_L (\vec{u} \times \vec{B}) \cdot d\vec{L} \qquad (9.15)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{u} \times \vec{B}) \qquad (9.16)$$

참고

II-4. Faraday 방정식

Eulerian Description

날아가는 야구공이 있을 때 야구공의 속도, 가속도는 야구공에서 측정, 혹은 계산한 값이다. 우리가 사용하는 d/dt의 미분자는 관찰자의 시선이 움직이는 야구공을 따라가며 측정, 혹은 계산한 시간에 대한 변화율이다.

시선을 움직이는 야구공에 고정하고 초기의 위치와 속도를 기억하여 지금의 속도를 계산하고 위치를 계산하는 것이 Lagrangian Description이다.

시선을 움직이지 않고 시간과 공간 좌표로 야구공이 나는 것을 관찰, 묘사하는 것이 Eulerian Description이다. 대부분의 경우 Field를 기술하는데 Eulerian Description이 편리하게 사용된다. 하지만 움직이는 야구공을 묘사하는 경우에는 Lagrangian Description이 편리하다.

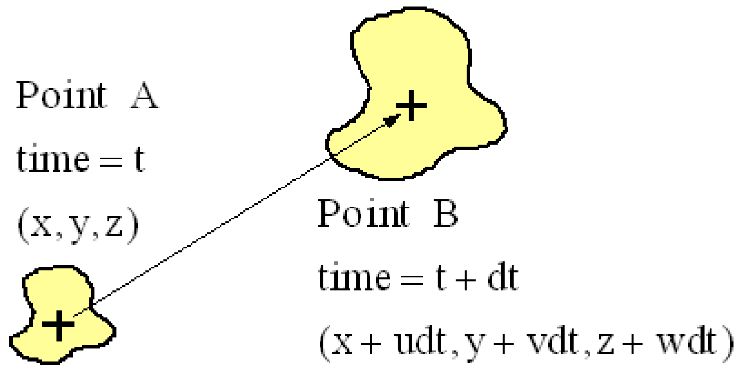


그림 2.8 Eulerian Description에서 Total Derivative d/dt 설명도. 입자들이 A에서 B로 이동.

그림 2.8에서 u, v, w는 좌표 x, y, z 방향의 속도이다. 시간이 t에서 t+dt로 지나가 입자들이 A에서 B로 이동했을 때 임의의 특성 변화를 dH라고 가정 한다. Euler Description에 의거 Taylor Series에 의해 dH는 방정식 (2.45)과 같이 나타낼 수 있다.

$$dH = \frac{\partial H}{\partial t}dt + \frac{\partial H}{\partial x}dx + \frac{\partial H}{\partial y}dy + \frac{\partial H}{\partial z}dz \qquad (2.45)$$

t의 미분 의 의미는 t 이외의 독립변수는 상수처럼 여기면서 미분하라는 수학적인 도구이며 현재 위치에서의 시간에 대한 변화율이다. 과거에 현재 위치에 있던 것은 그림 2.8에서 B로 갔고 지금 현재 위치에 있는 것은 다른 것이다. d/dt는 흐름을 따라가며 측정한 시간에 대한 변화율이다. dx=udt, dy=vdt, dz=wdt 이므로 방정식 (2.45)는 방정식 (2.46)이 된다.

$$\begin{aligned}\frac{dH}{dt} &= \frac{\partial H}{\partial t} + u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} + w\frac{\partial H}{\partial z} \\ &= \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)H \qquad (2.46)\end{aligned}$$

$\partial/\partial t = 0$ 인 것을 Steady State라 하고 $\partial/\partial t \neq 0$ 이면 Transient State라고 한다.

Faraday 방정식 유도

Faraday의 정성 어린 관찰의 결과 초전도 물질로 만든 폐회로를 통과하는 자장의 시간변화율에 비례하여 폐회로에 전류를 만드는 기전력이 커진다는 것이 알려졌다.

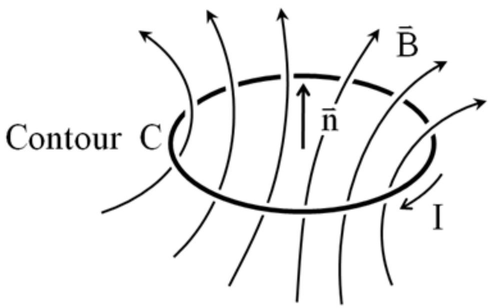


그림 2.9 Faraday 방정식을 유도하기 위한 설명도.

단위를 volt로 가지는 φ는 기전력이라 하고 F는 그림 2.9의 폐회로를 수직으로 관통하는 자장의 성분에 면적을 곱한 값이라고 하면 방정식 (2.47)을 얻는다. S는 Contour C로 둘러 쌓인 면적이다.

$$\varphi = -k\frac{\partial F}{\partial t} \qquad (2.47)$$

where $\varphi = \oint \vec{E}' \cdot d\vec{L}$

$F = \iint \vec{B} \cdot \vec{n} dS$

$\vec{E}'$은 실험실에서 실험자가 폐회로에 생긴 전장을 측정한 값이다. 상수 k가 1 이 되도록 단위를 조정한다. 폐회로가 움직이지 않으면서 측정한 전장을 $\vec{E}''$라고 하면 움직이지 않는 폐회로에 대한 방정식 (2.47)은 다음과 같이 정리 된다.

$$\oint \vec{E}'' \cdot d\vec{L} = \iint \nabla \times \vec{E}'' \cdot \vec{n} dS$$

$$= \frac{\partial}{\partial t}\iint \vec{B} \cdot \vec{n} dS$$

$$\nabla \times \vec{E}'' + \frac{\partial \vec{B}}{\partial t} = 0 \qquad (2.48)$$

방정식 (2.48)은 폐회로와 관계없는 일반적인 현상을 묘사하는 방정식이다. 폐회로가 움직이는 경우를 고찰한다. 폐회로가 그림 2.10과 같이 등속도 를 가지고 이동하며 폐회로에 생긴 전장 를 측정할 경우를 살핀다.

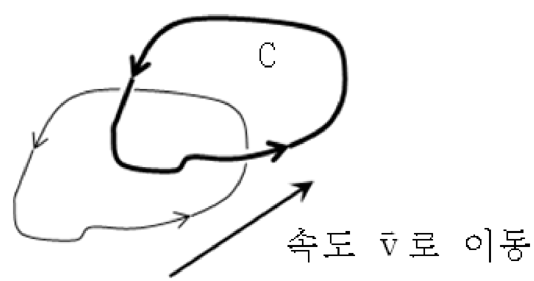


그림 2.10 폐회로가 속도 v로 이동.

방정식 (2.46)의 d/dt 의미를 새기면서 방정식 (2.47)의 우변을 정리한다.

$$\frac{d}{dt}\iint \vec{B}\cdot\vec{n}dS$$

$$= \iint \frac{d\vec{B}}{dt}\cdot\vec{n}dS$$

$$= \iint \left(\frac{\partial\vec{B}}{\partial t} + \vec{v}\cdot\nabla\vec{B}\right)\cdot\vec{n}dS$$

$$\leftarrow \nabla\times(\vec{B}\times\vec{v}) = \vec{v}\cdot\nabla\vec{B} - \vec{B}\cdot\nabla\vec{v} + \vec{B}\nabla\cdot\vec{v} - \vec{v}\nabla\cdot\vec{B}$$

$$= \iint \left(\frac{\partial\vec{B}}{\partial t} + \nabla\times(\vec{B}\times\vec{v}) + \vec{v}(\nabla\cdot\vec{B})\right)\cdot\vec{n}dS$$

$$= \iint \frac{\partial\vec{B}}{\partial t}\cdot\vec{n}dS + \iint \nabla\times(\vec{B}\times\vec{v})\cdot\vec{n}dS$$

$$= \iint \frac{\partial\vec{B}}{\partial t}\cdot\vec{n}dS + \oint(\vec{B}\times\vec{v})d\vec{L} \qquad (2.49)$$

방정식 (2.49)에 의하여 방정식 (2.47)을 정리하면 다음과 같다.

$$\nabla\times\left(\vec{E}' - \vec{v}\times\vec{B}\right) + \frac{\partial\vec{B}}{\partial t} = 0 \qquad (2.50)$$

일정하게 상대적으로 움직이는 2 개의 좌표계에서 각 관찰자가 관찰하는 물리적 현상은 같아야 한다는 Galilean Invariance에 의해 폐회로에 생긴 전장은 같아야 한다. 방정식 (2.48)와 (2.50)은 같다. $\vec{E}'' = \vec{E}$라 하면 방정식 (2.51)을 얻을 수 있다.

$$\oint \vec{E}''\cdot d\vec{L} = \iint \nabla\times\vec{E}''\cdot\vec{n}dS$$

$$= \frac{\partial}{\partial t}\iint \vec{B}\cdot\vec{n}dS$$

$$\nabla\times\vec{E}'' + \frac{\partial\vec{B}}{\partial t} = 0 \qquad (2.51)$$

다음과 같이 Faraday 방정식이 유도 되었다.

$$\boldsymbol{\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0} \qquad \mathbf{(2.52)}$$

참고: Vector identity

$$\nabla \cdot (\varphi \vec{u}) = \varphi \nabla \cdot \vec{u} + \vec{u} \cdot \nabla \varphi$$

$$\nabla \times (\varphi \vec{u}) = \varphi \nabla \times \vec{u} + \nabla \varphi \times \vec{u}$$

$$\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{v})$$

$$\nabla \times (\vec{u} \times \vec{v}) = \vec{v} \cdot \nabla \vec{u} - \vec{u} \cdot \nabla \vec{v} + \vec{v} \cdot \nabla \vec{u} - \vec{v} \cdot \nabla \vec{u}$$

$$\nabla (\vec{u} \cdot \vec{v}) = \vec{u} \cdot \nabla \vec{v} + \vec{v} \cdot \nabla \vec{u} + \vec{u} \times (\nabla \times \vec{v}) + \vec{v} \times (\nabla \times \vec{u})$$

$$\nabla \times (\nabla \varphi) = 0$$

$$\nabla \cdot (\nabla \times \vec{u}) = 0$$

$$\nabla \times (\nabla \times \vec{u}) = \nabla (\nabla \cdot \vec{u}) - \nabla^2 \vec{u}$$

$$\nabla \cdot (\nabla \varphi_1 \times \nabla \varphi_2) = 0$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{a})\vec{c}$$

$$\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = (\vec{a} \cdot \vec{c}) \cdot \left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right) \cdot \left(\vec{b} \cdot \vec{c}\right)$$

예제 9.1 그림 9.6의 Circuit에 유도되는 전압을 계산하라.

(a) 막대가 y=8 cm에 있고 $\vec{B} = 4\cos(10^6 t)\ \vec{a}_z\ \ mWb/m^2$
(b) 막대가 속도 $\vec{u} = 20\ \vec{a}_y\ \ m/sec$ 로 이동하고 $\vec{B} = 4\ \vec{a}_z\ \ mWb/m^2$
(c) 막대가 속도 $\vec{u} = 20\ \vec{a}_y\ \ m/sec$ 로 이동하고 $\vec{B} = 4\cos(10^6 t - y)\ \vec{a}_z\ \ mWb/m^2$

$$
\begin{aligned}
(9.15)\quad V_{emf} &= \oint_L \vec{E}\cdot d\vec{L} \\
&= \iint_S \nabla\times\vec{E}\cdot d\vec{S} \\
&= -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}
\end{aligned}
$$

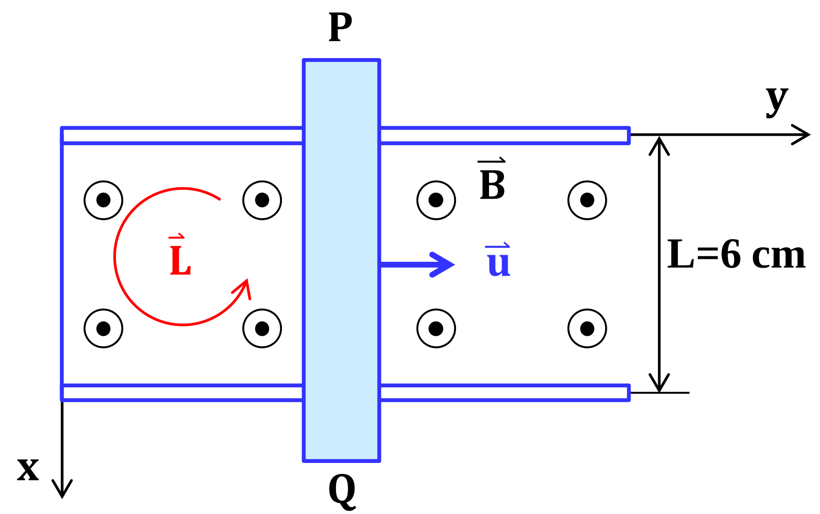


Fig 9.6 For example 9.1.

예제 9.1 그림 9.6의 Circuit에 유도되는 전압을 계산하라.

(a) 막대가 **y=8 cm**에 있고 $\vec{\mathbf{B}} = \mathbf{4cos(10^6 t)}\, \vec{\mathbf{a}}_z\ \ \mathbf{mWb/m^2}$

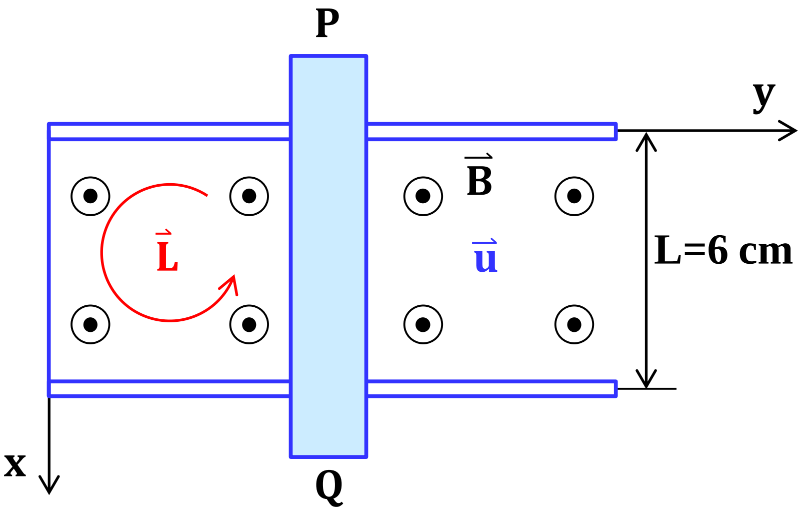
(9.15) $V_{emf} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint (\vec{u} \times \vec{B}) \cdot d\vec{L}$

$= -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$

$= \int_{y=0}^{0.08} \int_{x=0}^{0.06} 0.004 \times 10^6 \sin(10^6 t) dxdy$

$= 4000 \sin(10^6 t) \times (0.08 \times 0.06)$

$= 19.2 \sin(10^6 t)$ volt



예제 9.1 그림 9.6의 Circuit에 유도되는 전압을 계산하라.

(b) 막대가 속도 $\vec{\mathbf{u}} = \mathbf{20\,\vec{a}_y\ \ m/sec}$ 로 이동하고 $\vec{\mathbf{B}} = \mathbf{4\,\vec{a}_z\ \ mWb/m^2}$

$$(9.15)\ \ V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}$$

$$= \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}$$

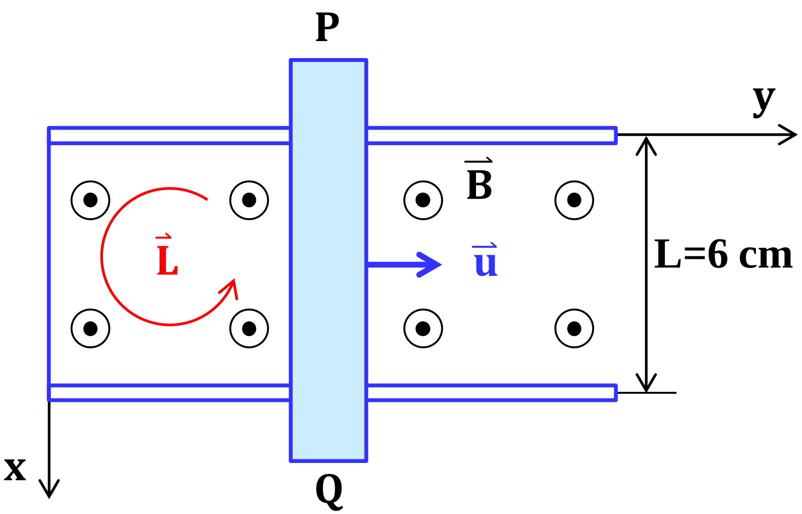
$$= \int_{x=\ell}^{0} (u\vec{a}_y \times B\vec{a}_z)\cdot dx\vec{a}_x$$

$$= \int_{x=\ell}^{0} (uB)\vec{a}_x \cdot \vec{a}_x dx$$

$$= \int_{x=\ell}^{0} (uB)dx$$

$$= -uB\ell = -20\times 0.004\times 0.06$$

$$= -4.8\ \ mV$$



예제 9.1 그림 9.6의 Circuit에 유도되는 전압을 계산하라.

(c) 막대가 속도 $\vec{\mathbf{u}} = \mathbf{20}\,\vec{\mathbf{a}}_{\mathbf{y}}$ **m/sec** 로 이동하고 $\vec{\mathbf{B}} = \mathbf{4cos(10^6t - y)}\,\vec{\mathbf{a}}_{\mathbf{z}}$ **mWb/m²**

방법 1

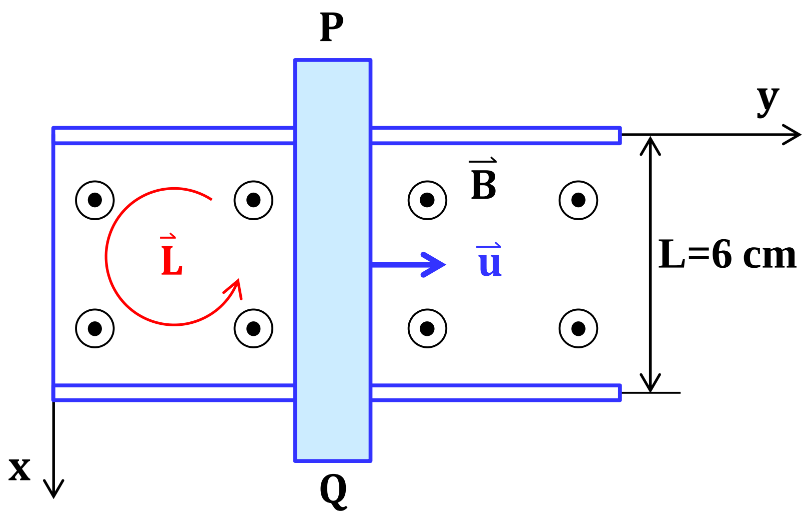
$$(9.15)\quad V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}$$

$$= \int_{x=0}^{0.06}\int_0^{y}(0.004)(10^6)\sin(10^6 t - y')dy'dx$$

$$+ \int_{0.06}^{0}[20\vec{a}_y \times (0.004)\cos(10^6 t - y)\vec{a}_z]\cdot \vec{a}_x dx$$

$$= \int_{x=0}^{0.06}\int_0^{y}(0.004)(10^6)\sin(10^6 t - y')dy'dx$$

$$+ \int_{0.06}^{0}[(20)(0.004)\cos(10^6 t - y)]\vec{a}_x \cdot \vec{a}_x dx$$



$$= 240\cos(10^6 t - y')\Big|_0^{y} \quad - (0.08)(0.06)\cos(10^6 t - y)$$

$$= 240\cos(10^6 t - y) - 240\cos(10^6 t) \quad - 0.0048\cos(10^6 t - y)$$

$$\approx 240\cos(10^6 t - y) - 240\cos(10^6 t)\ \text{volt}$$

예제 9.1 그림 9.6의 Circuit에 유도되는 전압을 계산하라.

(c) 막대가 속도 $\vec{\mathbf{u}} = \mathbf{20}\ \vec{\mathbf{a}}_{\mathbf{y}}\ \mathbf{m/sec}$로 이동하고 $\vec{\mathbf{B}} = \mathbf{4cos(10^6t - y)}\ \vec{\mathbf{a}}_{\mathbf{z}}\ \mathbf{mWb/m^2}$
방법 2

$$\psi = \int_S \vec{B} \cdot d\vec{S}$$

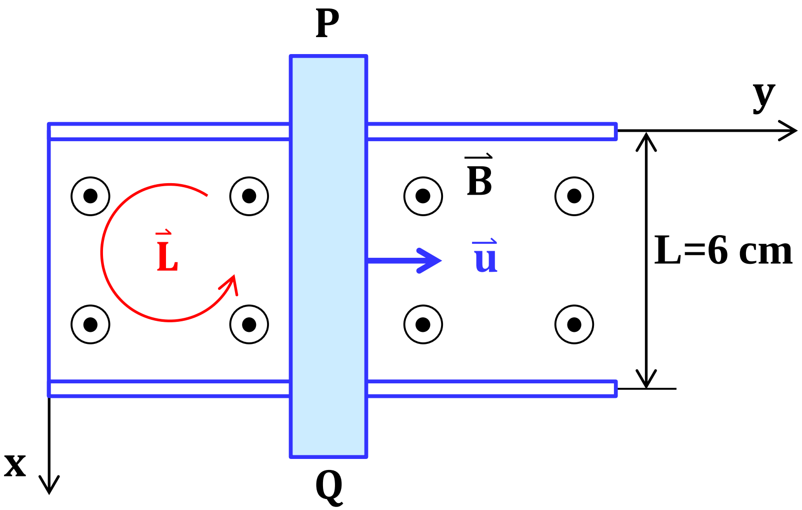
$$= \int_{y=0}^{y} \int_{x=0}^{0.06} 4\cos(10^6 t - y')dxdy'$$

$$= -4(0.06)\sin(10^6 t - y)\Big|_{y=0}^{y}$$

$$= -0.24\sin(10^6 t - y) + 0.24\sin(10^6 t)\ \ \text{mWb}$$

$$(y = ut = 20t)$$

$$= -0.24\sin(10^6 t - 20t) + 0.24\sin(10^6 t)\ \ \text{mWb}$$



$$V_{emf} = -\frac{\partial(0.001)\psi}{\partial t}$$

$$= 0.24(0.001)(10^6 - 20)\cos(10^6 t - 20t)\ \ - 0.24(0.001)(10^6 - 20)\cos(10^6 t)$$

$$\approx 240\cos(10^6 t - y) - 240\cos(10^6 t)\ \ \text{volt}$$

예제 9.2 그림 9.7에서 자기장 $\vec{\mathbf{B}} = \mathbf{50}\,\vec{\mathbf{a}}_x$ mWb/m^2 내에 loop가 존재 한다. loop가 50 Hz로 회전하며 자속과 쇄교 하며 t=0 에서 yz 평면에 있다.

(a) t=1 msec에서의 기전력

(b) t=3 msec에서 저항이 0.1 Ω일 때 유도 전류

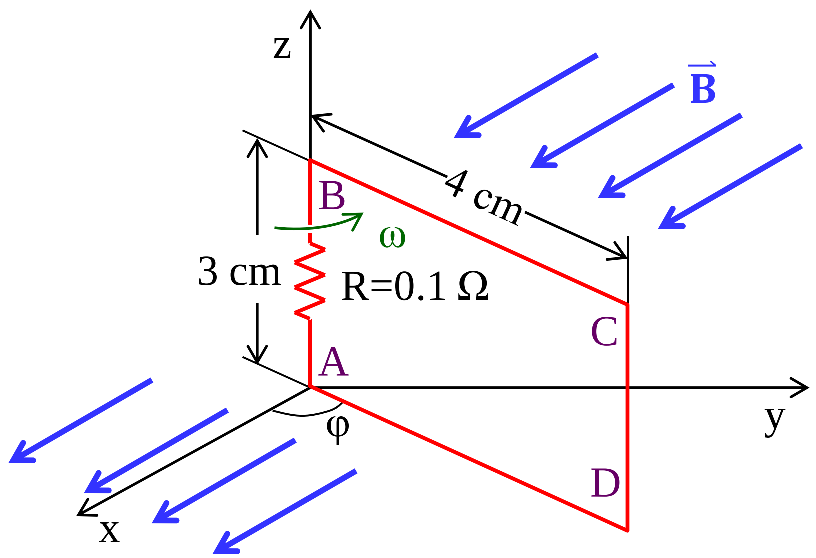


Figure 9.7 For example 9.2; polarity is for increasing emf.

예제 9.2 그림 9.7에서 자기장 $\vec{\mathbf{B}} = \mathbf{50}\,\vec{\mathbf{a}}_{\mathbf{x}}\ \mathbf{mWb/m^2}$ 내에 loop가 존재 한다. loop가 50 Hz로 회전하며 자속과 쇄교 하며 t=0 에서 yz 평면에 있다.
(a) t=1 msec에서의 기전력

$$(9.15)\quad V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L} = \oint_L (\vec{u}\times\vec{B})\cdot d\vec{L}$$

Line DC에서

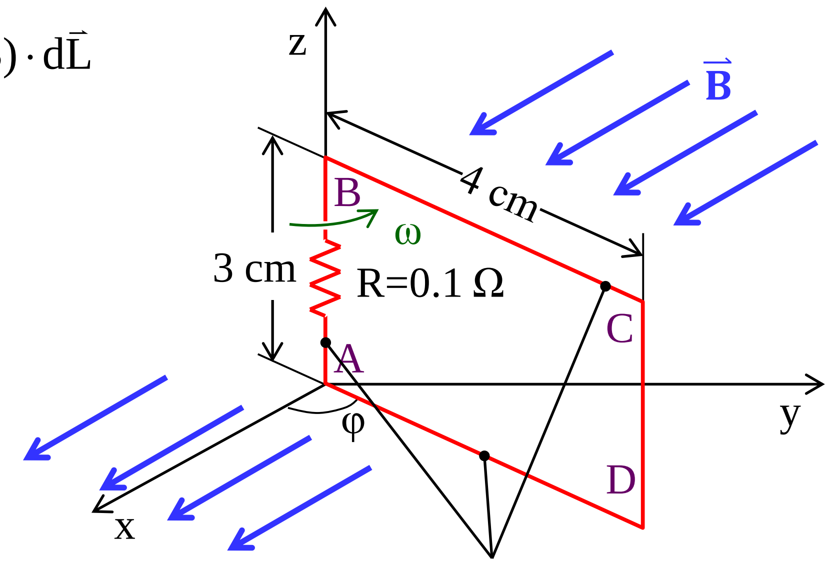
$$d\vec{L} = d\vec{L}_{DC} = dz\vec{a}_z$$

$$\vec{u} = \frac{d\vec{L}}{dt} = \frac{\rho d\phi}{dt}\vec{a}_\phi = \rho\omega\vec{a}_\phi$$

$$\omega = 2\pi f = 100\pi\ [\text{radian/sec}]$$

$$\vec{B} = B_0\vec{a}_x = B_0(\cos\phi\vec{a}_\rho - \sin\phi\vec{a}_\phi)$$

$$\vec{u}\times\vec{B} = \begin{vmatrix} \vec{a}_\rho & \vec{a}_\phi & \vec{a}_z \\ 0 & \rho\omega & 0 \\ B_0\cos\phi & -B_0\sin\phi & 0 \end{vmatrix} = -\rho\omega B_0\cos\phi\vec{a}_z$$



$$V_{emf} = \int_{z=0}^{0.03} (-\rho\omega B_0\cos\phi\vec{a}_z)\cdot dz\vec{a}_z$$

$$= \int_{z=0}^{0.03} (-\rho\omega B_0\cos\phi dz)$$

$$= -0.04\times 100\pi\times 50\times\cos\phi\times 0.03$$

$$= -6\pi\cos\phi\ \text{mV}$$

$$V_{emf} = -6\pi\cos\phi\ \text{mV}$$

$$\phi = \omega t + \pi/2$$

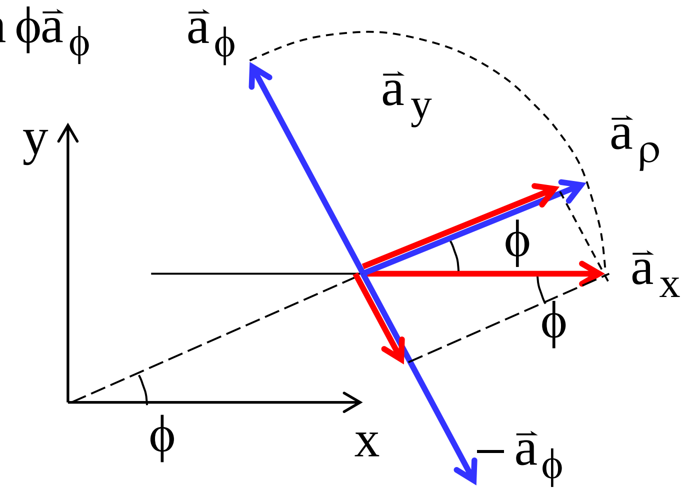
$$\phi_{t=0.001} = \pi/10 + \pi/2$$

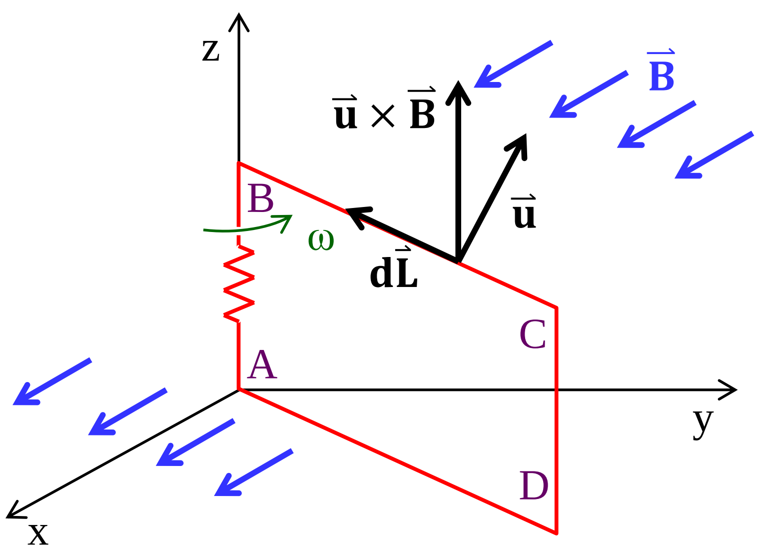
$$\cos(\pi/10 + \pi/2) = -\sin(\pi/10)$$

$$= 6\pi\sin(\pi/10)\ \text{mV}$$

$$= 5.825\ \text{mV}$$

$$\vec{a}_x = \cos\phi\,\vec{a}_\rho - \sin\phi\,\vec{a}_\phi$$





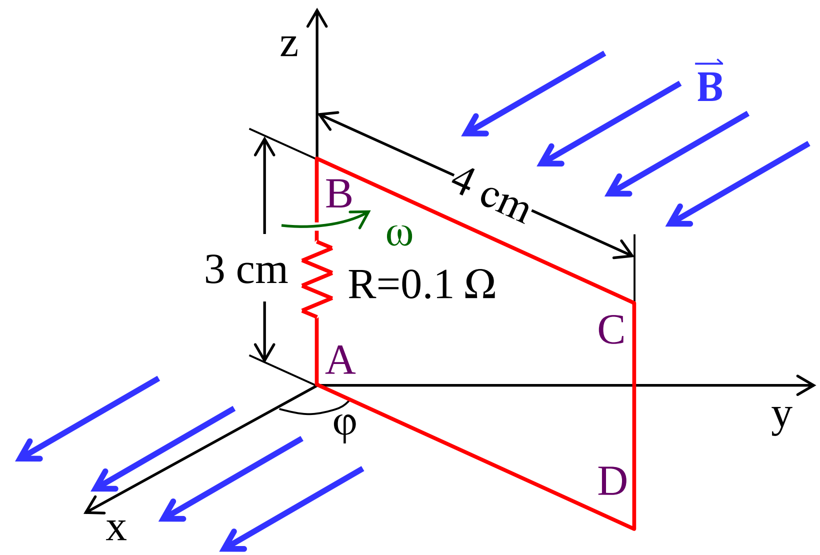
예제 9.2 그림 9.7에서 자기장 $\vec{\mathbf{B}} = \mathbf{50\,\vec{a}_x\,mWb/m^2}$ 내에 loop가 존재 한다. loop가 50 Hz로 회전하며 자속과 쇄교 하며 t=0 에서 yz 평면에 있다.
(b) t=3 msec에서 저항이 0.1 Ω일 때 유도 전류

$$V_{emf} = -6\pi\cos\phi \ \ mV$$

$$\begin{cases} \phi = \omega t + \pi/2 \\ \phi_{t=0.003} = 3\pi/10 + \pi/2 \quad \leftarrow \omega = 100\pi \\ \cos(3\pi/10 + \pi/2) = -\sin(3\pi/10) \end{cases}$$

$$= 6\pi\sin(3\pi/10) \ \ mV$$

$$I = \frac{V_{emf}}{R} = \frac{0.006\pi\sin(3\pi/10)}{0.1} = 0.1525 \ \ A$$



예제 9.3 그림 9.8에서 단면적 **10^{-3} m^2**, Turn **N_1=200회** 인 Coil에 전류 **$i_1=3\sin(100\pi t)$ A**가 흐를 때 **N_2=100회** 인 Coil에 유도되는 기전력을 구하라.
매질의 투자율은 **$\mu=500\mu_0$** 이다.

$$\psi = \frac{\Phi}{R} = \frac{N_1 i_1}{\ell/\mu S} = \frac{N_1 i_1 \mu S}{2\pi\rho_0}$$

$$V_2 = -N_2\frac{d\psi}{dt} = \frac{N_1 N_2 \mu S}{2\pi\rho_0}\frac{di_1}{dt} = \frac{N_1 N_2 \mu S}{2\pi\rho_0}300\pi\cos(100\pi t)$$

$$= -\frac{100\times200\times500\times(4\pi\times10^{-7})\times0.001\times300\pi\cos(100\pi t)}{2\times\pi\times0.1}$$

$$= -6\pi\cos(100\pi t)\ \text{volt}$$

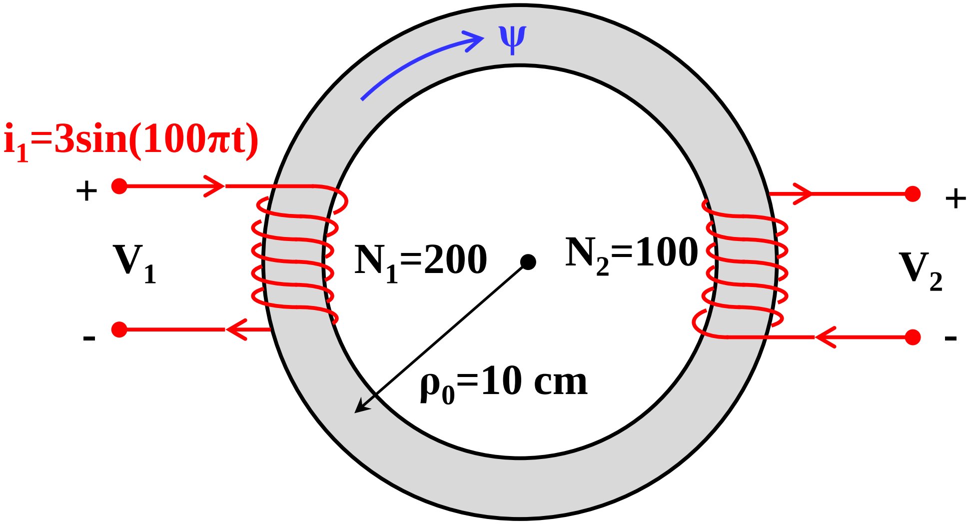


Fig 9.8 Magnetic circuit of example 9.3.

9.4 Displacement Current

$$\nabla \times \vec{H} = \vec{J} \qquad (9.17)$$

$$\begin{cases} \nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J} \\ \nabla \cdot \vec{J} = -\dfrac{\partial \rho_v}{\partial t} \neq 0 \end{cases}$$

$$\nabla \times \vec{H} \equiv \vec{J} + \vec{J}_d$$

$$\begin{cases} \nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d \\ \nabla \cdot \vec{J}_d = -\nabla \cdot \vec{J} = \dfrac{\partial \rho_v}{\partial t} = \dfrac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \dfrac{\partial \vec{D}}{\partial t} \\ \vec{J}_d = \dfrac{\partial \vec{D}}{\partial t} : \text{ displacement current} \qquad (9.22b) \end{cases}$$

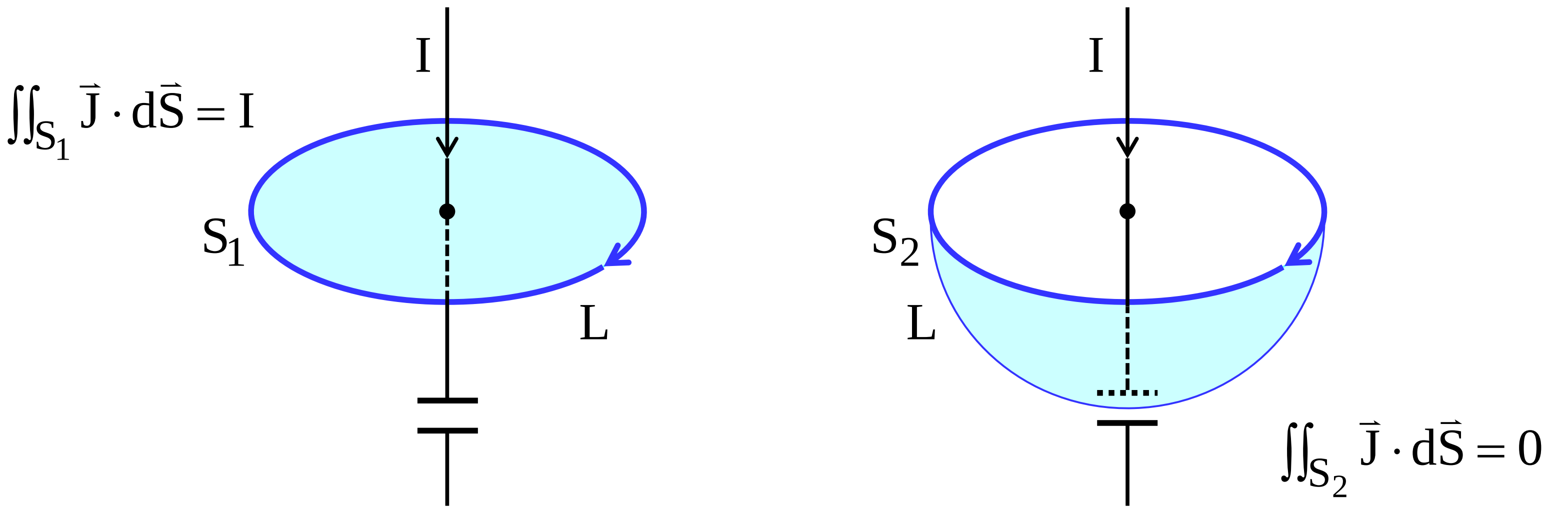
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad (9.23)$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\iint_S (\nabla \times \vec{H} = \vec{J}) \cdot d\vec{S}$$

$$\iint_S \nabla \times \vec{H} \cdot d\vec{S} = \iint_S \vec{J} \cdot d\vec{S}$$

$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S \vec{J} \cdot d\vec{S}$$



전자가 통과하지 않는다.

Fig 9.10 Two surfaces of integration showing the need for $\vec{\mathbf{J}}_{\mathbf{d}}$ in Ampère's circuit law.

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$$

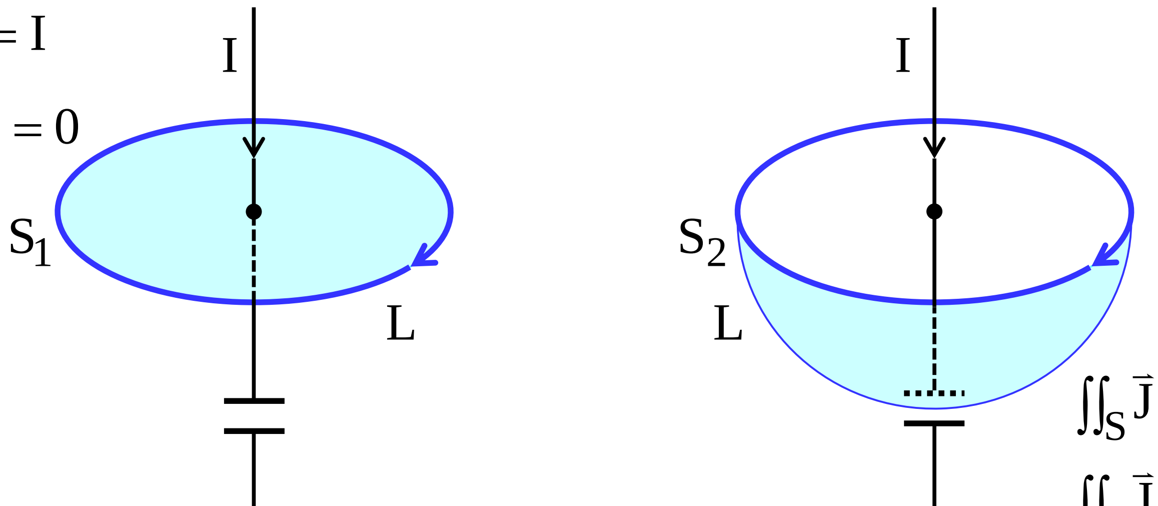
$$\iint_S (\nabla \times \vec{H} = \vec{J} + \vec{J}_d) \cdot d\vec{S}$$

$$\iint_S \nabla \times \vec{H} \cdot d\vec{S} = \iint_S (\vec{J} + \vec{J}_d) \cdot d\vec{S}$$

$$\oint_L \vec{H} \cdot d\vec{L} = \iint_S (\vec{J} + \vec{J}_d) \cdot d\vec{S}$$

$$\iint_S \vec{J} \cdot d\vec{S}_1 = I$$

$$\iint_S \vec{J}_d \cdot d\vec{S}_1 = 0$$



$$\iint_S \vec{J} \cdot d\vec{S}_2 = 0$$

$$\iint_S \vec{J}_d \cdot d\vec{S}_2 = I$$

Fig 9.10 Two surfaces of integration showing the need for $\vec{\mathbf{J}}_\mathbf{d}$ in Ampère's circuit law.

예제 9.4 극판 면적이 **5 cm²**, 사이 간격이 **3 mm** 인 평행 평판 Capacitor에 전압 **V=50sin(1000t)** 를 인가하였다. $\varepsilon=2\varepsilon_0$ 라고 할 때 변위전류 (Displacement Current)를 계산하라.

$$D = \varepsilon E = \varepsilon \frac{V}{d}$$

$$J = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{\partial V}{\partial t}$$

$$I = JS = \frac{\varepsilon S}{d}\frac{\partial V}{\partial t} = C\frac{\partial V}{\partial t}$$

$$= \frac{\varepsilon S}{d} 50000 \cos(1000t)$$

$$= \left(2 \times \frac{10^{-9}}{36\pi}\right) \times \frac{0.0005}{0.003} \times 50000 \cos(1000t)$$

$$= 147.4 \cos(1000t) \ \text{nA}$$

9.5 Maxwell's Equation in Final Forms

Table 9.1 Generalized Forms of Maxwell's Equations.

Differential form	Integral form	
$\nabla \cdot \vec{D} = \rho_v$	$\oiint_S \vec{D} \cdot d\vec{S} = \iiint \rho_v dxdydz$	Gauss Eq
$\nabla \cdot \vec{B} = 0$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$	Gauss Eq
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_L \vec{E} \cdot d\vec{L} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S}$	Faraday Eq
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint_L \vec{H} \cdot d\vec{L} = \iint_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$	Ampere Eq

- $\vec{H} = \vec{B}/\mu$: Magnetic field
- $\vec{B}$: Magnetic flux
- $\vec{D} = \varepsilon\vec{E}$: Electric flux density
- $\vec{E}$: Electric field
- $\mu_0 = 4\pi \times 10^{-7}$ H/m : Permeability (투자율)
- $\varepsilon_0 = 10^{-9}/36\pi$ F/m $= 8.854 \times 10^{-12}$ F/m
- $\vec{J}$: Current density
- ρ_v : Volume charge density

Continuity 방정식 유도

$$\iiint\left(\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla_r f+\frac{\vec{K}}{m}\cdot\nabla_v f=0\right)d^3v$$

$$\iiint\frac{\partial f}{\partial t}d^3v+\iiint(\vec{v}\cdot\nabla_r f)d^3x=-\iiint\left(\frac{\vec{K}}{m}\cdot\nabla_v f\right)d^3x$$

$$\frac{\partial}{\partial t}\iiint f d^3v+\iiint\nabla_r\cdot(\vec{v}f)d^3x=-\iiint\nabla_v\cdot\left(\frac{\vec{K}}{m}f\right)d^3x$$

$$\frac{\partial}{\partial t}\iiint f d^3v+\nabla_r\cdot\iiint(\vec{v}f)d^3x=-\oint\!\!\oint\vec{n}\cdot\left(\frac{\vec{K}}{m}f\right)dA$$

$$\frac{\partial n}{\partial t}+\nabla_r\cdot(n\vec{v})=0$$

$$\text{where}\quad n=\iiint f d^3v$$

Ohm 방정식

$$mn\frac{d\vec{v}}{dt} = -\nabla(nkT) + q\vec{E} + q(\vec{v}\times\vec{B}) - mn\nu\vec{v}$$

$$\vec{v} = \frac{q}{mn\nu}\vec{E}$$

$$qn\vec{v} = \frac{q^2}{m\nu}\vec{E}$$

$$J = \sigma\vec{E}$$

$$JA = I = \sigma A\frac{V}{L}$$

$$IR = V$$

$$\vec{F} = Q(\vec{E} + \vec{u} \times \vec{B}) \quad (9.28)$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}\vec{u} \quad (9.29)$$

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P} \quad (9.30a)$$

$$\vec{B} = \mu\vec{H} = \mu_0(\vec{H} + \vec{M}) \quad (9.30b)$$

$$\vec{J} = \sigma\vec{E} + \rho_v\vec{u} \quad (9.30c)$$

Control Volume을 이용하여 유도한다. 그림 2.7과 같이 dy와 dx 길이를 가지고 두께가 1 인 상상의 2차원 Control Volume을 설정한다. 2차원으로 Control Volume을 설정하는 이유는 도표에 나타내기가 쉽고 덜 복잡하기 때문이다. 이 Control Volume에 들어 있는 물질의 질량은 물질이 새롭게 생성되거나 없어지지 않는다면 안으로 흘러 들어가는 물리양의 Flux와 흘러 나가는 Flux의 차이 만큼 증가하게 된다.

$$\frac{\partial \rho}{\partial t} = dxdy \cdot 1$$

$$= \rho u dy \cdot 1 - \left[\rho u + \frac{\partial(\rho u)}{\partial x} dx\right] dy \cdot 1$$

$$+\rho v dx \cdot 1 - \left[\rho u + \frac{\partial(\rho v)}{\partial y} dy\right] dx \cdot 1 \quad (2.38)$$

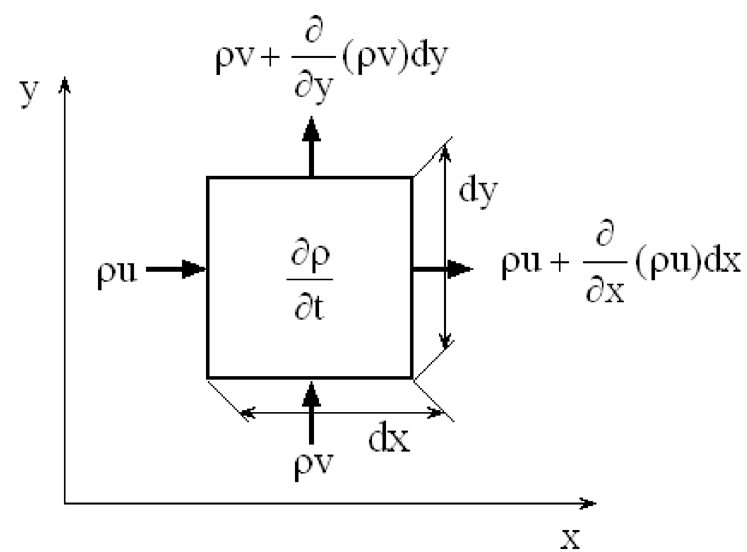


그림 2.7 연속 방정식을 유도하기 위한 Control Volume.

z 방향의 성분을 고려하여 이를 3차원으로 다시 정리하면 방정식 (2.39)와 같다.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad (2.39)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad (2.40)$$

$$\vec{E}_{1t} = \vec{E}_{2t} \quad \text{or} \quad (\vec{E}_1 - \vec{E}_2) \times \vec{a}_{n12} = 0 \qquad (9.31a)$$

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{K} \quad \text{or} \quad (\vec{H}_1 - \vec{H}_2) \times \vec{a}_{n12} = \vec{K} \qquad (9.31b)$$

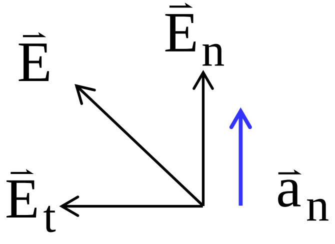
$$\vec{D}_{1n} - \vec{D}_{2n} = \rho_s \quad \text{or} \quad (\vec{D}_1 - \vec{D}_2) \cdot \vec{a}_{n12} = \rho_s \qquad (9.31c)$$

$$\vec{B}_{1n} - \vec{B}_{2n} = 0 \quad \text{or} \quad (\vec{B}_1 - \vec{B}_2) \cdot \vec{a}_{n12} = 0 \qquad (9.31d)$$

For perfect conductor

$\vec{E} = 0 \quad \vec{H} = 0 \quad \vec{J} = 0$

$\vec{E}_t = 0 \quad \vec{B}_n = 0$



(a) compatibility equations

$$\nabla \cdot \vec{B} = \rho^{m} = 0 \qquad (9.34)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \vec{J}_{m} \qquad (9.35)$$

ρ^{m}: Free magnetic density
자하밀도

J_m: magnetic current density
자기전류밀도

(b) constitutive equations

$$\vec{B} = \mu \vec{H} \qquad (9.36)$$

$$\vec{D} = \varepsilon \vec{E} \qquad (9.37)$$

(c) equilibrium equations

$$\nabla \cdot \vec{D} = \rho_{v} \qquad (9.38)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (9.39)$$

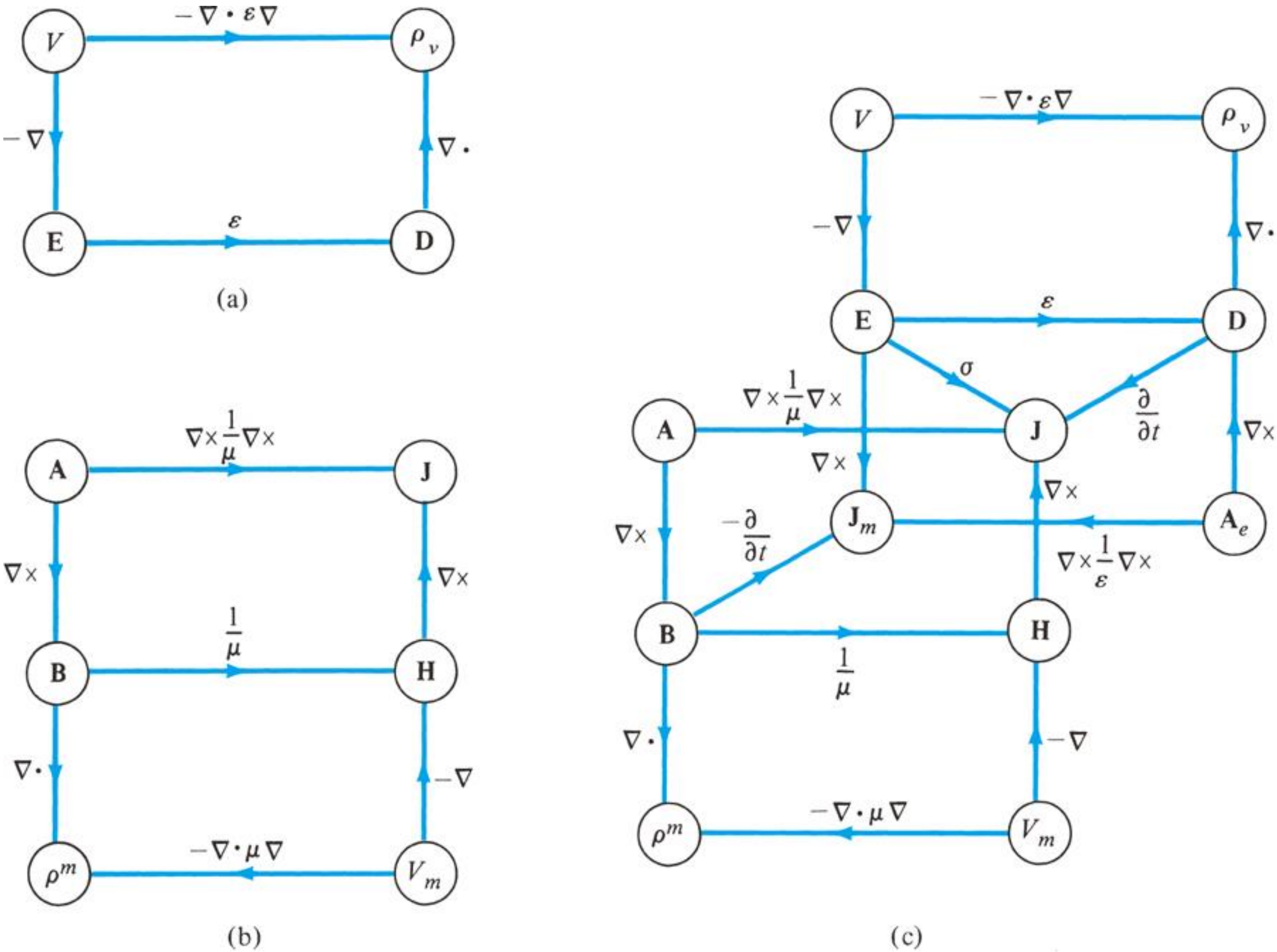


Figure 9.11 electromagnetic flow diagrams showing the relationship between the potentials and vector fields: (**a**) electrostatic system, (**b**) magnetostatic system, (**c**) electromagnetic system. [adapted with permission from the publishing department of the institution of electrical engineers.]

9.6 TIME-VARYING POTENTIALS

$$\vec{B} = \nabla \times \vec{A} \qquad \leftarrow \nabla \cdot \vec{B} = 0 \qquad (9.42)$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \qquad \leftarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (9.45)$$

$$\nabla \cdot \vec{A} = -\mu\varepsilon \frac{\partial V}{\partial t} \qquad \leftarrow \nabla \times \frac{\vec{B}}{\mu} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad (9.50)$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon} \qquad \leftarrow \nabla \times \frac{\vec{B}}{\mu} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J} \qquad (9.51)$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J} \qquad \leftarrow \nabla \times \frac{\vec{B}}{\mu} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J} \qquad (9.52)$$

참고: Vector identity

$$\nabla \cdot (\varphi\vec{u}) = \varphi\nabla \cdot \vec{u} + \vec{u} \cdot \nabla\varphi$$

$$\nabla \times (\varphi\vec{u}) = \varphi\nabla \times \vec{u} + \nabla\varphi \times \vec{u}$$

$$\nabla \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\nabla \times \vec{u}) - \vec{u} \cdot (\nabla \times \vec{v})$$

$$\nabla \times (\vec{u} \times \vec{v}) = \vec{v} \cdot \nabla\vec{u} - \vec{u} \cdot \nabla\vec{v} + \vec{v} \cdot \nabla\vec{u} - \vec{v} \cdot \nabla\vec{u}$$

$$\nabla(\vec{u} \cdot \vec{v}) = \vec{u} \cdot \nabla\vec{v} + \vec{v} \cdot \nabla\vec{u} + \vec{u} \times (\nabla \times \vec{v}) + \vec{v} \times (\nabla \times \vec{u})$$

$$\nabla \times (\nabla\varphi) = 0$$

$$\nabla \cdot (\nabla \times \vec{u}) = 0$$

$$\nabla \times (\nabla \times \vec{u}) = \nabla(\nabla \cdot \vec{u}) - \nabla^2\vec{u}$$

$$\nabla \cdot (\nabla\varphi_1 \times \nabla\varphi_2) = 0$$

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = (\vec{a} \cdot \vec{c})\vec{a} - (\vec{b} \cdot \vec{a})\vec{c}$$

$$\left(a \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = (\vec{a} \cdot \vec{c}) \cdot \left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right) \cdot \left(\vec{b} \cdot \vec{c}\right)$$

9.7 Time-Harmonic Fields

$$z = x + jy = r\angle\phi \qquad (9.57)$$

$$z = r\,e^{j\phi} = r(\cos\phi + j\sin\phi) \qquad (9.58)$$

$$r = |z| = \sqrt{x^2 + y^2} \qquad (9.59)$$

$$\phi = \tan^{-1}\frac{y}{x} \qquad (9.60)$$

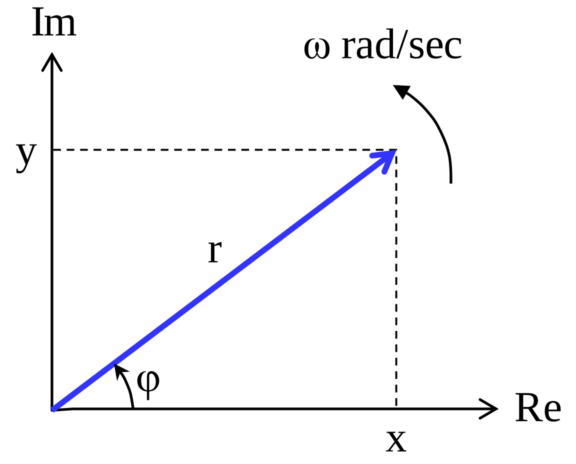


FIG 9.12 Representation of a phasor $z = x + jy = r\angle\varphi$.

$$z = \cos\theta + j\sin\theta$$

$$\frac{dz}{d\theta} = -\sin\theta + j\cos\theta$$

$$= j(\cos\theta + j\sin\theta)$$

$$= jz$$

$$\frac{dz}{z} = jd\theta$$

$$\ln z = j\theta + C$$

$$z = Ce^{j\theta} \quad \leftarrow 1 = Ce^{j0} = C$$

$$= e^{j\theta}$$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \qquad (9.61a)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \qquad (9.61b)$$

$$\begin{aligned} z_1 z_2 &= r_1 e^{j\phi_1} r_2 e^{j\phi_2} = r_1 r_2 e^{j(\phi_1 + \phi_2)} \\ &= r_1 r_2 (\angle\phi_1 + \angle\phi_2) \end{aligned} \qquad (9.61c)$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{r_1 e^{j\phi_1}}{r_2 e^{j\phi_2}} = \frac{r_1}{r_2} e^{j(\phi_1 - \phi_2)} \\ &= \frac{r_1}{r_2} (\angle\phi_1 - \angle\phi_2) \end{aligned} \qquad (9.61d)$$

$$\begin{aligned} \sqrt{z} &= \sqrt{re^{j\phi}} = \sqrt{r} e^{j\phi/2} \\ &= \sqrt{r} \angle\phi_1 / 2 \end{aligned} \qquad (9.61e)$$

$$z^* = x - jy = r\angle-\phi = re^{-j\phi} \qquad (9.91f\,)$$

$$\phi = \omega t + \theta \qquad (9.62)$$

$$re^{j\phi} = re^{j\theta}e^{j\omega t} \qquad (9.63)$$

$$\mathrm{Re}[re^{j\phi}] = r\cos(\omega t + \theta) \qquad (9.64a)$$

$$\mathrm{Im}[re^{j\phi}] = r\sin(\omega t + \theta) \qquad (9.64b)$$

Phasor current

$$I_s = I_0 e^{j\theta} = I_0 \angle\theta \qquad (9.65)$$

Phasor: obtained by dropping

the time factor $e^{j\omega t}$ in $I(t)$

$$I(t) = \mathrm{Re}[I_s e^{j\omega t}] \qquad (9.66)$$

$$\vec{A} = \mathrm{Re}(\vec{A}_s e^{j\omega t}) \qquad (9.67)$$

($\vec{\mathbf{A}}_s$: Phasor form of $\vec{\mathbf{A}}$)

If $\vec{A} = A_0 \cos(\omega t - \beta x)\vec{a}_y$

then $\vec{A} = \mathrm{Re}(A_o e^{-j\beta x}\vec{a}_y e^{j\omega t}) \qquad (9.68)$

$$\vec{A}_s = A_o e^{-j\beta x}\vec{a}_y \qquad (9.69)$$

$$\frac{\partial \vec{A}}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}[\vec{A}_s e^{j\omega t}]$$

$$= \mathrm{Re}[j\omega \vec{A}_s e^{j\omega t}] \qquad (6.70)$$

$$\frac{\partial \vec{A}}{\partial t} \rightarrow j\omega \vec{A} \qquad (6.71)$$

$$\int \vec{A} dt \rightarrow \frac{\vec{A}}{j\omega} \qquad (6.72)$$

Table 9.2 Time-Harmonic Maxwell's Equations Assuming Time Factor $e^{i\omega t}$

Point form	Integral form
$\nabla \cdot \vec{D}_s = \rho_{vs}$	$\oint\!\!\oint_S \vec{D}_s \cdot d\vec{S} = \iiint \rho_{vs} dxdydz$
$\nabla \cdot \vec{B}_s = 0$	$\oint\!\!\oint_S \vec{B}_s \cdot d\vec{S} = 0$
$\nabla \times \vec{E}_s = -j\omega \vec{B}_s$	$\oint_L \vec{E}_s \cdot d\vec{L} = -j\omega \iint_S \vec{B}_s \cdot d\vec{S}$
$\nabla \times \vec{H}_s = \vec{J}_s + j\omega \vec{D}_s$	$\oint_L \vec{H}_s \cdot d\vec{L} = \iint_S \left(\vec{J}_s + j\omega \vec{D}_s\right) \cdot d\vec{S}$

예제 9.5 다음 복소수를 계산하라.

(a) $z_1 = \dfrac{j(3-4j)^*}{(-1+6j)(2+j)^2}$ (b) $z_2 = \left[\dfrac{1+j}{4-8j}\right]^{1/2}$

$$(a)\ z_1 = \frac{j(3-4j)^*}{(-1+6j)(2+j)^2}$$

$$= \frac{j(3+4j)}{(-1+6j)(4+4j-1)}$$

$$= \frac{j(3+4j)}{(-1+6j)(3+4j)}$$

$$= \frac{3j-4}{-3+14j-24}$$

$$= \frac{-4+3j}{-27+14j}$$

$$= \frac{(-4+3j)(-27-14j)}{(-27+14j)(-27-14j)}$$

$$= \frac{150-25j)}{27^2+14^2}$$

예제 9.5 다음 복소수를 계산하라.

$$\text{(b) } z_2 = \left[\frac{1+j}{4-8j}\right]^{1/2}$$

$$\text{(b) } z_2 = \left[\frac{1+j}{4-8j}\right]^{1/2}$$

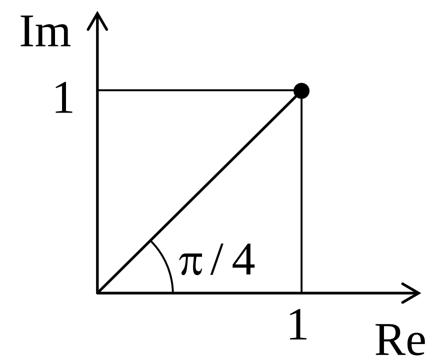
$$= \sqrt{\frac{\sqrt{2}e^{j\pi/4}}{4\sqrt{5}e^{-j1.11}}}$$

$$= \sqrt{\frac{\sqrt{2}}{4\sqrt{5}}}e^{j(\pi/4+1.11)/2}$$

$$= 0.3976e^{j0.9477}$$

$$= 0.3976e^{j(54.2^{o})}$$

$$1+j = \sqrt{2}e^{j\pi/4}$$



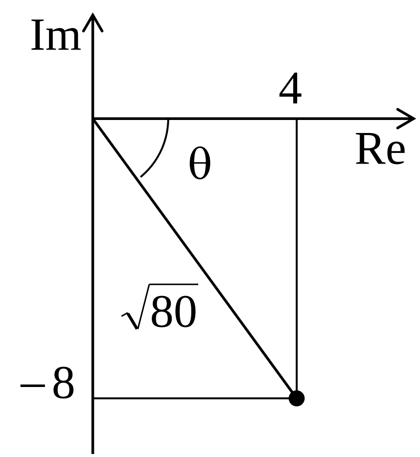
$$\tan\theta = -8/4$$

$$= -2$$

$$\theta = -63.4^{o}$$

$$= -1.11$$

$$4-8j = 4\sqrt{5}e^{-j1.11}$$



예제 9.6 $\vec{A} = 10\cos\left(10^8 t - 10x + \frac{\pi}{3}\right)\vec{a}_z$ 의 Phasor는?

$\vec{B}_s = (20/j)\vec{a}_x + 10e^{j2\pi x/3}\vec{a}_y$ 의 순시형 (instantaneous form)은?

$$\vec{A} = 10\cos(10^8 t - 10x + \pi/3)\vec{a}_z$$

$$= \mathrm{Re}\left[10e^{j(10^8 t - 10x + \pi/3)}\right]\vec{a}_z$$

$$= \mathrm{Re}\left[10e^{j(\omega t - 10x + \pi/3)}\right]\vec{a}_z$$

$$\vec{A}_s = 10e^{j(10^8 t - 10x + \pi/3)}\vec{a}_z e^{-j\omega t}$$

$$= 10e^{j(-10x + \pi/3)}\vec{a}_z$$

$$\vec{B}_s = \frac{20}{j}\vec{a}_x + 10e^{j\frac{2\pi x}{3}}\vec{a}_y$$

$$= -20j\vec{a}_x + 10e^{j\frac{2\pi x}{3}}\vec{a}_y$$

$$= 20e^{-j\frac{\pi}{2}}\vec{a}_x + 10e^{j\frac{2\pi x}{3}}\vec{a}_y$$

$$\vec{B} = \mathrm{Re}[\vec{B}_s e^{j\omega t}]$$

$$= \mathrm{Re}\left[\left(20e^{-j\frac{\pi}{2}}\vec{a}_x + 10e^{j\frac{2\pi x}{3}}\vec{a}_y\right)e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[20e^{-j\frac{\pi}{2}+j\omega t}\vec{a}_x + 10e^{j\frac{2\pi x}{3}+j\omega t}\vec{a}_y\right]$$

$$= 20\cos\left(-\frac{\pi}{2} + \omega t\right)\vec{a}_x + 10\cos\left(\frac{2\pi x}{3} + \omega t\right)\vec{a}_y$$

$$= 20\sin(+\omega t)\vec{a}_x + 10\cos\left(\frac{2\pi x}{3} + \omega t\right)\vec{a}_y$$

예제 9.7 $\vec{E} = \frac{50}{\rho}\cos(10^6 t + \beta z)\vec{a}_\varphi$ volt/m $\quad \vec{H} = \frac{H_0}{\rho}\cos(10^6 t + \beta z)\vec{a}_\rho$ A/m 일 때 Phasor Form으로 변형하고 Maxwell Eq을 만족 시키기 위한 상수 H_0, β를 구하라. Space Charge는 없다.

$$\vec{E} = \vec{E}_s e^{j\omega t} \qquad \vec{E}_s = \frac{50}{\rho} e^{j\beta z}\vec{a}_\phi$$

$$\vec{H} = \vec{H}_s e^{j\omega t} \qquad \vec{H}_s = \frac{H_0}{\rho} e^{j\beta z}\vec{a}_\rho$$

$$\begin{cases} \varepsilon\nabla\cdot\vec{E} = 0 \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{E}}{\partial t} \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \end{cases} \Rightarrow \begin{cases} \nabla\cdot\vec{E}_s = 0 \rightarrow \frac{1}{\rho}\frac{\partial}{\partial\phi}E_{\phi s} = 0 \\ \nabla\cdot\vec{H}_s = 0 \rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}H_{\rho s} = 0 \\ \nabla\times\vec{H}_s = j\omega\varepsilon\vec{E}_s \\ \nabla\times\vec{E}_s = -j\omega\mu\vec{H}_s \end{cases}$$

$$\nabla\times\vec{H}_s = \nabla\times\left[\frac{H_0}{\rho}e^{j\beta z}\vec{a}_\rho\right] = \frac{jH_0\beta}{\rho}e^{j\beta z}\vec{a}_\phi$$

$$\frac{jH_0\beta}{\rho}e^{j\beta z}\vec{a}_\phi = j\omega\varepsilon\frac{50}{\rho}e^{j\beta z}\vec{a}_\phi$$

$$\rightarrow H_0\beta = 50\omega\varepsilon$$

$$\begin{cases} \vec{E} = \vec{E}_s e^{j\omega t} & \vec{E}_s = \dfrac{50}{\rho} e^{j\beta z} \vec{a}_\phi \\ \vec{H} = \vec{H}_s e^{j\omega t} & \vec{H}_s = \dfrac{H_0}{\rho} e^{j\beta z} \vec{a}_\rho \end{cases}$$

$$\begin{cases} \varepsilon \nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{E}}{\partial t} \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \end{cases} \Rightarrow \begin{cases} \nabla \cdot \vec{E}_s = 0 \rightarrow \dfrac{1}{\rho} \dfrac{\partial}{\partial \phi} E_{\phi s} = 0 \\ \nabla \cdot \vec{H}_s = 0 \rightarrow \dfrac{1}{\rho} \dfrac{\partial}{\partial \rho} H_{\rho s} = 0 \\ \nabla \times \vec{H}_s = j\omega\varepsilon \vec{E}_s \\ \nabla \times \vec{E}_s = -j\omega\mu \vec{H}_s \end{cases}$$

$$\begin{pmatrix} \nabla \times \vec{E}_s = \nabla \times \left[\dfrac{50}{\rho} e^{j\beta z} \vec{a}_\phi \right] = -j\beta \dfrac{50}{\rho} e^{j\beta z} \vec{a}_\rho \\ -j\beta \dfrac{50}{\rho} e^{j\beta z} \vec{a}_\rho = -j\omega\mu \dfrac{H_0}{\rho} e^{j\beta z} \vec{a}_\rho \end{pmatrix}$$

$$\rightarrow \frac{H_0}{\beta} = \frac{50}{\omega\mu}$$

$$\left(\begin{aligned}&\nabla\times\vec{H}_s=\nabla\times\left[\frac{H_0}{\rho}e^{j\beta z}\vec{a}_\rho\right]=\frac{jH_0\beta}{\rho}e^{j\beta z}\vec{a}_\phi\\&\frac{jH_0\beta}{\rho}e^{j\beta z}\vec{a}_\phi=j\omega\varepsilon\frac{50}{\rho}e^{j\beta z}\vec{a}_\phi\end{aligned}\right)$$

$$\rightarrow \boxed{H_0\beta=50\omega\varepsilon}$$

$$\left(\begin{aligned}&\nabla\times\vec{E}_s=\nabla\times\left[\frac{50}{\rho}e^{j\beta z}\vec{a}_\phi\right]=-j\beta\frac{50}{\rho}e^{j\beta z}\vec{a}_\rho\\&-j\beta\frac{50}{\rho}e^{j\beta z}\vec{a}_\rho=-j\omega\mu\frac{H_0}{\rho}e^{j\beta z}\vec{a}_\rho\end{aligned}\right)$$

$$\rightarrow \boxed{\frac{H_0}{\beta}=\frac{50}{\omega\mu}}$$

$$\Rightarrow \boxed{\left(\begin{aligned}&\beta=\pm3.33\times10^{-3}\\&H_0=\pm0.1326\end{aligned}\right.}$$

참고

$$(3.28)\ \nabla V = \frac{\partial V}{\partial x}\vec{a}_x + \frac{\partial V}{\partial y}\vec{a}_y + \frac{\partial V}{\partial z}\vec{a}_z$$

$$(3.29)\ \nabla V = \frac{\partial V}{\partial \rho}\vec{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\vec{a}_\phi + \frac{\partial V}{\partial z}\vec{a}_z$$

$$(3.30)\ \nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a}_\phi$$

$$(3.39)\ \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$(3.40)\ \nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$(3.41)\ \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$(3.53)\ \nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$(3.55)\ \nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$(3.56)\ \nabla \times \vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

예제 9.8 $\vec{E} = 20\sin(10^8 t - \beta z)\vec{a}_y$ **volt/m** 이다.
$\sigma = 0,\ \ \mu = \mu_0,\ \ \varepsilon = 4\varepsilon_0$ 일 때 $\beta, \vec{H}$ 를 계산하라.

$$\cos(-\pi/2 + \omega t - \beta z) = \cos(-\pi/2)\cos(\omega t - \beta z) - \sin(-\pi/2)\sin(\omega t - \beta z)$$
$$= \sin(\omega t - \beta z)$$

$$\vec{E}_s = \vec{E}e^{-j\omega t} = 20e^{-j\pi/2}e^{-j\beta z}\vec{a}_y$$

$$\nabla \times \vec{E}_s = -j\omega\mu\vec{H}_s \rightarrow \vec{H}_s = \frac{\nabla \times \vec{E}_s}{-j\omega\mu} = \frac{1}{-j\omega\mu}\left[-\frac{\partial E_{ys}}{\partial z}\vec{a}_x\right] = \frac{20\beta}{\omega\mu}e^{-j\pi/2}e^{-j\beta z}\vec{a}_x$$

$$\nabla \times \vec{H}_s = j\omega\varepsilon\vec{E}_s \rightarrow \vec{E}_s = \frac{\nabla \times \vec{H}_s}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\frac{\partial H_{xs}}{\partial z}\vec{a}_y = \frac{20\beta^2}{\omega^2\mu\varepsilon}e^{-j\beta z}\vec{a}_y$$

$$\left(\begin{array}{l} \vec{E} = \vec{E}_s e^{-j\omega t} \quad \vec{E}_s = 20e^{-j\pi/2}e^{-j\beta z}\vec{a}_y \\ \vec{E}_s = \dfrac{20\beta^2}{\omega^2\mu\varepsilon}e^{-j\pi/2}e^{-j\beta z}\vec{a}_y \end{array}\right) \rightarrow 20 = \frac{20\beta^2}{\omega^2\mu\varepsilon} \rightarrow \beta = \pm 2/3$$

$$\vec{H}_s = \frac{20\beta}{\omega\mu}e^{-j\pi/2}e^{-j\beta z}\vec{a}_x = \pm\frac{1}{3\pi}e^{-j\pi/2}e^{-j\beta z}\vec{a}_x$$

$$\vec{H} = \mathrm{Re}\left[\vec{H}_s e^{j\omega t}\right]$$
$$= \pm\frac{1}{3\pi}\sin(10^8 t \pm \beta z)\vec{a}_x \ \ \mathrm{A/m}$$