Chapter 3 Active Filter : Part I

Filter :

Frequency Response $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$

$$\begin{cases} |H(j\omega)| & : \text{magnitude response (gain)} \\ \angle H(j\omega) & : \text{phase '' (phase shift)} \end{cases}$$

* Common Frequency Responses

- Lowpass
- Highpass
- Bandpass
- Band-reject (notch)

} ⇒ fig 3.1

- All-pass (delay filter.)

⇒
- Cutoff frequency, $\omega_c = 2\pi f_c$
- pass band (radio tuning circuitry)
- stop band (60Hz notch filter)
- transistion band

See fig 3.2 (effect of filtering)

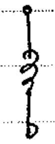
* Active filters : filters incorporating amplifiers.

cf) passive filters

* Discuss RC filter (LP & HP)

3.1 Transfer function

$\Rightarrow Z_c = \dfrac{1}{sC}$: complex impedance



$\Rightarrow Z_L = sL$: complex impedance

$s = \sigma + j\omega$: complex frequency

- $V = Z(s)\, I$: ohm's law
- KVL
- KCL
- Voltage and current division
- Superposition

X_i : input , X_o : output

$H(s) = \dfrac{X_o}{X_i}$: transfer function

$\boxed{(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt}$

$x_o(t) = \mathcal{L}^{-1}\{ H(s)\, X_i(s) \}$

In linear circuits,

$$H(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + \cdots + a_1 s + a_0}{b_n s^n + \cdots + b_1 s + b_0}$$

↪ rational function of s
(유리함수)

m and n : degree

n : order of filter.

$N(s)\Big|_{s=z_i} = 0 \Rightarrow s = z_i$: zero

$D(s)\Big|_{s=p_i} = 0 \Rightarrow s = p_i$: pole

$$H(s) = H_0 \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

$H_0 = \frac{a_m}{b_n}$: scaling factor

roots : critical frequency or
characteristic frequency

- real
- complex conjugate

* Example 3.1 ⇒ [illegible] (fig. 3.3)

* H(s) and Stability

BIBO ⇒ stable

CASE I. H(s) has a real pole.

$$\mathcal{L}^{-1}\left\{\frac{A_k}{s-\sigma_k}\right\} = A_k e^{\sigma_k t} u(t)$$

$\sigma_k < 0$: decay

$\sigma_k = 0$: constant

$\sigma_k > 0$: divergence

CASE II. H(s) has a complex conjugate poles

$$\mathcal{L}^{-1}\left\{ \frac{A_k}{s-(\sigma_k+j\omega_k)} + \frac{A_k^*}{s-(\sigma_k-j\omega_k)} \right\}$$
$$= 2\,|A_k|\, e^{\sigma_k t}\, u(t) \cos(\omega_k t + \angle A_k)$$

We need $\sigma_k < 0$ for stability.

" A circuit is stable if all poles lie in the left half of the s plane (LHP) "

* H(s) and Frequency Response

$\left\{ \begin{array}{l} \text{Stable circuit (all poles in LHP)} \\ \Downarrow \\ \text{transient response die out} \end{array} \right.$

$\Rightarrow$ We will study "steady-state response" only.

$x_i(t) = X_{im} \cos(\omega t + \theta_i)$

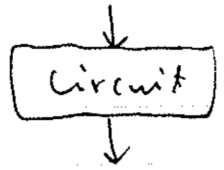


Fig. 3.4

$x_o(t) = X_{om} \cos(\omega t + \theta_o)$

$s = \sigma + j\omega$

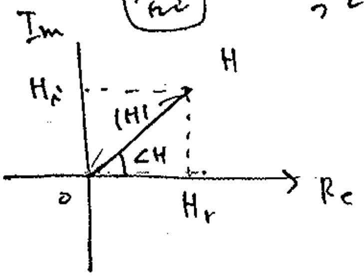
$\begin{cases} \sigma \Rightarrow \text{transient response} \\ \omega \Rightarrow \text{steady state response} \end{cases}$

$\Rightarrow$ We will compute $H(s)$ on $j\omega$-axis only to find out steady state response.

Let $s \rightarrow j\omega = j2\pi f$, then

$$\begin{cases} X_{om} = |H(j\omega)| \, X_{im} \\ \theta_o = \angle H(j\omega) + \theta_i \end{cases}$$

$\boxed{\text{참고}}$ 복소수 연산



$H = H_r + jH_i = |H| \angle{\angle H}$

$$\begin{cases} |H| : \text{magnitude or modulus} \\ \angle H : \text{argument or phase angle} \\ H_r : \text{real part} \\ H_i : \text{imaginary part} \end{cases}$$

$$|H| = \sqrt{H_r^2 + H_i^2}$$

$$\angle H = \tan^{-1} \frac{H_i}{H_r} \quad \text{if} \quad H_r > 0$$

$$\angle H = 180^\circ - \tan^{-1} \frac{H_i}{H_r} \quad \text{if} \quad H_r < 0$$

$$|H_1 H_2| = |H_1| \, |H_2|$$

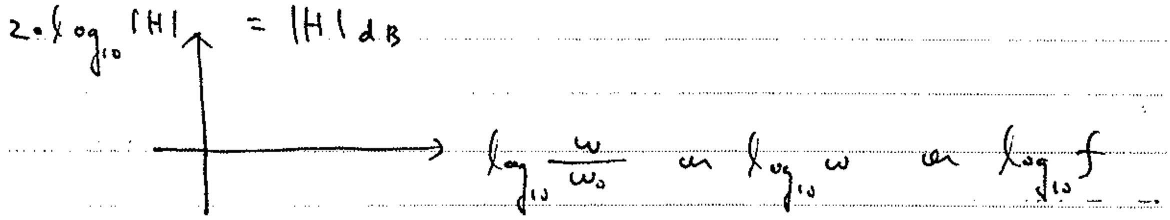
$$\angle(H_1 H_2) = \angle H_1 + \angle H_2$$

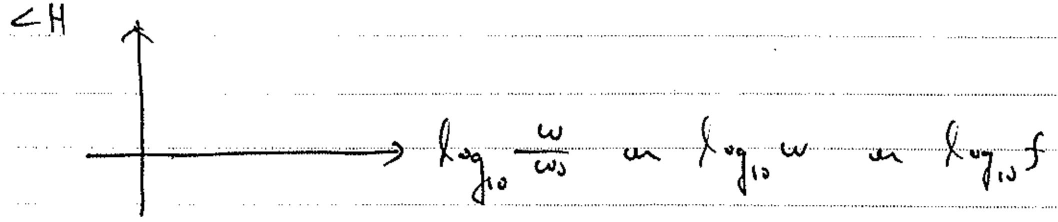
$$|H_1/H_2| = |H_1|/|H_2|$$
$$\angle(H_1/H_2) = \angle H_1 - \angle H_2$$

Cases. (주어지면 / 을 구하는 경우.)

① Circuit → plot $|H(j\omega)|$ & $\angle H(j\omega)$ (Bode plots)
② $H(j\omega)$ → find $H(s)$ & pole-zero plot
③ $H(j\omega)$ → design a circuit
④ Circuit → find $H(j\omega)$ experimentally

* Bode Plots





$$|H_1 H_2|_{dB} = |H_1|_{dB} + |H_2|_{dB}$$
$$|H_1/H_2|_{dB} = |H_1|_{dB} - |H_2|_{dB}$$
$$|1/H_1|_{dB} = -|H|_{dB}$$

⊖ Asymptotic approximation

$$\begin{cases} H \simeq H_r & \text{if } |H_r| \gg |H_i| \\ H \simeq jH_i & \text{if } |H_i| \gg |H_r| \end{cases}$$

3.2 First-Order Active Filter

$$Z_c = \frac{1}{sC} = \frac{1}{j\omega C}$$

$\lim_{\omega \to 0} Z_c = \infty$: for DC, cap. is open

$\lim_{\omega \to \infty} Z_c = 0$: for very high freq., cap. is short

* Differentiator

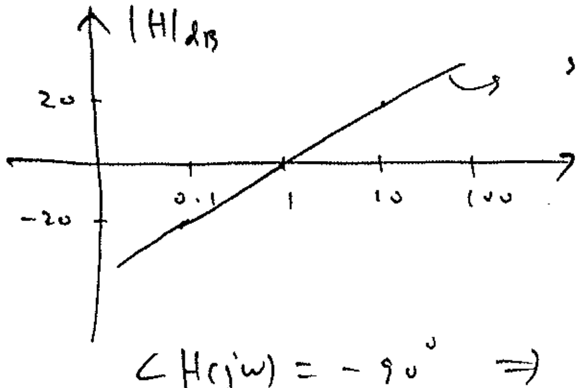
$$V_o = -\frac{R}{Z_c} V_i = -RCsV_i$$

$H(s) = -RCs$: a zero at the origin

$H(j\omega) = -j\omega RC$

let $\omega_0 = \frac{1}{RC}$ (scaling freq.),

$$H(j\omega) = -\frac{j\omega}{\omega_0} = \frac{\omega}{\omega_0} \angle{-90^\circ}$$

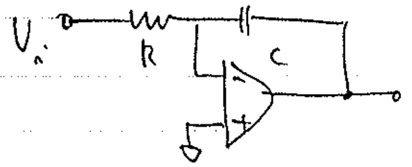


$\angle H(j\omega) = -90^\circ \Rightarrow 90^\circ$ phase lag

at $\omega = \omega_0$, $|Z_c| = R$ and $|H|_{dB} = 0$

$\Rightarrow$ ω_o is the unity-gain frequency

* Integrator



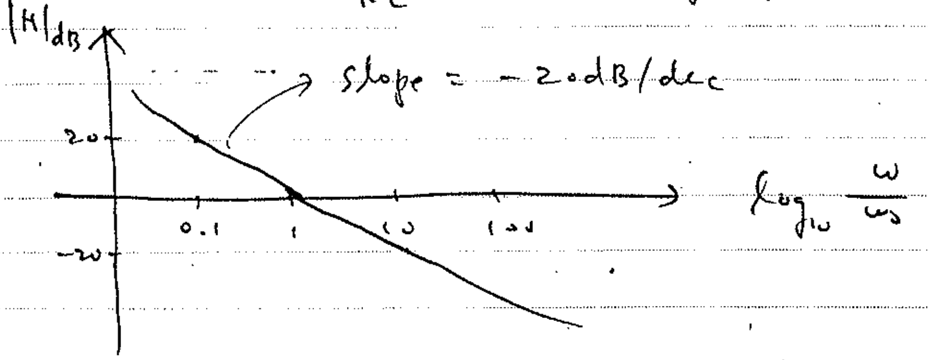
$$V_o = -\frac{Z_c}{R}V_i = -\frac{1}{RCs}V_i$$

(Miller integrator or inverting integrator)

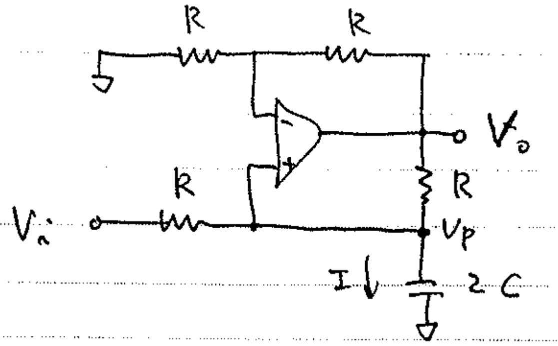
$$H(s) = -\frac{1}{RCs} \quad : \text{ a pole at the origin}$$

$$H(j\omega) = -\frac{1}{j\omega/\omega_o} = \frac{1}{\omega/\omega_o}\angle{+90^\circ}$$

$$\omega_o = \frac{1}{RC} \quad : \text{ unity-gain freq.}$$



$\angle H = +90^\circ$ $\Rightarrow$ 90° phase lead



: Deboo integrator

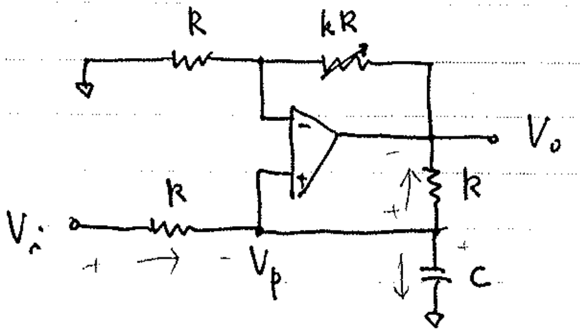
$$I = \frac{V_i}{R} \quad (\because \text{ Howland current pump})$$

$$V_p = \frac{1}{2sC} I = \frac{V_i}{2sRC}$$

$$V_o = \left(1 + \frac{R}{R}\right) V_p = \frac{V_i}{sRC}$$

$$H(s) = \frac{1}{sRC}$$

$$\angle H = -90^\circ \Rightarrow 90^\circ \text{ phase lag.}$$



$$\left(\begin{array}{l} \text{NIC} \Rightarrow -\frac{R}{kR} R = -\frac{R}{k}, \quad k \geq 0 \\ \text{Net resistance seen by } C = R \,/\!/\, (-R/k) \\ \qquad = \frac{R}{1-k} \end{array} \right)$$

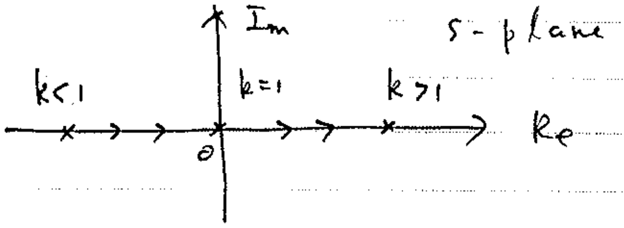
$$\begin{cases} \dfrac{V_i - V_p}{R} = sC\,V_p + \dfrac{V_p - V_o}{R} \\ V_o = \left(1 + kR/R\right) V_p = (1+k)\,V_p \end{cases}$$

$$\Rightarrow \quad V_o = \frac{1+k}{sRC + 1-k}\,V_i$$

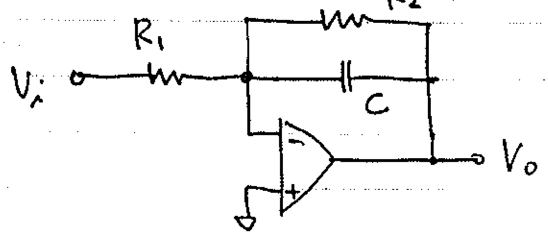
$$\therefore \; H(s) = \frac{1+k}{sRC + 1 - k}$$

$$p = -\frac{1-k}{RC}$$

natural response, $V_o(t) = V_o(0)\, e^{-t(1-k)/RC}\, u(t)$



* Low-pass Filter with Gain



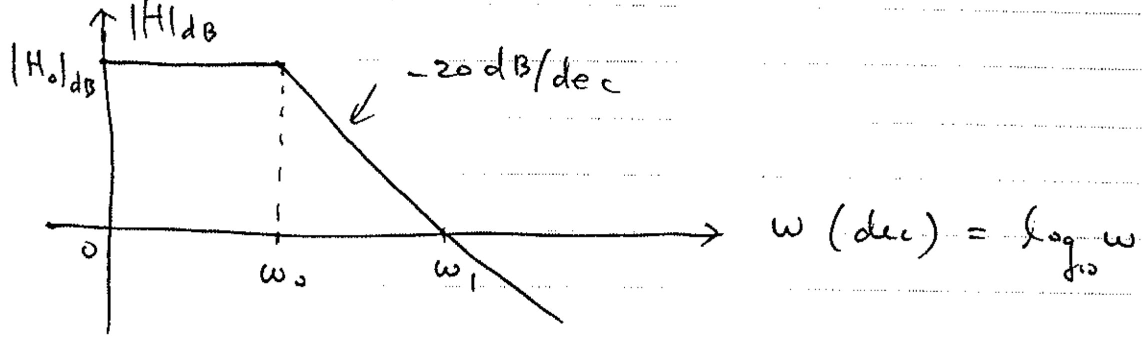
$$Z_2 = R_2 \,/\!/\, \frac{1}{sC} = \frac{R_2 \cdot \frac{1}{sC}}{R_2 + \frac{1}{sC}} = \frac{R_2}{1 + sR_2C}$$

$$H(s) = -\frac{Z_2}{R_1} = -\frac{R_2}{R_1}\,\frac{1}{1+sR_2C}$$

$$p = -\frac{1}{R_2 C}$$

$$H(j\omega) = H_0 \frac{1}{1 + j\omega/\omega_0}$$

dc gain $\leftarrow$ $H_0 = -\frac{R_2}{R_1}$, $\omega_0 = \frac{1}{R_2 C}$



at $\omega = \omega_0$, $|H| = |H_0| / |1+j| = \frac{1}{\sqrt{2}} |H_0|$

OR, $|H|_{dB} = |H_0|_{dB} - 3\ dB$

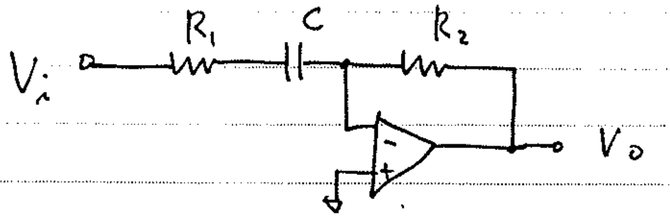
$\Rightarrow \omega_0$: $-3\,dB$ cutoff frequency

$0 \sim \omega_0$: $3\,dB$ bandwidth of LPF

记住, $\omega > \omega_0 \Rightarrow$ (lossy) integrator

Example 3.4

* High-pass Filter with Gain



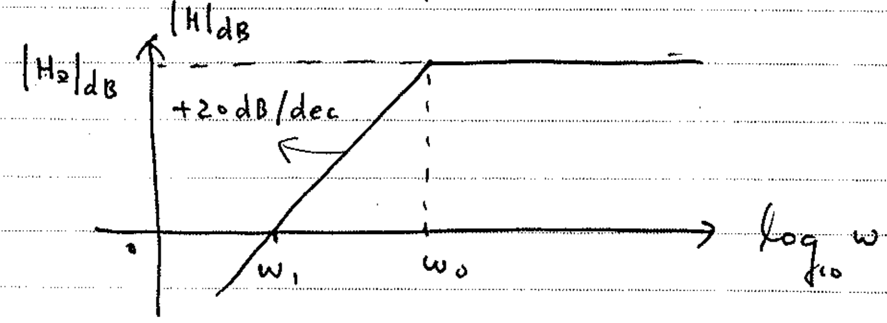
$$Z_1 = R_1 + \frac{1}{sC} = \frac{1 + sR_1C}{sC}$$

$$H(s) = -\frac{R_2}{Z_1} = -\frac{R_2\, sC}{1 + sR_1C} = -\frac{R_2}{R_1}\,\frac{sR_1C}{1 + sR_1C}$$

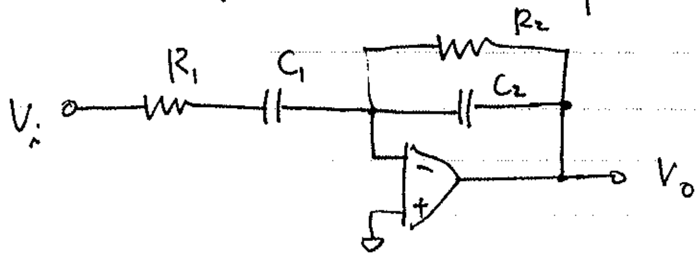
$$z = 0, \quad p = -\frac{1}{R_1C}$$

$$H(j\omega) = H_0 \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}.$$

high-freq. gain $\leftarrow$ $H_0 = -\frac{R_2}{R_1}$, $\omega_0 = \frac{1}{R_1C}$: -3dB cutoff freq.



* Wideband Band-pass Filter



$$Z_1 = R_1 + \frac{1}{sC_1} = \frac{1 + sR_1C_1}{sC_1}$$

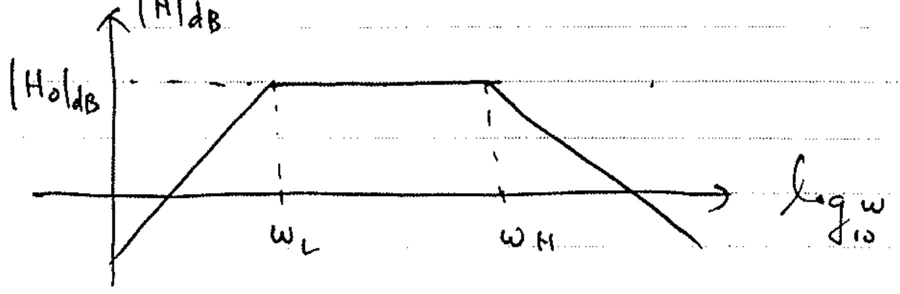
$$Z_2 = R_2 \,//\, \frac{1}{sC_2} = \frac{R_2}{1 + sR_2C_2}$$

$$H(s) = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1} \cdot \frac{R_1C_1 s}{1 + R_1C_1 s} \quad \frac{1}{1 + R_2C_2 s}$$

$$z = 0, \quad p = -\frac{1}{R_1C_1} \quad \& \quad -\frac{1}{R_2C_2}$$

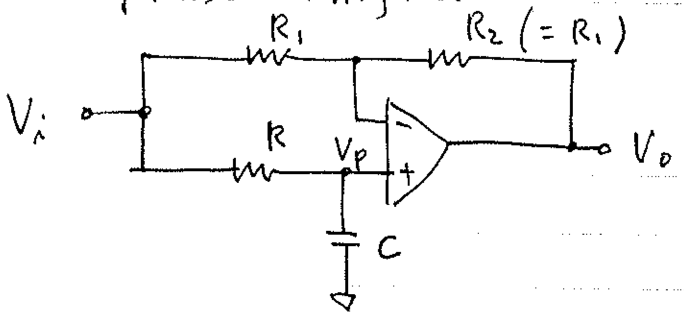
$$H(j\omega) = H_0 \frac{j\omega/\omega_L}{(1 + j\omega/\omega_L)(1 + j\omega/\omega_H)}, \quad \omega_H > \omega_L$$

$$\begin{cases} H_0 = -\dfrac{R_2}{R_1} & : \text{midfrequency gain (passband gain)} \\ \omega_L & : \text{low } -3\text{ dB cutoff freq.} \\ \omega_H & : \text{high } -3\text{ dB} \quad \text{"} \quad \text{"} \end{cases}$$



$\boxed{\text{Example 3.5}}$

* phase Shifter



$$V_p = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i = \frac{1}{1 + RCs} V_i$$

(R₂는 R₁와 같음)

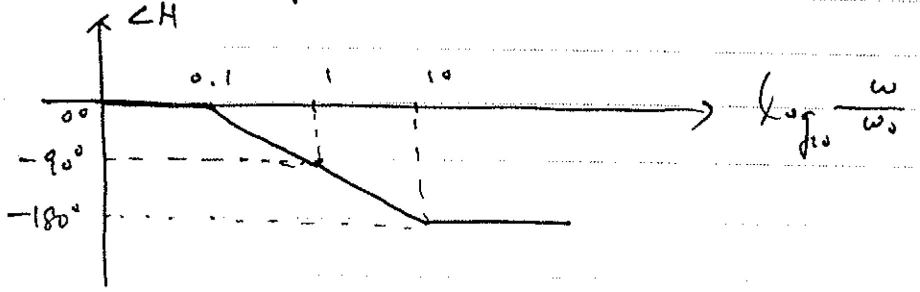
$$V_o = -\frac{R_2}{R_1} V_i + \left(1 + \frac{R_2}{R_1}\right) V_p = -V_i + 2V_p$$

$$= -V_i + \frac{2}{1 + RCs} V_i$$

$$= \frac{1 - RCs}{1 + RCs} V_i$$

$$H(s) = \frac{1 - RCs}{1 + RCs} \Rightarrow z = \frac{1}{RC}, \quad p = -\frac{1}{RC}$$

$$H(j\omega) = \frac{1 - j\omega/\omega_0}{1 + j\omega/\omega_0} = 1 \angle -2\tan^{-1}\frac{\omega}{\omega_0}$$



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phase shift ∝ freq. ⇒ "same time delay".

통과시간 = αt ⇒ $\sin\{\omega(t+\alpha t)\} = \sin(\omega t + \underbrace{\alpha\omega}_{\theta})$

$\frac{1}{1+j\omega/\omega_0}$, $\frac{j\omega/\omega_0}{1+j\omega/\omega_0}$, $\frac{1-j\omega/\omega_0}{1+j\omega/\omega_0}$: first order filter

LPF, HPF, All pass

3.3 Audio Filter Applications

fig 3.13
fig 3.14
fig 3.15
fig 3.16

3.4 Standard Second-Order Responses

1차필터의 >
H(jω)수식

$$H(s) = \frac{N(s)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$$

$$\begin{cases} N(s) & : \text{polynomial } (m \le 2) \\ \omega_0 & : \text{undamped natural freq.} \\ \zeta & : \text{damping ratio } (\underline{\text{zeta}}) \end{cases}$$

$$p_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_0$$

① $\zeta > 1$: over damped

$$\begin{cases} \text{real \& negative poles} \\ \text{natural response} = \text{two decaying exponentials} \end{cases}$$

② $0 < \zeta < 1$: underdamped

$$\begin{cases} \text{complex conjugate poles} : -\zeta\omega_0 \pm j\omega_0\sqrt{1-\zeta^2} \\ \text{natural response} = \text{damped sinusoids} \end{cases}$$

$$x_o(t) = 2|A|\, e^{-\zeta\omega_0 t} \cos\left(\omega_0\sqrt{1-\zeta^2}\, t + \angle A\right)$$

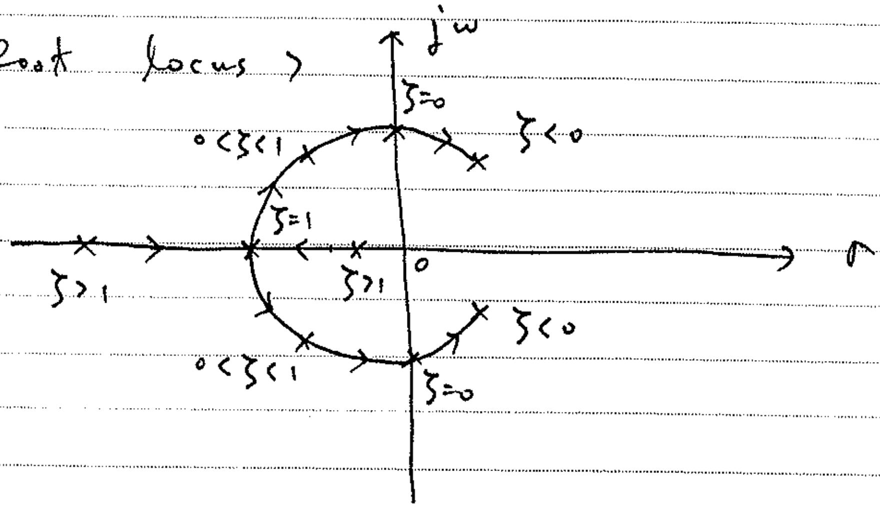
③ $\zeta = 0$: undamped

{ imaginary poles : $\pm j\omega_0$
{ natural response : sinusoids with ω_0

④ $\zeta < 0$: diverging

poles in RHP ⇒ unstable

< Root locus >



$$H(j\omega) = \frac{N(j\omega)}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

$$Q = \frac{1}{2\zeta}$$

* Lowpass Response, H_{LP}

$$H(j\omega) = H_{0LP}\, H_{LP}(j\omega) = H_{0LP} \frac{1}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

H_{0LP} : dc gain

① $\omega/\omega_o << 1$, $H_{lp} \to 1$ ∴ $|H_{lp}|_{dB} = 0$

② $\omega/\omega_o >> 1$, $H_{lp} \to -1/(\omega/\omega_o)^2$

$$\therefore |H_{lp}|_{dB} = 20\log_{10}\left[\frac{1}{(\omega/\omega_o)^2}\right]$$
$$= -40\log_{10}(\omega/\omega_o)$$

③ $\omega/\omega_o = 1$, $H_{lp} = -jQ$ ∴ $|H_{lp}|_{dB} = Q_{dB}$

$Q = 0.5 \sim 100$

$Q > 1 \Rightarrow$ peaking at $\omega = \omega_o$

$\boxed{\text{Fig 3.19(a)}}$

(i) Butterworth response : $\left\{\begin{array}{l} Q = \frac{1}{\sqrt{2}} = 0.707 \\ \text{maximally flat} \\ |H_{lp}|_{dB} = -3\,dB \end{array}\right.$

at $\omega = \omega_o$,

$\Downarrow$

ω_o : $-3\,dB$ cutoff freq.

(ii) $Q > \frac{1}{\sqrt{2}}$: Peaked response

$$|H_{lp}|_{max} = \frac{Q}{\sqrt{1-\frac{1}{4}Q^2}}$$

at $\omega/\omega_o = \sqrt{1-\frac{1}{2}Q^2}$

(iii) $Q > 5$, $\omega/\omega_o \simeq 1$, $|H_{lp}|_{max} \simeq Q$

(iv) $Q < 1/\sqrt{2}$, $|H_{lp}|_{max} = 1$ at $\omega = 0$ (dc)

* High-pass Response, H_{HP}

$$H(j\omega) = \underbrace{H_{oHP}}_{\text{high-freq. gain}} H_{HP}(j\omega) = H_{oHP} \frac{-(\omega/\omega_o)^2}{1-(\omega/\omega_o)^2+(j\omega/\omega_o)/Q}$$

$\boxed{\text{fig } 3.19(b)}$

H_{LP} 에서 $j\omega/\omega_o$ 를 $1/(j\omega/\omega_o)$ 로 대치 → H_{HP}

$|H_{LP}|$와 $|H_{HP}|$ 는 서로 mirror image.

* Band-pass Response, H_{BP}

$$H(j\omega) = \underbrace{H_{oBP}}_{\text{resonance gain or passband gain or midband gain}} H_{BP}(j\omega) = H_{oBP} \frac{(j\omega/\omega_o)/Q}{1-(\omega/\omega_o)^2+(j\omega/\omega_o)/Q}$$

① $\omega/\omega_o \ll 1$, $H_{BP} \to (j\omega/\omega_o)/Q$

$$\therefore |H_{BP}|_{dB} = 20\log_{10}[(\omega/\omega_o)/Q] = 20\log_{10}(\omega/\omega_o) - Q_{dB}$$

② $\omega/\omega_o \gg 1$, $H_{BP} \to -j/(\omega/\omega_o)Q$

$$\therefore |H_{BP}|_{dB} = -20\log_{10}(\omega/\omega_o) - Q_{dB}$$

③ $\omega/\omega_0 = 1$, $H_{Bp} = 1$ $\therefore |H_{Bp}|_{dB} = 0$

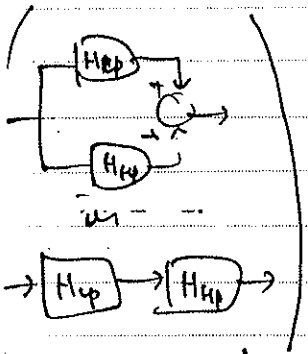
$|H_{Bp}|$의 최대치는 항상 $\omega = \underline{\omega_0}$

$\hookrightarrow$ peak freq. or resonance freq.

$\begin{cases} \omega\text{가 } \omega_0\text{와 크게 다르면 } \pm 20dB/dec \\ \omega\text{가 } \omega_0 \text{ 근처에서는 } Q\text{가 크면 slope이 급} \end{cases}$

$\boxed{\text{Fig 3.20}}$

$$BW = \omega_H - \omega_L$$



$$\begin{cases} \omega_L = \omega_0\left(\sqrt{1+1/4Q^2} - \frac{1}{2}Q\right) & : -3dB \text{ cutoff freq.} \\ \omega_H = \omega_0\left(\sqrt{1+\frac{1}{4}Q^2} + \frac{1}{2}Q\right) & : \text{ "} \\ \omega_0 = \sqrt{\omega_L\omega_H} \end{cases}$$

$$\Rightarrow \quad Q = \frac{\omega_0}{BW} \quad : \text{selectivity}$$

* Notch Response , H_N

$$H(j\omega) = \underbrace{H_{0N}}_{\hookrightarrow \text{ dc or high-freq. gain}} H_N(j\omega)$$

$$H_N(j\omega) = \frac{1-(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

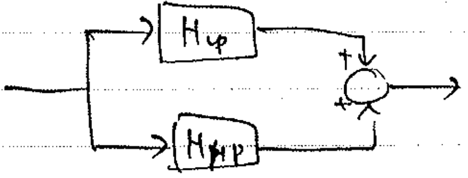
$\begin{cases} \text{pole pair :} \\ \text{zeros : } \pm j\omega_0 \rightarrow \text{notch freq.} \end{cases}$

Fig 3.21

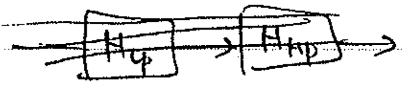
$$\begin{cases} \lim_{\omega \to 0} H_N \to 1 \quad \& \quad \lim_{\omega \to \infty} H_N \Rightarrow 1 \\ \lim_{\omega \to \omega_0} H_N \to 0 \\ Q\uparrow \Rightarrow \text{narrower notch} \end{cases}$$

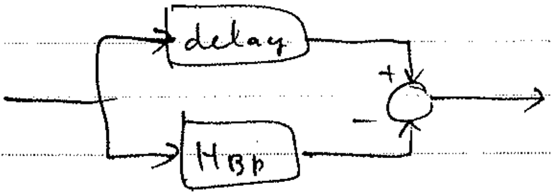
ω_0 = notch freq.

$$H_N = H_{LP} + H_{HP} = 1 - H_{BP}$$



or





* All-pass Response ; H_{AP}

$$H_{AP}(j\omega) = \underbrace{H_{0AP}}_{\text{gain}} H_{AP}(j\omega)$$

$$H_{AP}(j\omega) = \frac{1 - (\omega/\omega_0)^2 - (j\omega/\omega_0)/Q}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

$$Q > 0.5$$

$$\Downarrow$$

$N(j\omega) = D^*(j\omega)$ with complex and symmetric poles and zeros

$$\Downarrow$$

$$|H_{AP}| = 1$$

$$\angle H_{AP} = -2\tan^{-1}\frac{(\omega/\omega_o)/Q}{1-(\omega/\omega_o)^2}, \quad \omega/\omega_o < 1$$

$$= -360^\circ - 2\tan^{-1}\frac{(\omega/\omega_o)/Q}{1-(\omega/\omega_o)^2}, \quad \omega/\omega_o > 1$$

$\Rightarrow$ Fig 3.21.

$$H_{AP} = H_{LP} - H_{BP} + H_{HP} = 1 - 2H_{BP}$$

* Filter Measurements.

LPF

$H_{LP}(j0) = H_{oLP}$, $H_{LP}(j\omega_o) = -jH_{oLP}Q$

at $\omega = \omega_o$, we have 90° phase shift

$Q = |H_{LP}(j\omega_o)| / |H_{oLP}|$

BPF

$H_{BP}(j\omega_o) = H_{oBP}$

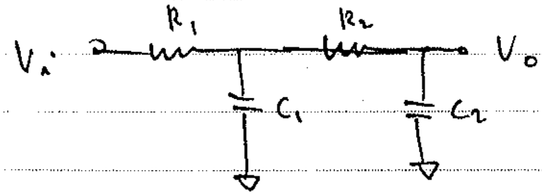
$\angle H_{BP}(j\omega_L) = \angle H_{oBP} - 45^\circ$

$\angle H_{BP}(j\omega_H) = \angle H_{oBP} - 135^\circ$

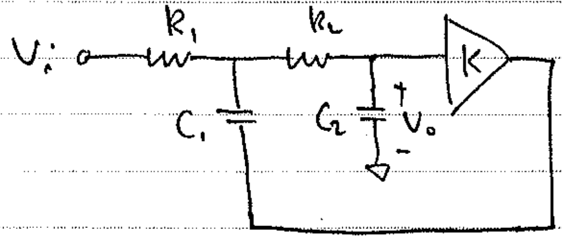
at $\omega = \omega_o$, 0° or 180° phase shift

$$Q = \frac{\omega_o}{\omega_H - \omega_L}$$

3.5 KRC Filters : $Q < 10$

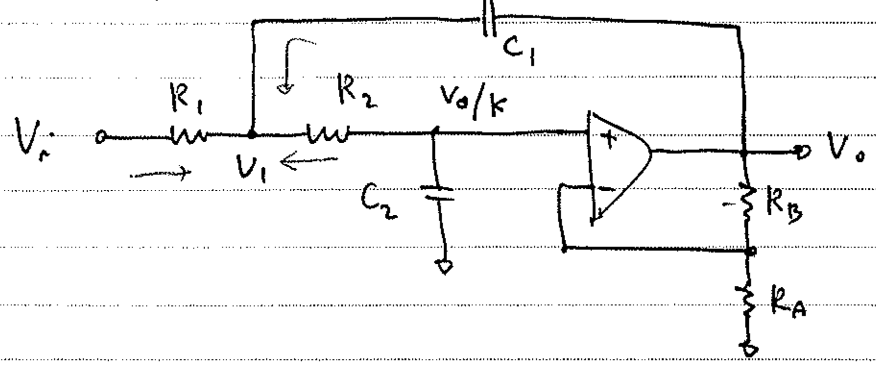


$\Rightarrow$ $Q < 0.5$
(passive filter)



$\Rightarrow$ $Q > 0.5$
(KRC filter or
Sallen-Key filter)

* Lowpass KRC Filter



$$K = 1 + \frac{R_B}{R_A}$$

$$V_o = K \frac{1}{1 + sR_2C_2} V_1$$

$$\frac{V_i - V_1}{R_1} + \frac{V_o/K - V_1}{R_2} + \frac{V_o - V_1}{1/sC_1} = 0$$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{K}{R_1C_1R_2C_2 s^2 + [(1-K)R_1C_1 + R_1C_2 + R_2C_2] s + 1}$$

$$H(j\omega) = K \frac{1}{1-\omega^2 R_1 C_1 R_2 C_2 + j\omega\left[(1-K)R_1C_1 + R_1C_2 + R_2C_2\right]}$$

$H(j\omega) = H_{0LP}\, H_{LP}(j\omega)$ 에서

$$H_{0LP} = K$$

$$\omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}}$$

$$\left(\because \left(\frac{\omega}{\omega_0}\right)^2 = \omega^2 R_1 C_1 R_2 C_2\right)$$

$$Q = \frac{1}{(1-K)\sqrt{R_1C_1/R_2C_2} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_2C_2/R_1C_1}}$$

설계 방법

① R_1 조정 $\Rightarrow$ ω_0 결정

② R_B 조정 $\Rightarrow$ Q 결정

③ K, R_1, C_1, R_2, C_2 의 결정

- 식 3개 미정
- equal component design
- unity-gain design

* Equal-component KRC Circuit $(Q < 10)$

$R_1 = R_2 = R$, $C_1 = C_2 = C$

$H_{0LP} = K$, $\omega_0 = \dfrac{1}{RC}$, $Q = \dfrac{1}{3-K}$

$\Rightarrow$ $RC = \dfrac{1}{\omega_0}$, $K = 3 - \dfrac{1}{Q}$, $R_B = (K-1)R_A$

ⓔ Example 3.8

* Unity-Gain KRC circuit

$K = 1$

$R_2 = R$, $C_2 = C$, $R_1 = mR$, $C_1 = nC$

$H_{0LP} = 1$, $\omega_0 = \dfrac{1}{\sqrt{mn}\,RC}$, $Q = \dfrac{\sqrt{mn}}{m+1}$

- [illegible]

[Q f ⇒ [illegible]

[Tuning [illegible]

For a given n, $m=1$ maximizes Q.

$(n = 4Q^2)$

$\boxed{\text{실제의 설계 방법}}$

① Choose C_1 & C_2 s.t. $\underline{n \geq 4Q^2}$

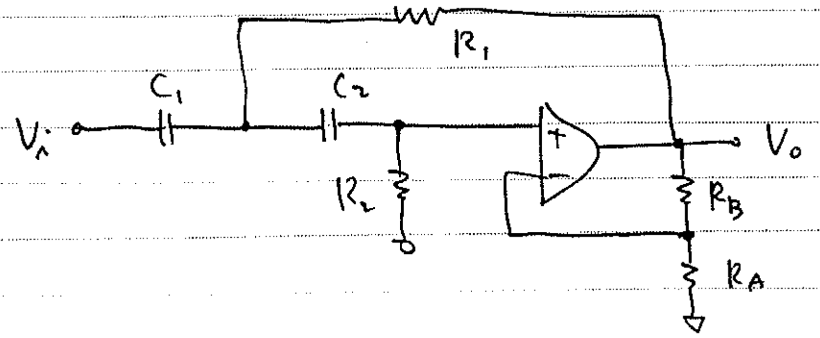
② $m = k + \sqrt{k^2-1}$

where $k = \dfrac{n}{2Q^2} - 1$

◎ Example 3.10

* High-pass KRC filter

⇒ LPF에서 R과 C를 서로 바꿈

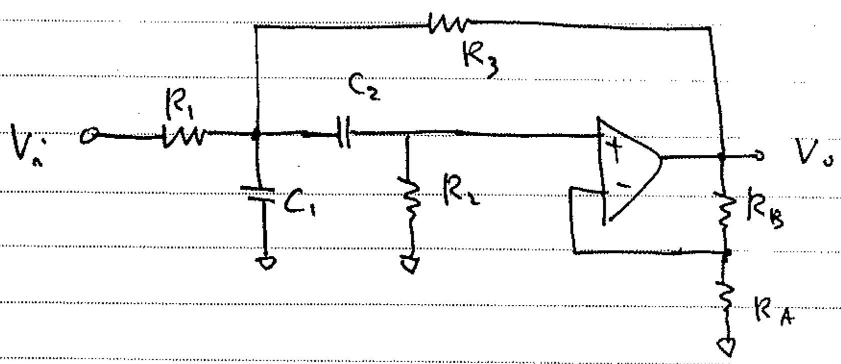


$H_{0HP} = K$, $\omega_0 = \dfrac{1}{\sqrt{R_1C_1R_2C_2}}$

$$Q = \frac{1}{(1-K)\sqrt{R_2C_2/R_1C_1} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_1C_1/R_2C_2}}$$

◎ Example 3.12

* Band pass KRC Filter



$$H_{0BP} = \frac{K}{1 + (1-K)R_1/R_3 + (1 + C_1/C_2)R_1/R_2}$$

$$\omega_0 = \frac{\sqrt{1 + R_1/R_3}}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q = \frac{\sqrt{1 + R_1/R_3}}{[1+(1-K)R_1/R_3]\sqrt{R_2C_2/R_1C_1} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_1C_1/R_2C_2}}$$

설계 방법 ① Adjust $R_1 \Rightarrow \omega_0$ 결정

② Adjust $R_B \rightarrow Q$ 결정

◎ $Q > \sqrt{2}/3$ 인 경우,

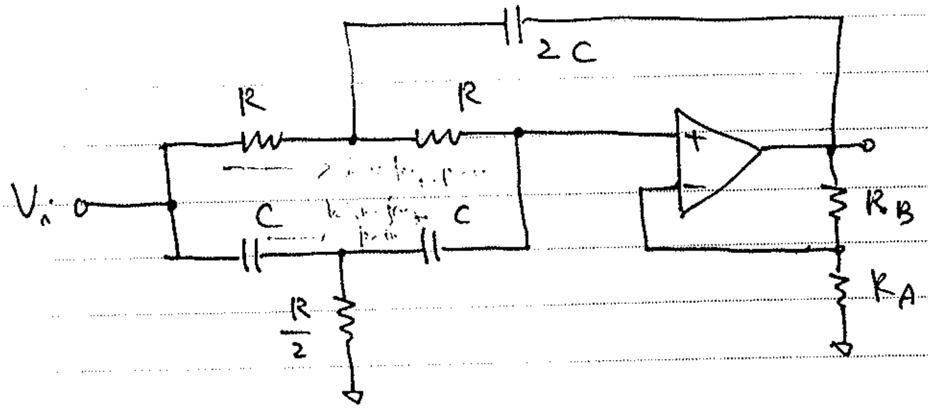
$$\begin{cases} R_1 = R_2 = R_3 = R \\ C_1 = C_2 = C \end{cases}$$

$$\Rightarrow H_{0BP} = \frac{K}{4-K}, \quad \omega_0 = \frac{\sqrt{2}}{RC}, \quad Q = \frac{\sqrt{2}}{4-K}$$

$$\Rightarrow RC = \frac{\sqrt{2}}{\omega_0}, \quad K = 4 - \sqrt{2}/Q, \quad R_B = (K-1)R_A$$

◎ Example 3.13

* Band-reject KRC filter



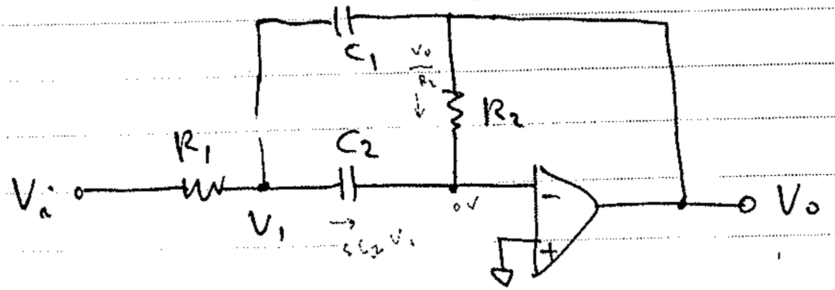
< Twin-T network >

$H_{0N} = K$, $\omega_0 = \frac{1}{RC}$, $Q = \frac{1}{4-K}$

◎ Example 3.14

3.6 Multiple feedback filters

* Band-pass filter (Delyiannis-Friend filter)



$V_o = -sR_2C_2V_1$ (KCL at negative input) $\frac{V_o}{R_2} + \frac{V_1}{1/sC_2} = 0$

$$\frac{V_i - V_1}{R_1} + \frac{V_o - V_1}{1/sC_1} + \frac{0 - V_1}{1/sC_2} = 0$$

$$H(j\omega) = \frac{-j\omega R_2 C_2}{1-\omega^2 R_1 R_2 C_1 C_2 + j\omega R_1(C_1+C_2)}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \frac{\sqrt{R_2/R_1}}{\sqrt{C_1/C_2}+\sqrt{C_2/C_1}}$$

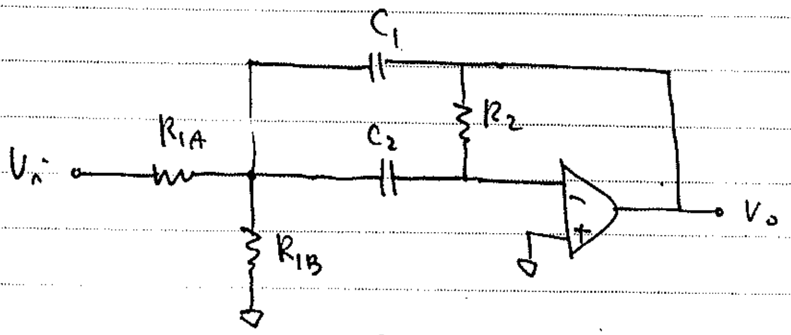
$$H_{oBP} = \frac{-R_2/R_1}{1+C_1/C_2}$$

$\boxed{\text{설계 방법}}$ $C_1 = C_2 = C$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2}\,C}, \quad Q = 0.5\sqrt{\frac{R_2}{R_1}}, \quad H_{oBP} = -2Q^2$$

$$\Rightarrow R_1 = \frac{1}{2\omega_0 Q C}, \quad R_2 = \frac{2Q}{\omega_0 C}$$

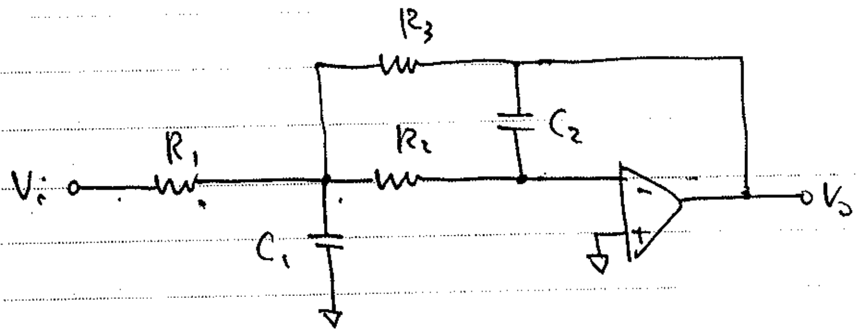
◎ $H_0 < 2Q^2$ 인 경우에는



$$R_{1A} = \frac{Q}{H_0 \omega_0 C}, \quad R_{1B} = \frac{R_{1A}}{\frac{2Q^2}{H_0}-1}$$

◎ Example 3.15

* Low-pass filter



$$H_{0LP} = -\frac{R_3}{R_1} \qquad \omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}}$$

$$Q = \frac{\sqrt{C_1/C_2}}{\sqrt{R_2 R_3/R_1^2} + \sqrt{R_3/R_2} + \sqrt{R_2/R_3}}$$

$$\begin{cases} R_3 \text{ 조정} \Rightarrow \omega_0 \text{ 결정} \\ R_1 \quad '' \quad \Rightarrow Q \quad '' \end{cases}$$

$\boxed{\text{설계 방법}}$

① C_2 를 결정

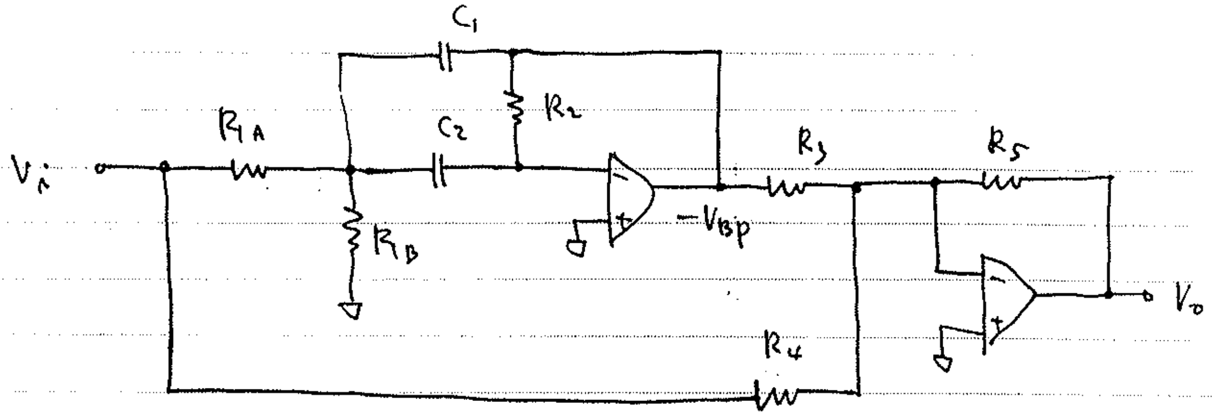
② $C_1 = n C_2$ with $n \geq 4Q^2(1+H_0)$

③
$$\begin{cases} R_3 = \dfrac{1+\sqrt{1-4Q^2(1+H_0)/n}}{2\omega_0 Q C_2} \\ R_1 = \dfrac{R_3}{H_0} \\ R_2 = \dfrac{1}{\omega_0^2 R_3 C_1 C_2} \end{cases}$$

단점 :
Q 와 H_0 가 [illegible] C_1 [illegible]
[illegible]
greater spread
of
Cap.

⓪ Example 3.16

* Notch filter



$$\frac{H_0 R_4}{R_3} = 1 \Rightarrow \begin{cases} H(j\omega) = H_{ON} H_N(j\omega) \\ H_{ON} = -\frac{R_5}{R_4} \end{cases}$$

◎ Example 3.17

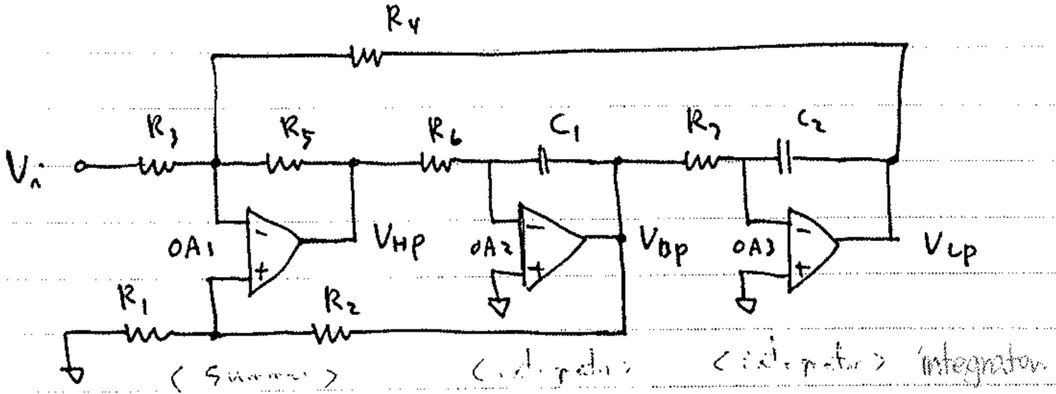
3.7 State-Variable and Biquad Filters

- multiple op amps
- easier to tune
- less sensitive to passive component values
- less component spreads
- more than one response (universal filter)

(universal filter)

* State-Variable (SV) filters Huelsman-Newcomb

- KHN filters (Kerwin-Huels-Newcomb)
- Two integrater and one summer
  - ⇒ 2nd order LPF, BPF, HPF
- Op amp 1개 추가 ⇒ notch, all-pass



중첩의 원리

$$\rightarrow V_{HP} = -\frac{R_5}{R_3} V_i - \frac{R_5}{R_4} V_{LP} + \left(1 + \frac{R_5}{R_3 \| R_4}\right) \frac{R_1}{R_1 + R_2} V_{BP}$$

$$= -\frac{R_5}{R_3} V_i - \frac{R_5}{R_4} V_{LP} + \frac{1 + R_5/R_3 + R_5/R_4}{1 + R_2/R_1} V_{BP}$$

$$\left.\begin{aligned} V_{BP} &= \frac{-1}{R_6 C_1 s} V_{HP} \\ V_{LP} &= \frac{-1}{R_7 C_2 s} V_{BP} \end{aligned}\right\} \Rightarrow V_{LP} = \frac{1}{R_6 C_1 R_7 C_2 s^2} V_{HP}$$

p.146 full eq.

① $\frac{V_{HP}}{V_i} = H_{0HP} H_{HP}$ 여기서

$$H_{0HP} = -\frac{R_5}{R_3}, \quad \omega_0 = \frac{\sqrt{R_5/R_4}}{\sqrt{R_6 C_1 R_7 C_2}}$$

$$Q = \frac{(1 + R_2/R_1)\sqrt{R_5 R_6 C_1 / R_4 R_7 C_2}}{1 + R_5/R_3 + R_5/R_4}$$

② $$\frac{V_{BP}}{V_i} = -\frac{1}{R_6 C_1 s}\,\frac{V_{HP}}{V_i} = H_{oBP}\,H_{BP}$$

$$\cancel{H_{oBP} = -\frac{R_5}{R_3}} \qquad H_{oBP} = \frac{1 + R_2/R_1}{1 + R_3/R_4 + R_3/R_5}$$

③ $$\frac{V_{LP}}{V_i} = -\frac{1}{R_7 C_2 s}\,\frac{V_{BP}}{V_i} = H_{oLP}\,H_{LP}$$

$$H_{oLP} = -\frac{R_4}{R_3}$$

장점: { Q ~ 수백이 가능
Q 가 저항값 변화에 둔감함

주의점: { metal-film resistor 사용
polystyrene 또는 polycarbonate cap. 사용
op amp. 의 전원에 bypass cap. 사용.

구현: $R_5 = R_4 = R_3$, $R_6 = R_7 = R$
$C_1 = C_2 = C$

$$\Rightarrow \quad \omega_o = \frac{1}{RC}, \quad Q = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right)$$

$$H_{oHP} = -1, \quad H_{oBP} = Q, \quad H_{oLP} = -1$$

Tuning: ① Adjust R_4/R_3 with R_3 for gain
② Adjust R for ω_o
③ Adjust R_2/R_1 for Q

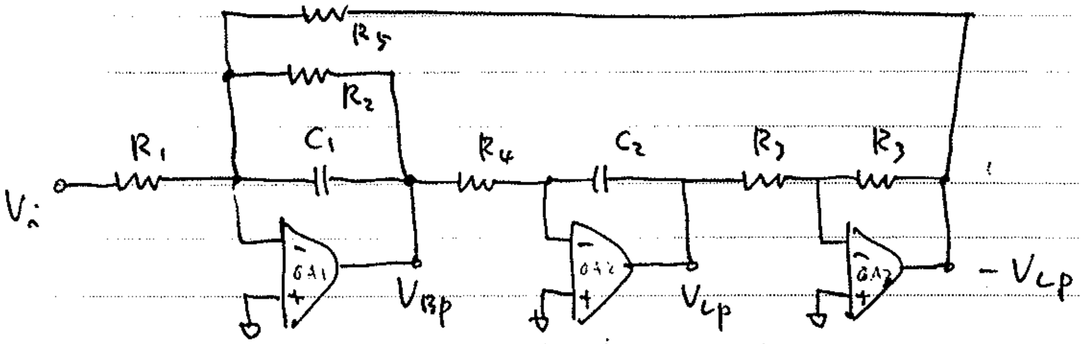
⊙ Example 3.18

개선 : ① R_3 대신에 voltage divider 사용
(입력의 dynamic range 증가)
② Noninverting (fig. 3. ~~35~~) 36
[illegible]

$$\omega_0 = \frac{1}{RC}, \quad Q = 1 + \frac{R_2}{2R_1}$$

$$H_{0HP} = \frac{1}{Q}, \quad H_{0BP} = -1, \quad H_{0LP} = \frac{1}{Q}$$

* Biquad filter (Tow - Thomas filter)



< leaky integrator > < integrator > < inverting amp >

[illegible]

$$\begin{cases} \dfrac{V_i}{R_1} + \dfrac{-V_{LP}}{R_5} + \dfrac{V_{BP}}{R_2} + \dfrac{V_{BP}}{1/sC_1} = 0 \\ V_{LP} = -\dfrac{1}{sR_4C_2} V_{BP} \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{V_{BP}}{V_i} = H_{0BP} H_{BP} \\ \dfrac{V_{LP}}{V_i} = -\dfrac{1}{sR_4C_2} \dfrac{V_{BP}}{V_i} = H_{0LP} H_{LP} \end{cases}$$

$$H_{0BP} = -\frac{R_2}{R_1}, \quad H_{0LP} = \frac{R_5}{R_1}, \quad \omega_0 = \frac{1}{\sqrt{R_4R_5C_1C_2}}$$

※ HP response도 얻음
※ common mode [illegible]
([illegible])

$$Q = \frac{R_2\sqrt{C_1}}{\sqrt{R_4 R_5 C_2}}$$

특징: $R_4 = R_5 = R, \quad C_1 = C_2 = C$

$$H_{0BP} = -\frac{R_2}{R_1}, \quad H_{0LP} = \frac{R}{R_1}$$

$$\omega_0 = \frac{1}{RC}, \quad Q = \frac{R_2}{R}$$

Tuning :
① Adjust R_4 and R_5 for ω_0
② Adjust R_2 for Q
③ Adjust R_1 for H_{0BP} and H_{0LP}

◎ Example 3.19

* Notch Response

구조: [SV + op amp] 또는 [Biquad + op amp]

회로: 그림 3.37

$$V_N = -\left[\frac{R_5}{R_2}(V_i - V_{BP}) \pm \frac{R_5}{R_4}V_{LP}\right]$$

$$\frac{V_N}{V_i} = -\frac{R_5\omega_z^2}{R_2\omega_0^2}\,\frac{1-(\omega/\omega_z)^2}{1-(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$$

$$\omega_0 = \frac{1}{RC}, \quad Q = \frac{R_1}{R}, \quad \omega_z = \omega_0\sqrt{1 \pm R_2/R_4Q}$$

↳ notch freq.

세 가지 설계의 경우

① R_4를 제거 $(R_4 = \infty)$

$\omega_z = \omega_0$, $H_{0N} = -\frac{R_5}{R_2}$

$\Rightarrow$ symmetric notch (그림 3.38 (b))

② S/W가 왼쪽 (+)

$\omega_z = \omega_0\sqrt{1 + \frac{R_2}{R_4 Q}}$, $H_{0LP} = -\frac{R_5\,\omega_z^2}{R_2\,\omega_0^2}$

$\downarrow$ dc gain

$(\omega_z > \omega_0)$

$H_{0HP} = -\frac{R_5}{R_2}$

$\Rightarrow$ low-pass notch (그림 3.38 (a))

③ S/W가 오른쪽 (−)

$\omega_z = \omega_0\sqrt{1 - \frac{R_2}{R_4 Q}}$, $H_{0HP} = -\frac{R_5}{R_2}$

$\hookrightarrow$ high-freq. gain

$(\omega_z < \omega_0)$

$H_{0LP} = -\frac{R_5\,\omega_z^2}{R_2\,\omega_0^2}$

$\Rightarrow$ high-pass notch (그림 3.38 (c))

설계 : $R = \frac{1}{\omega_0 C}$, $R_1 = QR$, $R_4 = \frac{R_2}{Q}\,\frac{\omega_0^2}{|\omega_0^2 - \omega_z^2|}$

$$\left\langle \begin{array}{ll} R_5 = R_2\left(\frac{\omega_0}{\omega_z}\right)^2 & \text{for } \omega_z > \omega_0 \\ R_5 = R_2 & \text{for } \omega_z < \omega_0 \end{array} \right.$$

◎ Example 3.20

3.8 Sensitivity

$$S_x^y = \frac{\partial y / y}{\partial x / x} = \frac{x}{y}\frac{\partial y}{\partial x}$$

$$\Rightarrow \underbrace{\frac{\Delta y}{y}}_{\text{fractional parameter change}} \simeq S_x^y \underbrace{\frac{\Delta x}{x}}_{\text{fractional component change}} \quad \text{for small } \Delta x$$

성질

$$\begin{cases} S_{1/x}^{y} = S_x^{1/y} = -S_x^{y} \\ S_x^{y_1 y_2} = S_x^{y_1} + S_x^{y_2} \\ S_x^{y_1/y_2} = S_x^{y_1} - S_x^{y_2} \\ S_x^{x^n} = n \\ S_{x_1}^{y} = S_{x_2}^{y} S_{x_1}^{x_2} \end{cases}$$

* KRC Filter Sensitivities

For LP KRC filter (Fig 3.23),

$$\omega_0 = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$S_{R_1}^{\omega_0} = S_{C_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

마찬가지 방법으로, 식 (3.94) ~ (3.95)

$$\begin{cases} \text{Equal component design} \Rightarrow S_K^Q \uparrow \text{ as } Q \uparrow \\ \text{Unity gain design} \Rightarrow \text{lower } S_x^Q \end{cases}$$

◎ Example 3.21

* Multiple Feedback Filter Sensitivities
회로는 그림 3.30
⇒ Sensitivity가 작음 (식 3.97 – 3.98)

* Multiple Op Amp Filter Sensitivities

- Biquad filter in Fig 3.26 : 이론의 생략.
  → low sensitivity
- SV filter : 이론의 생략
  ⇒ low sensitivity.