

## Final Exam (Fall 2011)

- (1) Derive two fundamental equations in impedance imaging.
- (a) From Maxwell's equations of  $\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}$  and  $\nabla \times \mathbf{H} = \tau\mathbf{E} = (\sigma + j\omega\epsilon)\mathbf{E} = \mathbf{J}$ , derive the first fundamental equation in impedance imaging, which is expressed by  $\mathbf{H}$  and  $\tau$ . Provide your interpretation of the derived equation.
- (b) Using that fact that  $\nabla \cdot (\mu_0\mathbf{H}) = 0$ , define a vector magnetic potential  $\mathbf{A}$ . From  $\nabla \times (\mathbf{E} + j\omega\mathbf{A}) = 0$ , define a scalar potential  $u$ . Noting that  $\nabla \cdot \mathbf{J} = 0$ , derive the second fundamental equation in impedance imaging, which is expressed by  $u$ ,  $\mathbf{A}$  and  $\tau$ . Provide your interpretation of the derived equation.
- (2) Consider an electrically conducting homogeneous cylindrical domain with a known conductivity  $\sigma=1$  S/m. You attached two electrodes with the same surface area of  $A$  at the middle positions of the cylinder on the left and right sides and injected dc current of  $I$  mA.
- (a) Denoting the voltage in the domain  $\Omega$  by  $u$ , describe a partial differential equation with proper boundary conditions.
- (b) On the cross-sectional plane at the center of the cylinder, draw current streamlines.
- (c) On the same cross-sectional plane, draw equipotential lines.
- (d) Derive expressions for the current density  $\mathbf{J}$  and magnetic flux density  $\mathbf{B}$ .
- (3) You are provided with a 16-channel EIT system using the neighboring data collection method.
- (a) Define a projection to collect the first set of boundary voltage data subject to the current injection between the first neighboring electrode pair.
- (b) Define the full set of boundary voltage data subject to 16 injection currents.
- (c) Derive the principle of the reciprocity and calculate the number of independent voltage data.
- (d) Assume that you designed an image reconstruction mesh with 100 elements

- (pixels). Derive an expression of the sensitivity matrix. Provide interpretations for each row and column of the sensitivity matrix.
- (e) Construct a linear time-difference conductivity image reconstruction algorithm using the truncated singular value decomposition of the sensitivity matrix.
- (f) Explain how to determine the rank of the sensitivity matrix and discuss its implication to the spatial resolution.
- (4) Inside an MRI scanner, you are conducting an MREIT imaging experiment using two pairs of surface electrodes.
- (a) For current injection along the horizontal pair of the electrodes, draw a spin-echo based pulse sequence to inject positive and negative current pulses. Explain why you need to inject both positive and negative currents.
- (b) Derive equations to extract the  $z$ -component  $B_z$  of the induced magnetic flux density  $\mathbf{B}=(B_x, B_y, B_z)$  from the following  $k$ -space data:
- $$S^\pm(m, n) = \iint M(x, y) e^{j\delta(x, y)} e^{\pm j\gamma B_z(x, y) T_c} e^{-j(xm\Delta k_x + yn\Delta k_y)} dx dy .$$
- (c) Using the  $z$ -component of the fundamental relation of  $\nabla^2 \mathbf{B} = -\mu_0 \nabla u \times \nabla \sigma$  where  $u$  is the voltage and  $\sigma$  is the conductivity at the position  $\mathbf{r}$ , derive the core equation between the two-dimensional gradient of  $\sigma$  and  $B_z$ . Assume that you know the voltage  $u$ .
- (d) Using two data sets of  $B_{z1}$  and  $B_{z2}$  subject to two injection currents, derive the core equation of the harmonic  $B_z$  algorithm in a matrix-vector form.
- (e) In practice, you do not know the voltage  $u$  since it is a nonlinear function of the unknown conductivity  $\sigma$ . Suggest an iterative method to recover the conductivity  $\sigma$ .