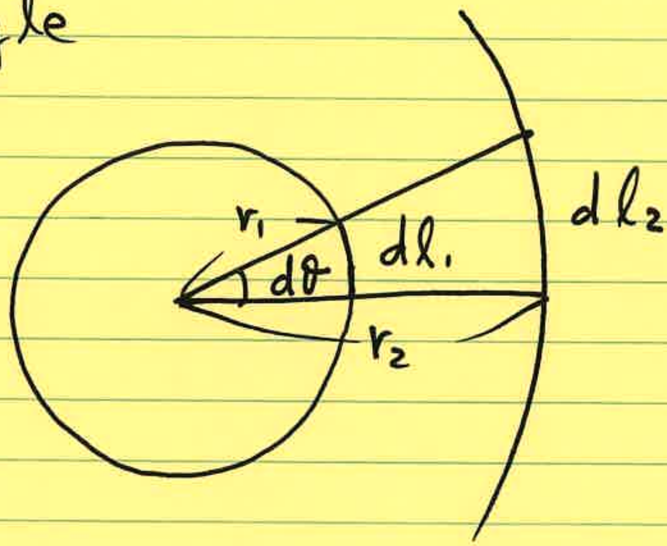
Solid Angle

* Angle

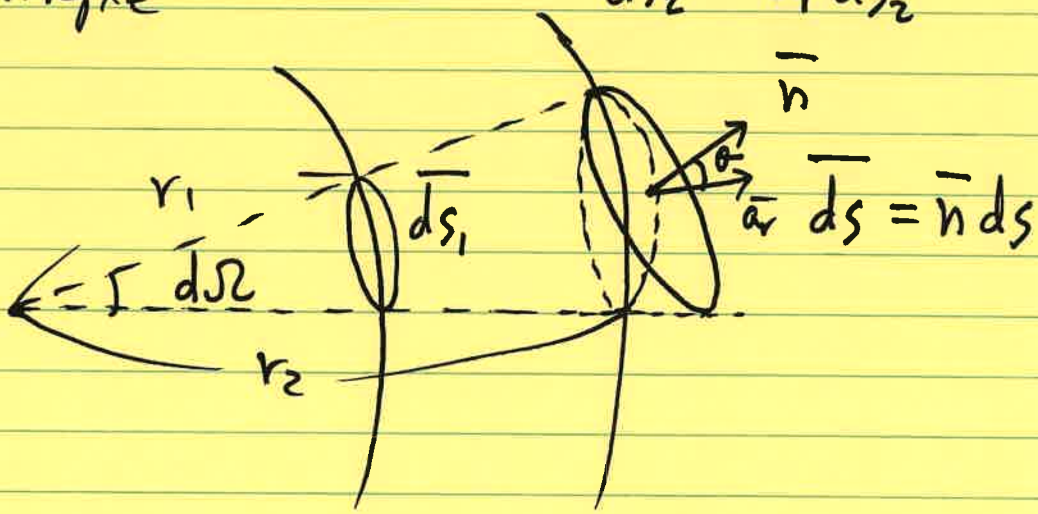


$$d\theta = \frac{dl_1}{r_1} = \frac{dl_2}{r_2} = \frac{dl}{r}$$

$$\theta = \frac{2\pi r}{r} = 2\pi \quad [\text{radian}]$$

* Solid Angle

$\overline{ds_2} = \bar{a}_r ds_2$



$$d\Omega = \frac{ds_1}{r_1^2} = \frac{ds_2}{r_2^2} = \frac{ds}{r^2}$$

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ [steradian]}$$

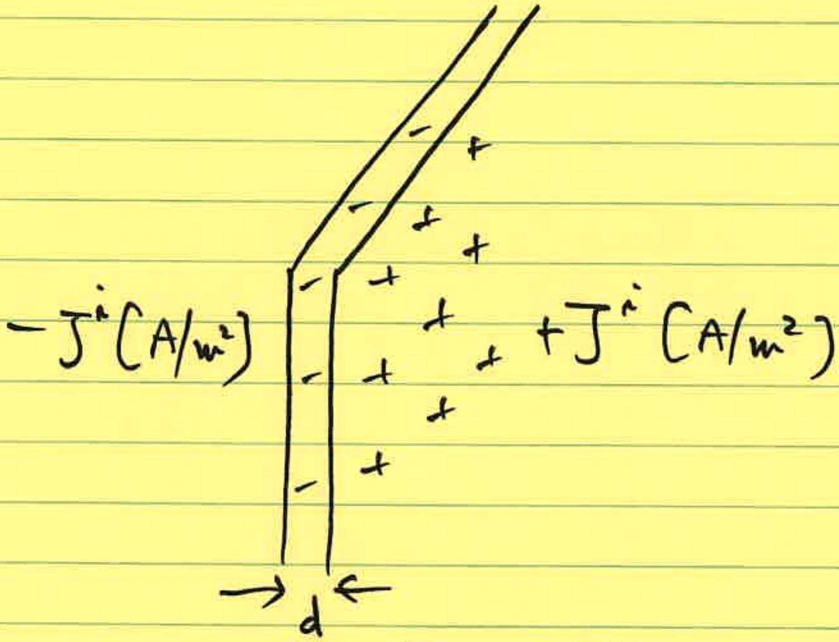
$$d\Omega = \frac{\overline{ds} \cdot \overline{a_r}}{r^2} = \frac{ds \cos\theta}{r^2}$$

$$= -\nabla\left(\frac{1}{r}\right) \cdot \overline{ds}$$

$$-d\Omega = \nabla\left(\frac{1}{r}\right) \cdot \overline{ds}$$

Charge Double Layer

– Distributed dipole source



$$\rightarrow \bar{p} = \bar{J}^i d$$

$$\phi_d = \phi_1 - \phi_2 = J^i \rho\, d$$

ρ: $[\Omega \cdot m]$, d: $[m]$

$$\underset{[A/m]}{\bar{p}} = \underset{[A/m^2]\,[m]}{\bar{J}^i d}$$

$$\bar{J}^i\, dv = \bar{J}^i d\, ds$$

$$= \bar{p}\, ds$$

$$= p\, \bar{n}\, ds = p\, \overline{ds}$$

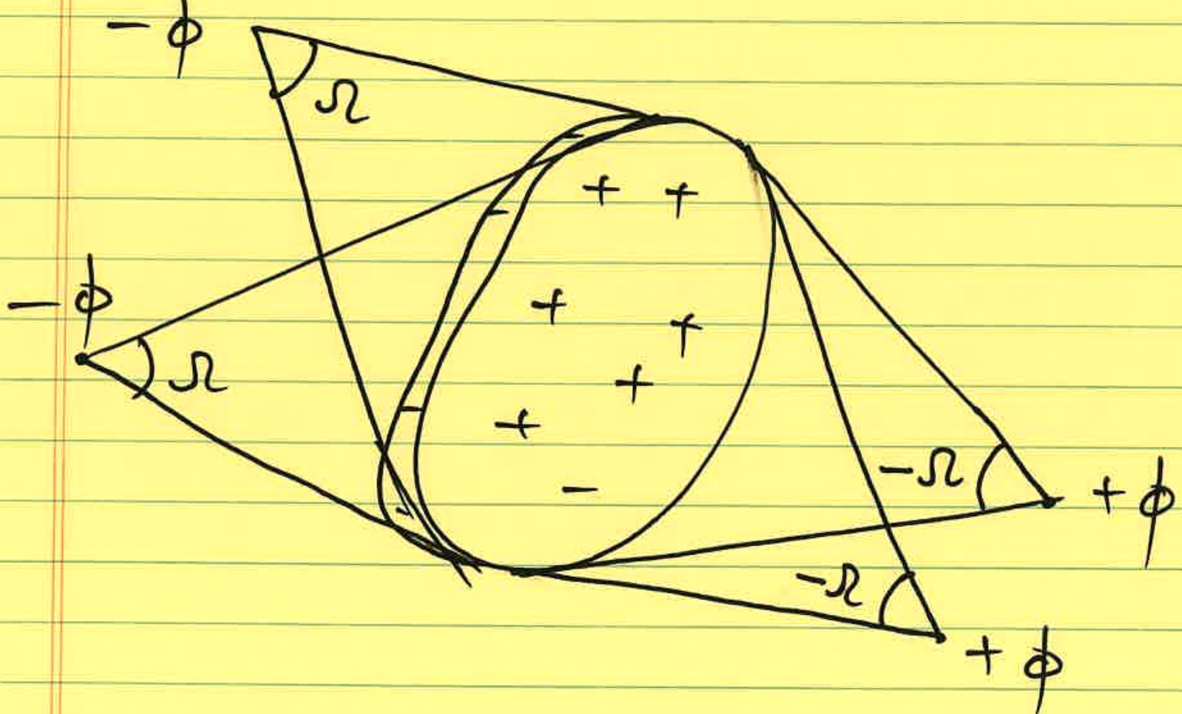
$$\phi = \frac{1}{4\pi\sigma} \int_v \bar{J}^i \cdot \nabla\left(\frac{1}{r}\right) dv$$

$$= \frac{1}{4\pi\sigma} \int_s p \, \nabla\left(\frac{1}{r}\right) \cdot \bar{ds}$$

$$= \frac{1}{4\pi\sigma} \int_s p \left(-d\Omega\right)$$

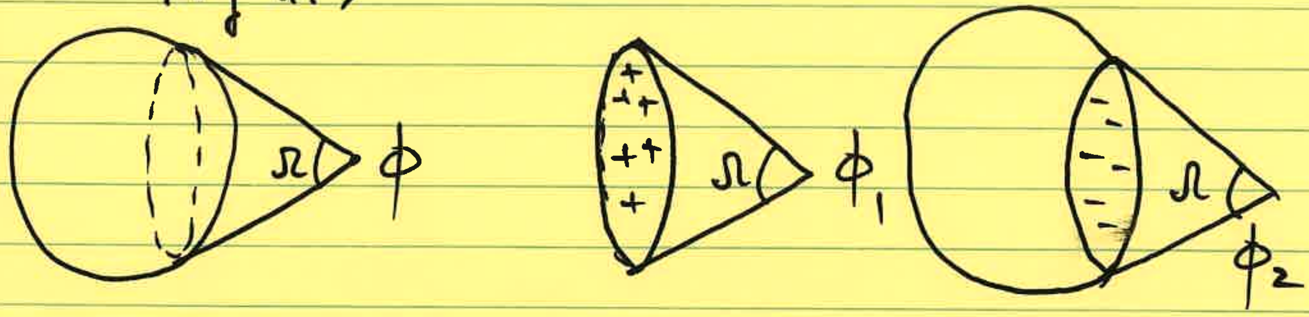
* Uniform double layer

$$\phi = \frac{1}{4\pi\sigma} p \left(-\Omega\right)$$



* closed double layer

Fig 11.3



$$\phi = \phi_1 + \phi_2 = 0 \quad , \quad \phi_1 = -\phi_2$$

- A closed uniform double layer generates a zero external potential.
- The potential of an open uniform double layer is determined by the rim of the openning.

See Fig 11. 4